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Proceedings of the American Academy of Arts and Sciences.

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TYPES OF ABNORMAL COLOR VISION.

By Louis Bell.

Investigations on Light and Heat published with Aid from the Rumford Fund.

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TYPES OF ABNORMAL COLOR VISION.

By Louis Bell.

Presented February, 11, 1914. Received, February 11, 1914.

Despite the fact that so-called color blindness has been studied for more than a century, little is yet known with respect to its real nature or the various forms under which it appears. The present investigation is a preliminary study of the latter made in the hope that it may give at least some slight clues to the former.

It is well known that about one man in 25 has a marked congenital color defect. Without going into detailed statistics, the older data show, from the work of Holmgren, B. Joy Jeffries, and the committee of the London Ophthalmological Society, 3.82 per cent of color deficient in 57398 males examined (Proc. Roy. Soc., LI, 319). This figure is based chiefly on tests with Holmgren's wools, and is materially increased in many later investigations made with the color lantern and the spectroscope. Donders for example found 6.6 per cent of color blind among 2300 railway employees in Holland, and even this figure has been much exceeded. Bearing in mind that minor color defects easily escape the wool test when applied in the ordinary course of testing men for the red and green vision required for signal lights, it is easy to realize that small variations from the normal, although perhaps of much theoretical significance, readily escape notice. These cases, too, have gone undetected on account of the frequent acceptance of Hering's theory, which associates defects in red and in green vision, a position now known in virtue of Burch's work on temporary induced color blindness (Phil. Trans., B **191**, 1) to be quite untenable. This work and the earlier experiments of Rayleigh (Nature, **25**, 64) have opened up a wide field for the study of abnormalities in color vision, which has of late years been somewhat explored. That such are exceedingly common is in no wise better shown than by a later paper of Burch (Phil. Trans., **199**, B, 231) in which beside giving various interesting cases of color defects, the author cites the detailed examination of eleven cases of practically normal color vision.

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The red-green and blue-green junctions found for these eleven persons are plotted as points in Figure 1 over the primary color sensation curves, reduced to equal area, as given by Exner (Wien. Sitz. II, **111**, 837). The arrows show the mean values adopted by Burch and agree with Exner's junctions to a close approximation. The extreme points at both junctions show a degree of variation in the sensation curves which connotes slight abnormality easily detectable with a spectroscopic test. The average red-green junction was at w. l. 5842 and the blue green junction at w. l. 4946. In Burch's earlier paper (loc. cit.) in the examination of 70 persons, all of whom passed the Holmgren wool test, still greater variations in the junction points appeared. And these differences merge again into those shown by patients recog-

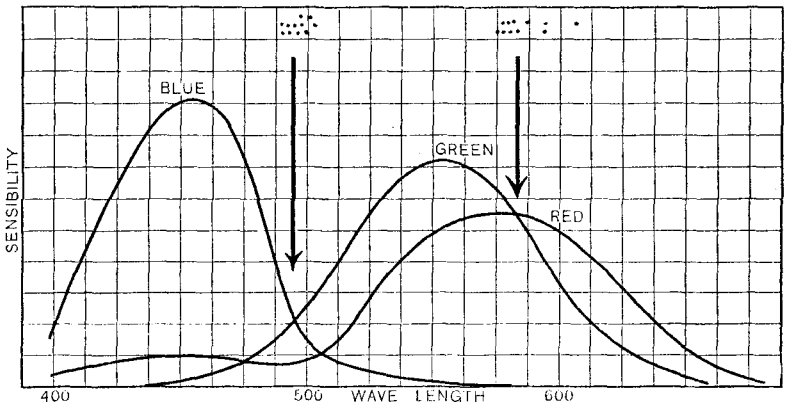

FIGURE 1.

nized as color-blind by simple tests, so that while one may define an average normal color-vision, from which large variations are rare, it is clear that the relations of the color sensations are subject to considerable variability. It is the purpose of this paper to classify such variations with reference to their bearing on the general color sense of the individual and to point out such of the more conspicuous abnormalities as have been detected in tests of color-sense.

The facts will be stated in terms of the ordinary trichromatic theory for the sake of simplicity. How far the observed facts regarding the typical color-sensation curves may be complicated by aberrancies of color perception referable to the cortex alone, and to what degree the independent violet sensation postulated by Burch (loc. cit.) may be

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involved with the residuum of the red and green sensations, the yellowing of the lens, and variations of macular pigmentation, are matters to be considered when the simpler facts of sensation can be better coordinated.

The relations of the three fundamental color sensations of the Young-Helmholtz theory, at least rest on a sound experimental basis and it is well understood that ordinary cases of color blindness imply an easily measurable deficit in one of these sensations.

In the normal or average eye the three fundamental sensations are related in a certain normal manner which may be schematically represented as

$$R,\quad G,\quad B,$$

in which each of the sensations has its typical average value. Similarly the ordinary case of red blindness may be written — R, G, B indicating a deficit from the normal value of the red sensation. To be rigorous one should give R a coefficient indicating the relative deficit of the red vision from its normal which may be anything from an amount just recognizable as a variant from normal to 100%. For example a certain case of partial red blindness examined by the writer would have been expressed

$$-.64R,\quad G,\quad B.$$

And similarly one may set down as the variants from normal involving change in a single color sensation curve the following

$$\begin{array}{ll} & R,\ G,\ B, \\ (1) & +R,\ G,\ B, \\ (2) & R,+G,\ B, \\ (3) & R,\ G,+B, \\ (4) & -R,\ G,\ B, \\ (5) & R,-G,\ B, \\ (6) & R,\ G,-B, \end{array}$$

Of these (4), the −R type, is the ordinary case of "color blindness"; (5) is the rather rare green blindness of which typical cases are reported by Abney (Proc. Roy. Soc., Vol. 83A); (6) is the still rarer blue blindness (Abney, Colour Vision, p. 73). Of the plus variants (1) is the plus red, the first of the type detected, (Rayleigh, loc. cit.). Burch (Physiological Optics, p. 119) cites several instances of (2). The plus blue variation is apparently very uncommon but Burch cites a case

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(Phil. Trans., **199**, B, p. 250) which appears to be a good example. There seems to be no intrinsic reason why plus variations from the normal should be less frequent than the minus variations, and they probably are not so in fact, the apparent reason being the wrong diagnosis likely to be made with the ordinary method of testing. Thus (1) and (5) are quite certain to be confused in the worsted tests most commonly employed, likewise (2) and (4) while plus or minus variations in the blue unless very marked would almost certainly escape detection.

The spectroscope and the color mixing apparatus give the only reliable methods of testing for minor variations, and the writer has used in his own experiments a simple form of spectroscopic test which may be worth describing here. The chief test is in principle the converse of Rayleigh's, and consists in matching a synthetic yellow and a synthetic blue-green, of the same hues as the spectral colors corresponding to the red-green and blue-green junctions for the normal eye, by shifting a pure spectrum which occupies the lower half of a slit in the focal plane of the eyepiece while the synthetic color occupies the upper half.

The apparatus is shown in diagram in Figure 2. The basis is a simple constant deviation spectroscope mounted in a capacious wooden box to avoid stray light. The prism P is mounted as usual on a turntable b, rotated by the milled head D with a screw bearing on a steel plane on the turntable. On the screw shaft is a wide pinion f, engaging a narrow rack ending in a pointer g, moving over the scale h. The rack works under the turntable freely through guides. The main slit, S, has the usual collimating lens c, of 4 cm. diameter and 15 cm. focus. The observing telescope O, of about 4 cm. diameter and 35 cm. focus has a compound eyepiece slit e, a slider with an adjustable slit and a clear aperture for viewing the spectrum. Long screws k, k, position the slider so that when slipped to the right the slit can be brought anywhere in the spectrum, and when slipped to the left the field is wholly or partly clear.

Beyond the turntable is a second collimator with adjustable slit S' and lens c' 12 mm. in diameter. This ranges over the prism P which is 25 mm. thick and the beam from c' is turned into the upper part of O by the mirror m. A small 60 degree prism can also be placed in front of c' to furnish a reference spectrum. Condensing lenses L, L' direct the light from sources l, l' upon the slits.

In using the apparatus the filter i is placed in front of S'. For synthetic yellow this is made of opposed wedges of cobalt and selenium

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glass, which by adjustment of S', L' and l' can be made to give a pretty good match to the normal eye with pure spectral yellow, brought to the same luminosity by the main slit. The patient is then, the spectrum having been widely displaced by turning D, required to bring it back for a match. A very slight degree of red blindness causes him to match the synthetic yellow with a green, while a plus red or minus

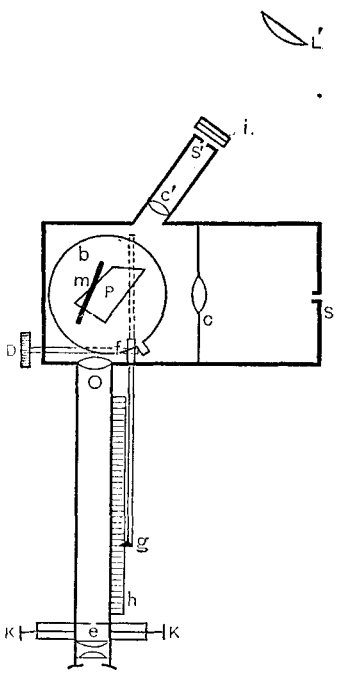

FIGURE 2.

green color-abnormality will produce a reddish match. Filter i is then replaced by a synthetic blue-green, produced by cobalt chloride in acetone solution combined with a uranine filter, and the test repeated. Here a green blind observer would match with blue and a blue blind case with green. This blue-green junction supplements the other in the diagnosis. Finally for further evidence of + or − red

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and blue sensations, the red and violet end points are determined after resting the eye. This requires care in keeping the slit width and illumination constant and in resting the eye for ten minutes or so before the test to approach a steady condition of adaptation. The two junction point filters should be modified by neutral tint glass if needed, so that one need not adjust their luminosity with the slits, but may be able to pass quickly from one standard filter to the other. Both directions of moving the spectrum should be tried to eliminate fatigue effects.

This method gives quick and certain qualitative diagnosis of any of the cases of simple color aberration (1) to (6). The settings at the red-green and blue-green junctions instantly disclose even a very slight variation in color sense and the end point readings in conjunction show at once whether this variation is due to weakness or abnormal strength of red or blue sensations.

Proceeding further in the analysis of variations one must recognize the probability of variants in which two of the three color sensations are abnormal instead of one as in (1) to (6). These fall into three groups as follows: two sensations weak; two sensations strong; one weak and another strong.

	(7) $-$R, $-$G, B
	(8) $-$R, G, $-$B
	(9) R, $-$G, $-$B
Also	(10) $+$R, $+$G, B
	(11) $+$R, G, $+$B
	(12) R, $+$G, $+$B
and finally	(13) $-$R, $+$G, B
	(14) $-$R, G, $+$B
	(15) R, $-$G, $+$B
	(16) R, $+$G, $-$B
	(17) $+$R, G, $-$B
	(18) $+$R, $-$G, B

These twelve variations are less easy of diagnosis than (1) to (6) since they depend on more complex quantitative relations.

Take for example (7). Here the red-green junction may be absolutely normal, but the blue-green junction will yield a bluish match showing either weak green or strong blue. The red end point, or for that matter the blue end point, tells the story. Case (8) can be dis-

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tinguished from case (2) by the end points, and so on, assuming that the abnormalities are not vanishingly small.

The red end point gives fairly definite information but the blue end point is less satisfactory. It is so much modified at times by variation in pigmentation of the macula and by yellowing of the lens as to be somewhat confusing. In the old the lens may be so altered in color that the solar H and K lines cannot be seen. As to pigmentation a case is cited by Abney (Researches in Colour Vision, p. 349) in which much of the blue end of the spectrum was absorbed. Such cases are separated from a genuine −B case by their giving, with R and G normal, a normal blue-green junction.

It is pertinent to inquire in how far these twelve binary variations actually occur. The ordinary cases of color blindness reported have not been so tested as to show them easily. The writer has never noted them personally in congenital color blindness, but one (7) is typical of color fatigue due to the mercury-arc and may easily be detected, as found by Williams and the writer (Electrical World, Sept. 2, 1911). A case reported by Edredge-Green (Colour Blindness, p. 154, F. A.) probably belongs to this type, since with a considerably shortened red spectrum his red-green junction was toward the green and he classified violet, blue, and bluegreen together.

Type (8) probably corresponds to a case described by Burch (Phil. Trans., **199**, B, p. 250, XVII).

Of the next group there is much difficulty in obtaining definite information since for example R,+G,+B and the ordinary −R, G, B could be distinguished only with some difficulty and would certainly escape ordinary tests. Any case in which some of the ordinary mistakes of the red blind are made with the confusion colors, while the ordinary end points are retained should be looked into carefully. A plus sensation will rarely be detected by the end points unless the abnormality is very marked. Burch's fatigue tests (Phil. Trans., **191**, B, p. 1 et seq.) used to supplement the junction and end point tests probably will prove to give the clearest diagnosis of this group. Luminosity tests might be useful since there is evident increase in general sensibility but on account of difficulties due to adaptation and the uncertainty of readings, the experiments are troublesome unless with experienced observers.

The next group (13) to (18) is easier to deal with and more examples may be found in the literature. (13) is well shown by Burch (Phil. Trans., **199**, B, p. 240, VII) in a case of marked but not complete red-blindness with slightly hypernormal green. Type (14) is also clearly

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described in Burch's very next case (loc. cit. p. 241, VIII), and (15) is also described in the same paper as case XII. To (16) should probably be referred a case described by Edridge-Green (Colour Blindness, p. 140, C. A.) in which the red end point was normal, and the blue considerably shortened while the ordinary yellow was encroached upon by the green and the blue-green junction shifted towards the blue. The remaining two types of this group do not appear in any recorded cases examined by the writer. (17) and (9) might easily be confused, as also (18), with some degrees of ordinary green blindness.

Finally there must be recognized a group of abnormalities in which all three primary sensations are affected. In the notation here used one may have +R, +G, +B and −R, −G, −B, as well as the normal R, G, B. In other words some persons undoubtedly have a generally strong color sense, and others a generally weak color sense, in each case *without peculiarities*. From each of these types obviously may spring a group of color variants with a single abnormality, corresponding to the (1) to (6). In these there is simple variation of one sensation with a general sensibility graded up or down.

Likewise there will be groups corresponding to variations, + or −, of two color sensations in the same direction, giving simple binary variations graded up or down from (7, 8, 9) and (10, 11, 12).

If one sensation remain + or −, with the other two abnormal relatively in opposite directions, there results a group like (13–18) but starting from a different plane of sensibility; and, since the + and − are referred to the normal as the datum point, the types are sometimes sharply marked. This group is, in effect made up of ternary color aberrations in which all three primary sensations show abnormal values. It comprises the following types.

(19) +R,+G,+B,	(23) −R,−G,−B,
(20) +R,+G,−B,	(24) −R,+G,−B,
(21) +R,−G,+B,	(25) −R,−G,+B,
(22) +R,−G,−B,	(26) −R,+G,+B,

Of this ternary group several of the types are to be found more or less clearly described in the literature. (21) for example, is substantially Burch's Case XIII, (Phil. Trans., **199**, B, cit.) where red and blue sensations were abnormally strong with marked deficiency in green. A rather clear case of (23) is described by Edridge-Green (Colour Vision, p. 158, G. A.) Here the spectrum was clearly shortened at both ends, especially the red, while some of the color matches indicated weak green sensation as well. The patient evidently had a general

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low degree of color sense with particular weakness of red sensation. (24) corresponds with a description by Burch (Physiological Optics, p. 119, of a case of predominant green sensation so marked that vision was almost monochromatic, which makes it probable that the red and blue sensations were weakened. Indeed it seems likely that great exaggeration of one sensation may be associated with weakness of one or both the remaining sensations. (25) seems to agree well with a case cited by Edridge-Green (Colour Blindness, p. 203) as examined by Sir Wm. Ramsay. In this case there was remarkable exaggeration of the blue, and degradation of the other sensations, so that vision was almost monochromatic.

All the color abnormalities here noted are such as belong to the simple trichromatic theory assuming that the several color sensation curves retain their shapes and their normal position in the spectrum, varying only in area. So little is known of the mechanism of color vision that one cannot even predicate whether shifts and changes of shape are or are not likely to take place. Indeed these could hardly be differentiated from other variations except they chanced to be very marked indeed. Even the interesting case described by Abney and Watson (Proc. Roy. Soc., **89**, A, p. 232) as showing a shift of the green sensation toward the red should be tested by the junction and end points and by Burch's fatigue method before forming a final judgment.

If this shifting, or a change of shape in the sensation curve should prove real, still further classes of variants would be formed, but however that may be, the mere variations of sensation curve area which are known to take place must give rise, assuming a certain relation between them as normal, to the definite groups of color variants here noted.

Of the 26 abnormal types of congenital color vision in this list 16 are fairly represented in recorded cases. Seven of the remaining 10 are types having two sensations + and thus varying from a simple deficit of the remaining sensation only in the degree of luminosity of the other two. The best method of differentiation here seems to be careful study of the end points, and fatigue tests. The remaining three involve abnormal blue vision combined with abnormal green vision, both of which separately seem to be relatively rare. Whether they are so in fact is somewhat dubious since with the commoner tests all the smaller variations in the blue are likely to be missed or merged in variations of pigmentation or color in the lens, while certainly most of the + G types are placed with the more familiar −R. In all the

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study of minor variation in color sensations the color fields ought to be more thoroughly investigated than is usual.

In conclusion the following list shows the tabulation of all types of abnormality here considered, those which the writer has observed or found recorded being denoted by an asterisk.

Simple	*Binary*	*Ternary*
(1) +R, G, B*	(7) −R,−G, B*	(19) +R,+G,+B
(2) R,+G, B*	(8) −R, G,−B*	(20) +R,+G,−B
(3) R, G,+B*	(9) R,−G, B	(21) +R,−G,+B*
(4) −R, G, B*	(10) +R,+G, B	(22) +R,−G,−B
(5) R,−G, B*	(11) +R, G,+B	(23) −R,−G,−B*
(6) R, G,−B*	(12) R,+G,+B	(24) −R,+G,−B*
	(13) −R,+G, B*	(25) −R,−G,+B*
	(14) −R, G,+B*	(26) −R,+G,+B
	(15) R,−G,+B*	
	(16) R,+G,−B	
	(17) +R, G,−B	
	(18) +R,−G, B	

The writer will be grateful for notes on any of the missing types in the table which may have escaped notice in the very scattered literature of this intricate subject.

A study of these types inevitably leads to the question as to whether any remedial measures can help the victims of abnormal color vision. Within restricted limits the answer may be affirmative. The method which has to be followed is precisely that which has already been tried with considerable success in modifying the color of artificial illuminants to obtain normal daylight values of color viewed by them. An ordinary gas flame, for example, is in effect partially blue-blind and the normal eye will see colored objects under such a light very much as the partially blue blind would see them in daylight. The necessary correction has been found to be the interposition of absorbing media which reduce the green and red elements in the same degree as the deficit of the blue element in the source. The penalty of doing this is the loss of considerable luminosity. The selective screens for this purpose are highly effective subject to this limitation. By a process exactly analogous it should be possible to provide a certain proportion of the partially red blind with spectacles which would give them at least an approximation to normal color vision, although at the expense of con-

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siderable luminosity, in amount depending on the extent of the red sensation deficit. Those in whom one sensation is nearly or quite absent are of course beyond the chance of help, since balance would require an almost complete obscuration of the remaining sensations, but the theory of the correction rests on a substantial basis and can be put into practice in not too severe cases of partial color deficiency. The method to be followed would be substantially that of Abney in obtaining the sensation curves of the individual and the problem then would resolve itself into making approximate corrections to reduce the curves as nearly as possible to normal relations. In bright light the corrected vision would then show colors in approximately their true relations, though somewhat dulled.

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CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORIES OF HARVARD UNIVERSITY. No. LXXIII.

LABOULBENIALES PARASITIC ON CHRYSOMELIDAE.

By Roland Thaxter.

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CONTRIBUTIONS FROM THE CRYPTOGAMIC LABORATORIES OF HARVARD UNIVERSITY. No. LXXIII.

LABOULBENIALES PARASITIC ON CHRYSOMELIDAE.

By Roland Thaxter.

Presented May 13, 1914. Received April 1, 1914.

During the past eight years I have obtained from various sources a considerable number of Laboulbeniales parasitic on beetles belonging to family Chrysomelidae. The first which came under my notice was found on a species of *Lactica* near Buenos Aires, and was included in my recent paper on Argentine Laboulbeniales.[1] The forms considered in the present Contribution have been obtained on alcoholic material, from various regions in the Tropics, which I owe to the kindness of numerous correspondents; or have been collected by myself in Trinidad; while a considerable number are derived from the dry material in the collections of the Museum of Comparative Zoölogy at Cambridge. In this connection I desire to acknowledge my great indebtedness to Mr. W. M. Mann for the privilege of examining the insects collected by him in Brazil during the Leland Stanford University Expedition in 1911, to the Rev. Geo Schwab, Mr. C. S. Banks, the late Prof. Kellerman, and others, who have been so kind as to have collecting done for me: as well as to Mr. F. C. Bowditch for numerous determinations, and to Mr. Samuel Henshaw for freedom to examine the Museum collections and for other favors.

In June, 1912, a paper was published by Spegazzini on Argentine Laboulbeniales, in which several new species parasitic on this group of beetles are included. Of these one, a species very common on members of the genus *Lema*, is referred by him to *Sphaleromyces*: but since its structure corresponds in all respects to that of a typical *Laboulbenia*, it should undoubtedly be removed to this genus. The three remaining forms considered by this author, are placed by him in a new genus, *Laboulbeniella*, which is said to be distinguished from *Laboulbenia* by the fact that cells III and IV of the last mentioned genus are here replaced by a single cell, and that cell VI, the 'stalk-

[1] These Proceedings, **48** (1912).

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cell' of the perithecium, is absent. Having had an opportunity to examine abundant material of these species, as well as of others belonging to the same type, I am unable to follow Spegazzini in making this separation; since in all cases I find that cell VI, as well as the usual basal cells of the perithecium, are present and variably developed, as in *Laboulbenia;* and the fusion of cells II–IV appears to be altogether too insignificant a character to form the basis of a new genus. It is true, however, that this replacement, or fusion, is characteristic of various forms parasitic on Chrysomelidae, and in most cases appears to have become a fixed condition. In other cases, however, the normal type is found and does not vary; while in still others both may be associated in the same species. I have called attention to the last mentioned condition in Part II of my monograph, and as will be seen by reference to fig. 11 of Pl. XIV, have figured a variety of *L. decipiens* in which, although the '*Laboulbeniella*' type predominates, the typical number and arrangement of the cells of the receptacle also occur. I have been disinclined, and I think rightly, to give generic value to variations in the cell numbers, especially of that portion of the receptacle in *Laboulbenia,* cells III–V and the insertion-cell, which corresponds to the base of the primary appendage in such genera as *Corethromyces.* Such departures from the normal type are seen in a majority of the aquatic species of *Laboulbenia,* in *L. proliferans, L. variabilis* and a number of the forms which occur on *Clivinae,* in all of which a multiplication of the cells in this region has been effected, instead of a reduction, as in the present instance. I have therefore placed in the genus *Laboulbenia* not only such of the chrysomelid forms as correspond exactly to the type, or in which the '*Laboulbeniella*'-condition occurs only occasionally, but also those in which cells III and IV are normally replaced by a single cell (designated as cell III + IV). Another type is also included, illustrated by the single species *L. partita,* in which, on certain hosts, the number of cells in the receptacle may become normally multiplied through the secondary division of the subbasal cell, a condition not previously observed except in abnormalities.

Although as will be seen a great majority of the chrysomelid parasites belong to *Laboulbenia,* two other genera are represented by well marked forms; *Dimeromyces* contributing four species from Mexico, the West Indies and the Straits Settlements; while seven species of *Ceratomyces* are included, six of them parasitic on 'flea beetles' from the West Indies and Brazil, the seventh a very peculiar form from Kamerun and Madagascar.

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Dimeromyces Homophoetae nov. sp.

Male individual amber-yellow. Receptacle consisting of three or four cells each of which may bear an antheridium: the primary appendage two celled, the basal cell shorter and broader, separated by a constriction from the distal which terminates in a long sharp spine and may often become one or more times septate, the terminal cell proliferating distally to form an antheridium which thus appears to be borne on a long several celled stalk on which the spine is lateral. Antheridia two to five, the stalk well developed, rather slender; the venter rather abruptly distinguished, though relatively narrow and slightly shorter than the nearly straight stout neck which tapers but slightly above its somewhat enlarged base. Total length to tip of appendage 60 μ; to tip of antheridium 80 μ, of proliferous antheridium 112 μ; normal antheridium including stalk about 35–40 $\times$ 6 μ.

Female individual: receptacle suffused with amber-brown below, distally amber-yellow; consisting normally of four cells obliquely superposed, terminated by a two-celled primary appendage, the terminal cell of which is narrower than the basal and about as long; sometimes more or less inflated at its base, and ending in a long straight sharp spine: the basal cell becoming rather deeply suffused and extending upward against the base of the lower appendage which is somewhat broader than the sub-basal cell from which it springs, and slightly geniculate: the third cell giving rise to the normally single perithecium, the fourth to a second somewhat smaller appendage; the two appendages slender, simple, tapering, somewhat divergent on either side of the perithecium, which is subfusiform with no clearly defined stalk; the stalk-region hyaline, the rest of the perithecium amber-brown, the tip distinguished by two successive indentations below the hyaline three-lobed apex, the lateral lobes of which are symmetrical and smaller than the much more prominent median lobe. Spores 32 $\times$ 3.5 μ. Perithecium, including stalk-portion, 100–190 $\times$ 20–35 μ. Total length to tip of primary appendage 70 μ; to tip of perithecium 140–250 μ. Appendages, longest, 90–110 μ.

On *Homophoeta aequinoctialis* Linn., No. 1577, Guatemala, on inferior prothorax; No. 2066, Grenada, W. I., on antennae, and No. 2475, Port of Spain, Trinidad, W. I. The specimens from Guatemala are somewhat larger than those from Grenada, but correspond in all other respects. The species appears to be more nearly allied to *D. Forficulae* than to any of the other described forms.

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Dimeromyces Aulacophorae nov. sp.

Male individual: hyaline with purplish brown shades. Receptacle consisting of three to four very obliquely superposed cells terminated by a simple appendage, the basal cell of which is long and slightly inflated, bearing distally a three or four-celled terminal portion from which it is separated by a rather deep constriction associated with a conspicuous blackened septum. Antheridial branches one to three in number arising unilaterally, one from each cell of the receptacle; the stalk cell narrower below, suffused, rather long; the antheridium hyaline, its basal cells rather large and long; the body rather narrow, not symmetrically related to the rather broad long nearly isodiametric neck, the apex of which is nearly truncate. Receptacle 32 × 10 μ exclusive of foot; basal cell of appendage 16 × 6 μ, the distal part 30–40 × 4 μ. Antheridium including stalk 30–40 × 5–6 μ, the stalk sometimes 11 μ long.

Female individual: purplish or amber-brown. Receptacle consisting of usually six or seven obliquely superposed cells; flattened, except the terminal one which is separated from the basal cell of the terminal appendage by a horizontal septum; the subterminal cell producing the solitary perithecium; all the rest, except usually the small basal cell, giving rise to appendages which form a unilateral series: basal cells of the lateral appendages short, somewhat inflated and curved upward, bearing one to rarely four simple hyaline or slightly suffused straight or flexuous elongate branchlets, which for the most part taper but slightly and are distinguished at the base by a conspicuous broad blackened septum and constriction: the terminal appendage similar to that of the male, except that its basal cell often bears two branchlets distally. Perithecium relatively very large, hyaline or yellowish below, distally more or less suffused with purplish brown, straight or less often curved, slightly broader just above the short hardly distinguishable stalk, distally tapering rather rapidly to the rather broad apex which, in anterior view, is symmetrically three lobed, the middle lobe prominent and broader. Perithecium when well developed 175–250 × 24–28 μ, sometimes smaller (100 × 18 μ). Spores 32 × 3 μ. Receptacle about 65 × 20 μ. Basal cell of primary appendage 20 × 6.5 μ. Longer appendages 130 × 3.5 μ.

On the elytra of *Aulacophora postica* Chap., Perak, Straits Settlements. M. C. Z., No. 2510.

The material of this species, although sufficiently abundant, is not

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in very good condition, there being no perfect individuals. It is well distinguished by its enormous perithecia and by the development of secondary branches from the basal cells of the secondary appendages. It seems to be more nearly related to *D. Homophoetae* than to any other species, the conformation of the apex of the perithecium being somewhat similar, as well as the long-stalked antheridia, but it is otherwise very distinct.

Dimeromyces Hermaeophagae nov. sp.

Male individual: nearly hyaline. Receptacle consisting of two superposed cells, the basal larger, laterally related to the foot; the subbasal giving rise to the single slender antheridium, the stalk-cell and basal cells of which form a relatively long stalk below the narrow only slightly enlarged venter, which is about as long as the rather stout outcurved neck. Primary appendage consisting of a basal cell nearly as long as the subbasal cell of the receptacle and bearing terminally a two-celled slightly inflated portion of about the same length, from which it is separated by a clearly defined dark septum. Total length to tip of appendage, 35 μ; to tip of antheridium 50 μ. Antheridium-stalk (stalk- and basal cells) 16 μ; venter and neck 18 μ; appendage 17 μ.

Female individual: nearly hyaline or faintly yellowish. Perithecium arising from the third cell of the receptacle, its stalk-cell small, hardly distinguishable, its form rather elongate, tapering slightly, the tip distinguished by a very slight elevation; the spreading funnel-shaped apex distinguished by a constriction. Receptacle four-celled, the basal slightly larger, with inferior lateral small foot; the second and fourth bearing secondary appendages the basal cells of which are rather stout and long, and distinguished from the tapering several-celled terminal portion by a distinct dark septum which does not reach as far as the tip of the perithecium. Primary appendage like that of the male, stouter. Perithecium, exclusive of stalk-cell, 60–70 μ. Secondary appendages 60–70 μ. Primary appendage 20–25 μ. Total length to tip of perithecium 75–90 μ.

On antennae of *Hermaeophaga insularis* Jac., No. 2066, Grenada, B. W. I. (Brues).

This species seems most nearly allied to *D. minutissimus* and differs from all other known species in the peculiar funnel-shaped apex of the perithecium.

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Dimeromyces Longitarsi nov. sp.

Male individual: hyaline or faintly yellowish; receptacle consisting of three obliquely superposed cells, the two upper bearing antheridia, and subequal, the basal as long as these two combined; the distal bearing terminally the unicellular appendage, which is twice as broad as the base of a hyaline usually straight subulate process that terminates it and nearly equals it in length. Antheridia normally two, the stalk-cell broader than long; the rather stout venter somewhat longer than broad, abruptly distinguished from the rather stout neck which is of about the same length and distally bent abruptly outward. Total length to tip of spinous process 60–64 μ; to tip of distal antheridium 70 μ, the foot included. Appendage, including spinous process, 14–15 μ. Basal cell of receptacle 18 μ. Antheridia 22 $\times$ 2 μ.

Female individual: tinged with yellowish, the perithecium deeper, faintly tinged with amber-brown, bent inward basally and distally, rather long and narrow, slightly broader distally below the paler tip, which is rather clearly distinguished above a slight constriction most prominent on the inner side; the apex distinguished by an often deep constriction, its base inflated to form rounded prominences above which it is compressed, asymmetrical, tapering to a blunt point: the stalk-cell small subtriangular, free only externally. Receptacle consisting of three obliquely superposed cells, the two upper subequal, angular, the basal longer; primary appendage as in the male, a minute cell distinguishable at the base of the spinous process: secondary appendage single, its basal cell similar to the subbasal cell of the receptacle from which it arises and with which it is obliquely associated; the rest of the appendage simple, slender, whip-like, tapering; seldom extending beyond the tip of the perithecium. Perithecium 80–120 $\times$ 20–25 μ. Secondary appendage 85–100 μ: primary appendage, including spine, 25 μ.

On the elytra of *Longitarsus testaceus* Mels., Fayetteville, Arkansas No. 1801; of *L. subcinctus* Har., No. 2455 and of *Aphthona Deyrollei* Baly, No. 2454, Port of Spain, Trinidad, B. W. I.

This species is most nearly related to *D. Homophoetae* from which it is distinguished by its unicellular primary appendage and single secondary appendage, as well as by various other points of difference. The material from Arkansas appears to differ in no respect from that obtained in Trinidad.

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Laboulbenia Bruchii (Speg.).

Sphaleromyces Bruchii Speg., Cont. al Est. d. l. Laboulbeniomycetas Argentinas; Ann. d. Mus. Nat. d. Hist. Nat. de Buenos Aires, XXIII, p. 195, with figure, 1912.

This form, which is very common on various species of *Lema* in Tropical America, has been referred to *Sphaleromyces* by Spegazzini. Its structure, however, is that of a typical *Laboulbenia,* and the number and arrangement of the cells of its receptacle are both normal. Although the structure is sufficiently clear in younger individuals, cells II and III usually become very deeply suffused, except along their inner edges, and are indistinguishable from cell IV, the variable prolongation of which forms the curious spur-like termination regarded by Spegazzini as a development from cell II, and as similar to that which arises from the distal cell of the two-celled receptacle of *Corethromyces,* in which it is a development from this cell only. This characteristic varies very greatly in different individuals of the present species, the spur in many cases being but slightly developed, or at least far less prominent than it is in the typical form represented by the figures of Spegazzini. The species is a very striking and variable one, closely allied to the three following forms which occur on the same host genus, and in the characteristic development of cell IV resembles *L. producta* of my second Monograph (Plate LXIV, fig. 13). The longitudinal dehiscence of the perithecium, which is given by Spegazzini as a distinguishing character of this species, must be regarded as an accidental splitting, which is not infrequently seen in dried material that has been taken from museum specimens and suddenly swelled by mounting.

Material has been examined from the following sources. No. 1643, on *Lema* sp. Guatemala, (Kellerman); No. 1774, on *L. Sallei* Jac. Mexico (Biologia Coll.); No. 1775, on *L. Albini* Lac., Mexico (Biologia Coll.); No. 1776, on *L. dimidiaticornis* var., Mexico, (Biologia Coll.); No. 2212, on *L. gracilis* Jac., Para, Brazil (Mann): Nos. 2476–2477, on several species of *Lema,* Port of Spain, Trinidad.

Laboulbenia Papuana nov. sp.

Receptacle short and subtriangular, the basal cell nearly hyaline, except distally and along its posterior margin; short, abruptly broader distally, subgeniculate; its base clasped by a bluntly pointed

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upgrowth from the foot on either side, which is black, contrasting and slightly oblique: cell II much smaller, flattened, dirty olivaceous, extending upward anteriorly and separated by a slightly oblique partition from cell VI; which is smaller, concolorous and separated by a strongly oblique septum from the basal cells of the perithecium, which are relatively large, clearly defined and paler dirty yellowish: cells III–V forming a clearly distinguished, somewhat darker translucent grey olive-colored region, the inner walls of which are nearly vertical, but curve abruptly outward where they are in contact with cell II: cell III larger than cell II, but somewhat similar in shape, except for its curved base, separated from cell IV by a strongly oblique curved septum; cell IV nearly as large as cells II and III combined, externally concave, projecting beyond the insertion-cell to form a broad rounded prominence distally slightly compressed; cell V well defined, triangular, externally convex. Insertion-cell clearly defined, translucent. Basal cell of outer appendage about as long as broad, distally much broader, and bearing a terminal series of three radially arranged branches, themselves once branched above their basal cells; the outer curved outward and externally more deeply suffused with blackish brown externally at the base; the branchlets curved more or less strongly inward, distally hyaline, with olive brown suffusions below, seldom reaching to the tip of the perithecium. Perithecium relatively long, curved slightly outward, wholly free above its basal cells; the inner half rich brown, sometimes contrasting with the often much paler dirty yellowish brown outer half, which is concolorous with the long compressed slightly darker, not abruptly distinguished tip, the concave outer margin of which is abruptly much darker; the lips small but well defined. Perithecia 100–125 × 25–28 μ. Appendages, longest 110 μ. Receptacle including protrusion 85–100 × 50–60 μ, exclusive of foot. Total length, including foot, 200–228 μ.

On the elytra of *Lema* sp., New Guinea, M. C. Z. No. 2511.

This species is so closely related to *L. Bruchii* that I have hesitated to separate it specifically, and it may prove to be merely a well marked variety when a large series becomes available for examination. The type material includes a half dozen specimens in good condition, all of which show the characteristics above described.

Laboulbenia rhinoceralis nov. sp.

Receptacle short and stout, subtriangular, the basal cell larger than the subbasal, longer than broad, hyaline becoming tinged with

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olivaceous; cells II and III subequal; cell IV forming a short blunt prolongation external to and beyond the insertion-cell; cells II–IV becoming almost or quite opaque and indistinguishable, except along their inner margins; cell V relatively large and rounded, suffused with olivaceous brown, as is cell VI, which lies obliquely below it, is of about the same size, and is separated by a deep indentation from the outer lower basal cell of the perithecium immediately above it, which is quite hyaline, bulging asymmetrically outward and separated from the cell above it by a deep indentation. Insertion-cell rather thick, black; lying horizontally in the deep depression formed between the base of the perithecium and the projecting termination of cell IV. Outer appendage deeply suffused at the base and externally, curved outward, consisting of usually two larger basal and subbasal cells; the former giving rise to a subterminal inner branch, and the latter to a subterminal inner and two or three terminal short branches, the outer black and abortive; the basal cell of the inner appendage nearly as large as that of the outer, bearing a branch on either side which is very similar to the outer appendage and externally suffused; all the branches rather short and stout, more or less suffused about the base. Perithecium wholly free, the upper basal cells forming a very short stalk, relatively large, long and narrow; the venter hardly inflated, opaque or nearly so; the tip translucent, not distinguished from the venter, except by a clean cut transverse line of demarcation; the hyaline apex subtended externally by two superposed long hyaline slightly outcurved horn-like appendages, the lower an extension of a laterally misplaced lip-cell, the upper a subterminal projection from the outer of the three remaining lip-cells which surround the sulcate pore; the latter turned slightly inward. Perithecia 90–120 × 20–25 μ, its horn-like appendages about 18 × 6 μ. Spores 35 × 3.5 μ (measured in perithecium). Receptacle average 52 × 28 μ. Appendages longer 52 μ. Total length to tip of perithecium 125–175 μ.

On the elytra of *Lema gracilis* Jac., No. 2212, Para, Brazil (Mann), on *Lema* sp., Port of Spain, Trinidad, No. 2477; on *Lema* sp., Suriname, No. 2480 (Rorer).

A very peculiar species distinguished from its near ally, *L. Bruchii*, by the peculiar horn-like processes which subtend the apex. The lower of these processes is formed by the abnormal termination or the left posterior series of wall-cells which, instead of forming a lip-cell, bends abruptly outward as it traverses the tip, crossing the series external to it and projecting as a free appendage. The two horns are sometimes malformed and somewhat misplaced, but are usually well developed and symmetrically placed one above the other;

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the whole tip thus armed strongly suggesting the head of a two-horned rhinoceros. The species was common in the vicinity of Port of Spain, but individuals were found only in the depressions near the bases of the elytra. The receptacle is twisted one quarter in relation to the perithecium, in most specimens; so that, when the latter lies flat, the receptacle is usually viewed edgewise.

Laboulbenia Hottentottae nov. sp.

Receptacle rather slender, sometimes slightly geniculate between cells II and III; cell I usually quite hyaline; cells II–III becoming more or less deeply suffused, or nearly opaque; cell II sometimes abruptly broader than cell I, but always shorter; separated from cell VI by a horizontal, from cell III by a very oblique partition; cells III and IV subequal, separated by a very oblique partition; cell V narrow, triangular, nearly hyaline; cell VI suffused, distinguished above and below by an indentation, obliquely separated from the cells above, which form a broad almost stalk-like base to the perithecium. Insertion-cell suffused, but not blackened, rather thick. Basal cell of outer appendage somewhat longer than broad, brown, giving rise to a short hyaline erect subterminal branch on the inner side; the sub-basal cell colored like the basal, somewhat longer; the rest of the appendage hyaline, few-celled, short; basal cell of the inner appendage much smaller than that of the outer, concolorous; producing a branch on either side, each usually once branched; the branches sometimes exceeding the perithecium in length, hyaline, except the brownish basal cells. Perithecium wholly free, rather long and slender, often nearly symmetrically inflated, deep rich brown, tapering evenly distally; the tip hardly if at all distinguished, darker along the inner side; the apex blunt, the lip-edges subhyaline about the pore, sometimes slightly oblique outward; a slight projection usually visible next the pore. Perithecia, exclusive of basal cells, 90–125 × 18–25 μ. Receptacle 90–110 × 25–32 μ. Appendages, longest 120 μ. Total length to tip of perithecium 160–200 μ.

On elytra etc. of *Lema Hottentotta* Lac., Zanzibar (M. C. Z.): crowded near tips of elytra.

This species usually has a somewhat falcate or slightly curved habit, and is rather slender in form. Although apparently allied to *L. Bruchii*, its appendages and receptacle separate it at once from any of the forms of that species. The material available is abundant and in good condition.

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Laboulbenia Braziliensis nov. sp.

Receptacle often straight and narrowly wedge-shaped, sometimes slightly curved and less regular, but tapering more or less regularly from summit to base; hyaline, becoming yellowish or tinged with brown, deeply along the posterior margin especially immediately below the insertion-cell; cell III longer than cell IV which is abruptly prominent below the insertion-cell, sometimes forming a rounded projection beyond it: cell V relatively large, sometimes as large as cell IV, and in contact with cell III: cell VI relatively very large, exceeding cell III in length. Insertion-cell thick and rather narrow; basal cell of the outer appendage rather small somewhat rounded, tinged with brown externally, separated by an oblique blackened septum from a distal and external furcate branch, within which one or more additional erect branches arise, not thus separated; basal cell of the inner appendage much smaller, also pale brownish, giving rise to branches on either side which are usually once branched, and, like those of the outer, are rather stout hyaline or faintly brownish, erect, slightly tapering, seldom reaching beyond the tip of the perithecium. Perithecium deep rich brown becoming opaque or nearly so, wholly free; the base of the venter somewhat narrower than the basal cell-region below it, the outer margin nearly straight, the inner convex; the tip bent outward, not distinguished except on the inner side, the apex symmetrically rounded or almost truncate; the lip-edges broadly hyaline, contrasting. Perithecia 130–158 × 40–48 μ. Receptacle, average, 175 × 52 μ. Appendages, longest, 160 μ. Total length to tip of perithecium 350–380 μ.

On a chrysomelid allied to ?*Oedionychus*. Rio de Janeiro, Brazil, M. C. Z. 'Mrs. Munro,' No. 1786, on the legs and elytra.

This species seems to be very well distinguished from any of the other species on Chrysomelidae. The host is a stout chrysomelid, the elytra dark bluish black, with a red margin all around. More than twenty mature individuals and numerous younger specimens have been examined, in which the measurements seem unusually constant and the variations in other respects slight, except that in some specimens cell IV protrudes in a fashion which recalls less well developed individuals of *L. Bruchii*.

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Laboulbenia idiostoma nov. sp.

Receptacle evenly suffused with olive-brown, somewhat darker distally, short and compact, relatively small; cell I slightly larger than cell II and nearly triangular, or symmetrically pointed distally between cells III and VI; cells III and IV subequal, the latter forming a rounded prominence distally, which turns the insertion-cell so that it is very oblique, or even vertical, in position, the appendages being thus turned so that they cross the base of the perithecium obliquely, or at right angles; cell VI rounded and prominent, the basal cells of the perithecium also small and bulging, especially externally. Perithecium relatively large, long, straight or but slightly curved, often somewhat broader distally, rich purplish brown, almost opaque, free; the tip abruptly attenuated, indented externally, and hyaline distally, the apex prominently bilobed, the lobes rounded and symmetrical when viewed anteriorly or posteriorly. Appendages relatively long, straight or symmetrically curved; the outer appendage simple, its basal cell large, externally convex, its distal septum broad and blackened; the basal cell of the inner appendage half as large; a group of antheridial and sterile branchlets arising on either side, which may be branched near the base so as to form a tuft in which about six of the branchlets are apt to be sterile and project across the base of the perithecium; all the sterile branchlets subcylindrical, rigid, rather remotely septate, sometimes furcate near the blunt tip; the antheridia relatively large and long, with a narrower base below the rounded venter, usually in pairs, and sometimes on long branchlets. Perithecia, average, 122 × 30–35 μ. Receptacle 70–85 × 35–42 μ. Appendages, longest, 200 μ. Total length to tip of perithecium, average 175 μ.

On antennae of *Haltica Jamaicensis* Fab. Ennery, Hayti (Mann), No. 2491.

A very distinct species observed only on the antennae of its host. The peculiar tip of its perithecium is not unlike that of *L. leptostoma* Speg.

Laboulbenia fuliginosa nov. sp.

Receptacle and appendages dirty yellow olive-brown; the former of normal form and structure, more deeply suffused with age; cells IV and V often subequal; cell III slightly larger than cell VI; cells I and II of nearly equal length; the whole tapering more or less regularly to

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the base. Insertion-cell broad and thick; basal cell of the outer appendage externally more deeply suffused and slightly convex, separated by an oblique blackened septum from its outer branch which is short, submoniliform, and usually four-celled; a second branch arises within the outer, its basal cell about equal to that of the outer, but bearing two branchlets, radially placed; consisting of four or five short cells like those of the outer branch, extending usually not more than two thirds the length of the perithecium, towards which, like the outer, they are usually slightly bent; basal cell of the inner appendage slightly smaller than that of the outer, bearing a branch on either side consisting of a single cell terminated usually by a pair of antheridia, which may also be associated with one or two short stout branchlets like the outer. Perithecium free, except its basal cells, becoming quite opaque; convex on both sides, more abruptly so externally; tapering distally to the rather narrow tip; the apex sometimes slightly apiculate; the lip-edges hyaline, or translucent. Perithecia (above basal cells) 100–125 × 40–48 μ. Spores about 55 × 5.5 μ. Receptacle 150–200 × 45–52 μ. Longest appendages 90 μ (usually broken). Total length to tip of perithecium 60–90 μ.

On the elytra and under surface of *Haltica plebeia* Oliv. No. 1785, Hayti (M. C. Z.); on *Haltica* sp. Cuba, (M. C. Z.), No. 1784; and 1787; No. 1770, on *Haltica amethystina* Oliv., Vera Paz, Biologia Coll. on *H. Jamaicensis* Fab., No. 2491. Ennery, Hayti (Mann); No. 2506, Jamaica.

This is a very ordinary looking species, of more or less uniform dark, dirty olive-brown color, apparently very common on its hosts in Hayti and Cuba. It is perhaps as nearly allied to *L. partita* as to any of the other species on Chrysomelidae, but never appears to deviate in structure from the ordinary type of the genus. The material is usually poor, even on hosts preserved in alcohol, and the outer branch of the appendage is seldom found in adults.

Laboulbeni Halticae nov. sp.

Dull olivaceous, the perithecium and distal portion of the receptacle somewhat darker. Receptacle normal, or occasionally cells III and IV replaced by a single cell, slender, but little broader distally; cells I and II subequal, or I slightly longer; cells III and IV subequal; cell V reaching down to, or nearly to, cell III and sometimes equaling it in size. Insertion-cell thick, deeply suffused, but not opaque:

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basal cell of outer appendage large, longer than broad; its subbasal cell slightly smaller, and normally bearing two long branchlets, both simple, or the outer once branched: basal cell of the inner appendage very small, bearing a branch on either side which may be simple or once branched and bearing scanty antheridia; all the branchlets becoming tinged with olivaceous, especially at the base and externally, the longest sometimes twice as long as the perithecium or even longer. Perithecium four fifths or more free, rather narrow, the venter but slightly inflated; the tip hardly distinguished, broad; the apex broad, rounded or subtruncate, externally unevenly oblique, hyaline about the pore, beside which one of the lip-cells usually forms a distinct, though minute prominence. Perithecium 70–90 × 25–30 μ. Receptacle 85–125 × 25–30 μ. Appendages, longest, 150 μ; in one young specimen 227 μ. Total length to tip of perithecium 125–210 μ, average 175 μ.

On *Haltica* sp. Kamerun (Schwab), No. 2449 (type): on *Systena Deyrollei* Boh., Port of Spain, Trinidad, No. 2467.

The specimens from Kamerun and Trinidad do not appear to differ essentially. The species is not distinguished by striking peculiarities, but does not seem referable to any described form. The material is abundant and in good condition. A small percentage of the specimens from Kamerun are of the '*Laboulbeniella*' type, cells III and IV being replaced by a single cell. This does not seem to be the case, however, in the material from Trinidad.

Laboulbenia Nodostomae nov. sp.

Short and stout. Cell I slightly longer than cell II which is slightly longer than broad, its anterior margin strongly convex; cell III broader than long, shorter than cell IV; cell V small triangular, cell VI somewhat smaller than cell II; cells II–V dirty yellow, deeply tinged with brown, the rest dirty yellow; the walls, only, tinged with brown. Insertion-cell broad thick and blackish; appendages almost or quite hyaline. Basal cell of the outer appendage somewhat longer than broad; bearing distally two simple branches, obliquely related, the outer stout tapering, elongate and incurved; basal cell of the inner appendage about half as large as that of the outer, producing a single simple short stout branch on either side, each with one or two antheridia at the base. Perithecium a little less than one third free, rather short and stout, dark blackish olive, paler below, and subterminally;

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the tip deep black, short, broad, abruptly distinguished; the lip-edges broadly hyaline, contrasting; the apex more or less rounded, two indistinct papillae above the external pore. Perithecia 60–65 × 25 μ. Spores as long as venter, about 40 × 4 μ. Receptacle 80 × 35 μ. Appendages, longest 175 μ. Total length to tip of perithecium 130 μ.

On tip of the elytron of *Nodostoma* sp., Mindanao, Philippines, No. 2386, received through the kindness of Mr. Oakes Ames.

An insignificant species very similar to some of the forms on Carabidae allied to *L. flagellata.*

Laboulbenia Philippina nov. sp.

Receptacle varying in length from the variable elongation of cell II; hyaline, becoming tinged with straw-yellow; cells I and II relatively large, of about the same diameter, cell II often far longer; cells III, IV and VI subequal and subisodiametric; cell V very small, triangular. Basal cells of the perithecium hyaline, well defined, contrasting with the venter; that lying externally above cell VI usually prominent; venter rich contrasting translucent purplish brown; slightly, often symmetrically, inflated; the tip more or less well defined, rather abruptly so externally where a narrow margin is deeply suffused; otherwise quite hyaline, except for a black patch which subtends the rather prominently rounded inner lip-cell; the lip-edges hyaline, outwardly oblique. Appendages wholly hyaline, becoming yellowish; the basal cell of the outer appendage giving rise to an outer and an inner branch; the basal cell of the inner appendage much smaller, producing a usually simple branch on either side; all the branches rather stout, elongate, somewhat divergent or even slightly recurved, slightly tapering. Insertion-cell contrasting with all the cells about it. Perithecium about five sixths free, 85–100 × 28–35 μ. Spores 50 × 3.5 μ. Receptacle 100–225 × 38–42 μ. Appendages longest 210–420 μ. Total length to tip of perithecium 150–335 μ.

On the elytra and legs of a chrysomelid near *Rhembastus:* No. 2451 Manila, Philippines (Banks).

Very like some forms of *L. polyphaga,* and distinguished from related forms on Chrysomelidae by its often elongate form, hyaline or evenly pale yellow color, its stout elongate concolorous appendages and deeply suffused contrasting perithecium.

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Laboulbenia Oedionychi nov. sp.

Receptacle normal, hyaline, becoming faintly tinged with olivaceous, especially distally where it is indistinctly punctate; cells I and II subequal, relatively large, the remaining cells small; cell III slightly larger than cell IV; the outer margin of the latter curved evenly outward and downward from the insertion-cell; cell V triangular, small, not in contact with cell IV. Insertion-cell black contrasting, rather thick, not very broad. Outer appendage almost invariably simple, slender, divergent, horizontal or recurved, long, slender and slightly tapering, often geniculate at the base in the region of the subbasal cell; which, like the basal, is more or less tinged with blackish olive externally, and is often prominent distally on the inner side: basal cell of the inner appendage somewhat smaller than that of the outer, producing a short, usually simple, branch on either side, each bearing a single antheridium near the base. Perithecium united to cells IV and V at its base, transparent olivaceous, usually straight; the venter long, narrow, slightly and evenly inflated; the tip slightly distinguished, very slightly bent outward, broad, more deeply blackened below the hyaline lip-edges; the rather coarse lips outwardly oblique. Perithecia 80–85 × 20–24 μ. Spores 48 × 4 μ. Receptacle 90–140 × 28 μ. Appendage, longest, outer 175–230 μ, the inner, 35–80 μ. Total length to tip of perithecium 140–210 μ.

On the elytra of *Oedionychus* nov. sp., No. 2415 and 2450, Manila, Philippines (Banks).

Abundant material of this species in perfect condition has been examined. The species like *L. Halticae* to which it is allied, is not unlike some of the many forms allied to *L. polyphaga* and *L. flagellata* which occur on Carabidae. Specimens from the legs (No. 2450) of the same host are somewhat smaller, the outer appendage more deeply blackened externally, both the basal and subbasal cells bearing distally on the inner side an erect hyaline branch; the receptacle short, cell I longer than cell II; the perithecium more deeply suffused.

Laboulbenia Hermaeophagae nov. sp.

Receptacle normal, faintly punctate, olivaceous, darker distally; the basal cell paler or hyaline below, narrow, somewhat longer than the subbasal; cells III and VI subequal, the latter strongly convex externally; cell IV prominent below the insertion-cell; cell V rounded

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or irregularly quadrangular, its base resting on cell II; all the cells distinguished from one another by more or less distinct indentations, owing to the convexity of their margins which may be pronounced. Insertion-cell thick and deeply suffused. Outer appendage consisting of a divergent series of three cells, successively smaller and externally more deeply suffused with blackish olive, the terminal one bearing distally a pair of short branchlets radially placed; the basal and subbasal each bearing a stout subterminal simple branch on the inner side; basal cell of the inner appendage much smaller, bearing a branch on either side which may be once branched; all the branchlets rather short and stout, never reaching to the tip of the perithecium. Perithecium wholly free, almost subfusiform, the small but prominent basal cells forming an indistinct stalk, often hyaline and contrasting with the deep olive brown venter above; which is evenly inflated, more strongly convex externally, but sometimes almost symmetrical, tapering slightly to the truncate apex which is surmounted by a pair of minute prominences. Perithecia, average, 80×25 μ. Spores 50×5 μ. Receptacle $70\text{–}80 \times 30$ μ. Appendages longest 70 μ. Total length to tip of perithecium 150–175 μ.

On tips of elytra of *Hermaeophaga* sp., No. 2466, Port of Spain, Trinidad.

Several specimens of a species very similar to this, were obtained on a species of *Hermaeophaga* from Jamaica. The cells of the receptacle in this form bulge more prominently than in the type, but there is otherwise slight difference between the two. The perithecium appears to be more often twisted about one fourth, so that the appearance of the apex as above described is that of an anterior or posterior view.

Laboulbenia Manobiae nov. sp.

Receptacle uniformly hyaline tinged with yellow, normal in structure; cells I and II subequal relatively large, somewhat more than twice as long as broad; cells III, IV and VI subequal and subisodiametric, the latter, as well as the large external basal cell just above it, bulging prominently. Insertion-cell broad, rather thin and black. Basal cell of outer appendage somewhat longer than broad, externally somewhat suffused, bearing a stout simple branch subterminally which projects outward; its basal cell, except distally, deeply suffused, the remaining cells nearly or quite hyaline; a second similar but wholly hyaline branch arising terminally: basal cell of the inner

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appendage somewhat smaller, bearing one or two hyaline branches similar to those of the outer; on either side the external one is subtended by a short stiff characteristically blackened branchlet: all the branches stout, hyaline, elongate, tapering but slightly. Perithecium nearly free, rich blackish brown, the suffusion involving part of its basal cells; rather symmetrically inflated, a contrasting paler region just below the well distinguished rather broad deeply suffused tip; the lips rather coarse and prominent, the two inner lips bearing distally a well defined papilla. Perithecium 75 × 25 μ. Appendages about 140 μ. Receptacle 70–80 × 28 μ. Total length to tip of perithecium 145 μ.

On the tips of the elytra of *Manobia abdominalis* Jac., M. C. Z., No. 2505.

Although this species represents a somewhat ordinary type of the genus, it differs from others that are known, through the presence of a short black subulate branchlet which subtends the stout external branch of the inner appendage on either side. The branches of the appendage are unusually stout, almost hyaline, and diverge almost horizontally in the two types, which are in good condition.

Laboulbenia partita nov. sp.

Receptacle very variably developed; elongate, or short and stout, the structure normal, or often abnormal through the secondary division of cell II, which may be replaced by a series of from two to rarely ten or often nine superposed cells, many of which may be distinguished, singly or in pairs, by slight constrictions; the series of about the same diameter throughout, except the terminal cell which may rarely be once or twice longitudinally divided; the distal portion of the receptacle, cells III–VI, usually normal, rarely of the "*Laboulbeniella*-type," the whole hyaline and abruptly contrasting with the dark perithecium, of more or less suffused, with yellowish brown, especially at the margins of cells III–IV; cells III and VI usually subequal, cell IV protruding below the black, well defined insertion-cell. Appendages variable, the basal cell of the outer slightly longer than broad, somewhat inflated and externally suffused, bearing distally one to three branches, usually two, radially placed the basal cell of the outer often suffused externally, bearing distally usually two or three short simple branchlets radially placed, blunt and hyaline; basal cell of the inner appendage much smaller than the outer, bearing a branch on

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either side which may be multiplied by successive branching so as to form a rather dense tuft, none of the branchlets extending beyond the tip of the perithecium. Perithecium free, erect, straight, deep slightly olivaceous brown above its contrasting basal cells, the tip bent slightly outward; lip-cells hyaline, tipped by two minute papillae. Spores about 40 × 3.5 μ. Perithecia, suffused part, average 75–85 × 32 μ. Appendages, longest, 85 μ. Receptacle, average, 135 × 25 μ, (85–175 μ). Total length to tip of perithecium 140–250 μ.

On *Nisotra dilecta* Dej. and *Nisotra* sp. Kamerun, German W. Africa, (Schwab) usually on elytra: On *N. Chapuisi* Jac., Madagascar, M. C. Z., No. 2504.

The abnormal septation of cell II is more common than the normal type on *N. dilecta* while on the second species, a uniformly steel blue form, it is comparatively rare. Specimens in which the septation is pronounced might well be mistaken for a species of *Misgomyces*. Specimens also occur having the simplified structure of "*Laboulbeniella*" as separated by Spegazzini. The material from Madagascar includes only the normal type and the individuals are smaller and less well developed.

Laboulbenia Dysonichae (Speg.).

Laboulbeniella Disonychae Speg., Contr. al Est. d. l. Lab. Argent. p. 188. 1912.

This species has been made the type of a new genus *Laboulbeniella* by Spegazzini, which he based on the fact that cells II and IV are replaced by a single cell. As I have already pointed out above, this character seems to me too trivial to form the basis of a new genus, for the reasons mentioned. It appears, however, to be the normal condition in many of the chrysomelid forms and is found in all the following species, as well as occasionally in several of those above described, as for example in *L. Halticae*. *L. Tucumanensis* described by Spegazzini on a similar host, which is said to possess stouter perithecia, does not appear to differ in any essential respect from the present form, as far as can be determined from the figures and description of the author.

Material of this species has been examined from Mexico, Biologia Coll., No. 1768 on *Disonycha figurata* Jac., and was obtained by myself in Trinidad on *D. austriaca* Schf., Nos. 2474, 2474b and 2474B, from the vicinity of Port of Spain and from Sangre Grande. It is

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distinguished by its very slender and long appendages, which, although they are only about 4 μ in diameter, may reach a length of more than 300 μ. In the specimens examined, the diameter of the perithecium varies from 21–35 μ.

In the following descriptions all of which, except that of *L. Podontiae*, relate to forms of the "*Laboulbeniella*"-type, the cell which replaces cells III and IV in the receptacle is spoken of as "cell III + IV."

Laboulbenia arietina nov. sp.

Receptacle relatively small, much shorter than the perithecium; olive brown, except the basal cell which is pale and slightly larger than the subbasal; cell III + IV about as long as cell II and half as wide; cell V minute; cell VI well developed, obliquely placed. Insertion-cell olivaceous, small, not deeply blackened. Appendages similar to those of *L. Disonychae*, the small bases of the outer and inner, opaque, prominent and persistent; giving rise to slender branches two or three times successively branched, olivaceous, their lower septa dark, bent rather abruptly toward and across the base of the perithecium and more or less circinate about it. Perithecium wholly free, relatively long and large, bent toward the appendages, dark brown, the base and tip paler olivaceous; the tip not otherwise distinguished; the apex oblique and prominent, externally oblique; one of the inner lip cells extending upward to form an erect blunt hyaline tipped appendage, its base concolorous with the dark olivaceous apex. Perithecia 100–140 × 20–25 μ; the terminal appendage 18–25 μ. Receptacle 60–100 × 24–28 μ. Appendages longest about 150–175 μ. Total length to tip of perithecium 160–240 μ.

On the elytra of *Disonycha recticollis* Jac., No. 1843, Guatemala, Kellerman; and of *D. austriaca* Schf., No. 2474b, Port of Spain, Trinidad.

The four individuals of this peculiar species which have been examined correspond in all respects; the Mexican being only slightly larger than the Trinidad specimens. It is very closely allied to *L. Disonychae* with which it occurred, but is at once distinguished by the terminal appendage of its perithecium.

Laboulbenia Podontiae nov. sp.

Receptacle usually small and short, but variable; dull dark brown, the basal cell conspicuously paler dirty yellowish or yellowish brown,

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relatively large, sometimes nearly as long as the rest of the receptacle; cells II and VI subequal in length; cell II usually broader; cells III and IV often about equal their septa oblique, their external margins convex; cell V small, extending to cell III; cell VI and the basal cell above it, externally convex. Insertion-cell free, rather thick, black, pointed above. Basal cell of the outer appendage rather deeply suffused with olive-brown, somewhat long and narrow; bearing distally, or distally and externally, a series of two or three branches, similar, radially arranged with blackened basal septa; their basal cells broader distally, short and stout and bearing one to three secondary branches in a similar fashion, this branching being variably repeated; the cells of these branches and branchlets similar, hyaline, with distal external suffusions; the ultimate branchlets few-celled, hyaline, curved toward the tip of the perithecium which they barely reach; all the septa of the partly suffused cells deeply blackened: basal cell of the inner appendage about half as large, becoming pushed somewhat sidewise and bearing distally a series of two or three branches radially arranged, or slightly oblique, which are similar to those of the outer appendage. Perithecium arising opposite the inner upper angle of cell III; concolorous with the receptacle, or more olivaceous; usually rather strongly curved, and tapering from near the base to the rounded apex; which is usually narrow, but hardly differentiated mostly long and slender, slightly enlarged at the apex, the lip edges not prominent, the inner hyaline: the wall-cells indicated by dark lines, the longitudinal ones having a spiral tendency sometimes lacking, so that the apex may be seen in anterior or posterior view and appear broad and subtruncate. Perithecia 70–100 × 18–20 μ. Spores very slender, the distal half about as long as the basal, attenuated to a fine point bent abruptly sidewise, about 55 × 3 μ. Receptacle 70 × 24–28 μ; larger form 100 × 28 μ. Appendages about 70 μ. Total length to tip of perithecium about 140–160 μ; the larger type sometimes 230 μ.

On the elytra of *Podontia lutea* Oliv., No. 2507, Hong Kong; No. 2508 on *P. 14-punctata* Linn., Himalayas, both in M. C. Z.

Although the material of this species is abundant, no specimens are in perfect condition, the appendages apparently breaking very readily. It varies very greatly in size and form, many pigmy individuals occurring near the middle and base of the elytra, in depressions of their surface. In these small stout forms the receptacle is compacted and reduced, cell I forming a short hyaline stalk; cells III and IV prominent at the side; while the rest are hardly distinguishable from the base of the stout, small perithecium. The material from *P. 14-punc-*

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tata is in part similar to that from *P. lutea*, but one specimen of the latter species bears a distinctly larger, rather slender, straight form, in which the twist of the wall cells of the straight perithecium is more distinct; so that its tip is nearly always seen at right angles to the normal position, and appears broad and flattened and quite unlike the more typical form, which occurs on other specimens of the same species. The form is most nearly related to *L. orientalis*, the fundamental characters of its appendages being very similar. The latter may prove to be longer than above indicated when perfect specimens become available for examination.

Laboulbenia Diabroticae nov. sp.

Perithecium relatively large, its basal cells and basal wall-cells concolorous with the receptacle, dirty yellowish, the rest rich translucent brown, except the hyaline outer half of the apex, the right anterior lip-cell forming distally and externally a more or less distinct truncate projection, its fellow usually shorter and rounded. Receptacle variable, stout, sometimes short, sometimes elongate through the elongation of cells I and II; cell III + IV relatively small, bulging somewhat externally; cell VI nearly as large as cell III + IV, sometimes larger; the basal cells of the perithecium distinct above it; all the cells distinguished by more or less evident constrictions. Insertion-cell nearly opaque, as are the bases of the appendages; basal cell of the outer appendage bearing a double row of close set radially disposed branches; which may be twice branched, hyaline within or deeply suffused throughout, some or all of their tips bent abruptly outward or sidewise, or strongly recurved, the curved portion tapering to a blunt point; the branches of the inner appendage similar to those of the outer; the branchlets as a whole successively longer from without inward, seldom reaching to the tip of the perithecium, the septa oblique. Perithecium 100–225 × 20–28 μ. Spores about 50 × 5 μ. Appendages, longest 120–140 μ. Receptacle to insertion-cell, longest, 265 μ; average 175 × 35 μ. Total length to tip of perithecium 300–390 μ.

On elytra of *Diabrotica Fairmairei* Baly (Types), No. 1771, Biologia Coll., Jalapa, Mexico: of *Diabrotica* sp., No. 2471, Port of Spain, Trinidad: on legs of *Diabrotica* sp., No. 1641, Los Amates, Guatemala, (Kellerman).

This well marked species varies considerably in the dimensions of

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its perithecium and receptacle. In the Mexican material the latter is comparatively short and stout, while the former is much elongated; but in the Guatemala material the reverse is the case. What appears to be an immature condition of the same species was also found on a species of *Luperodes* from Porto Velho, Amazon, collected by Mr. Mann, but none of the specimens have developed perithecia. The hooked terminations of the branches in this form are somewhat different from the ordinary type.

Laboulbenia Monocestae nov. sp.

Receptacle relatively very small and compact, the basal cell usually quite hyaline, often curved, as large as the remainder of the receptacle combined; cell II and III + IV deep contrasting translucent brown, the former very broad and short, obliquely separated from cell I as well as from cells III + IV and VI; which is obliquely placed, subtriangular, hyaline or partly suffused; cell III + IV mostly suffused, broad and short, externally prominent; cell V very small, triangular, its upper surface partly free between the base of the perithecium and the insertion-cell. Basal cells of the perithecium hyaline and contrasting, as are the lower external wall-cells of its main body; which is elongate, relatively very large, more or less curved toward the appendages, dark rich brown, hardly inflated, tapering very slightly at the broad apex; which is subtruncate or slightly rounded; the lip-edges slightly suffused. Insertion-cell thick, narrow, opaque and indistinguishable from the externally blackened basal cell of the outer appendage, which is variably developed through proliferation that takes place distally from the inner side and results in a variably developed crest-like series of from three to eight radially placed branches, each usually once, less often twice branched, externally blackened as are the outer branchlets, except distally; the lower septa all suffused; all the branchlets curved toward or past the perithecium, rather stout, slightly tapering: basal cell of the inner appendage brown, erect, longer than broad, bearing distally a pair of short branches usually consisting of a single cell terminated by a pair of antheridia. Perithecia 85–125 × 25 μ. Receptacle 50–55 × 25–30 μ. Appendages, longest, 175 μ. Total length to tip of perithecium 140–175 μ.

On the legs of *Monocesta atricornis* Clk., Manaos, Amazon, (Mann) No. 2221.

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A species well distinguished by its contrasting coloration, minute receptacle and large dark perithecium, which is characteristically hyaline externally at the base. The material is abundant and in good condition and does not seem to vary towards any of the varieties of *L. Homophoetae* which is its nearest ally.

Laboulbenia armata nov. sp.

Hyaline becoming faintly olivaceous, except the basal cell. Receptacle much smaller than the perithecium, the hyaline basal cell larger than the rest of the receptacle, from which it is separated by a horizontal septum; cell II about as large as cell III + IV, rounded, separated by symmetrical and oblique septa from cell III + IV and cell VI, which is small and somewhat oblique; cell V small, lying opposite the insertion-cell, against the base of the perithecium, the inner half of which it forms. Insertion-cell narrow, opaque and indistinguishable from the basal cells of the appendages. Basal cell of the outer appendage small, quite opaque, longer than broad; producing a terminal and a subterminal inner branch, each usually once branched; the outer branchlet of the terminal branch short, curved outward, externally blackened; the other branchlets rather long, slightly flexuous and tapering; basal cell of the inner appendage very small, rounded, prominent, hyaline, free, bearing a short branch of two or three small cells on either side. Perithecium diverging from the appendages, relatively large long and hardly inflated, wholly free; the tip well distinguished on both sides, darker; the apex paler, surmounted by an outcurved, purplish, tooth-like external or sublateral process, formed by one of the lip-cells; the other lip-cells hyaline, somewhat prominently rounded and subtending this process on the inner side; the blackened insertion of the trichogyne usually distinct on the left side of the purplish brown tip. Perithecium 120–125 × 24–30 μ; the horn-like process about 12–14 μ long. Receptacle 70–80 × 25 μ. Appendages, longest, 175–210 μ. Total length to tip of perithecium 175–210 μ.

On the elytra of *Oedionychus sublineatus* Jac., No. 1772. (Biologia Coll.) Teapa, Mexico.

The appendages of this species are fundamentally similar in the origin and arrangement of their branches to those of *L. Homophoetae*, but the form is otherwise quite peculiar.

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Laboulbenia Homophoetae (Speg.).

Laboulbeniella Homophoetae Speg. Cont. al Est. d. l. Laboulbeniomycetas Argentinas, p. 191.

The material of this species studied by Spegazzini appears to have been in very bad condition; but although it is not possible to form an accurate idea of the appendages from the figures given, there can, I think, be little doubt that the forms which I have assembled under this name are correctly referred. The species is the commonest one which is found on Chrysomelidae, with the possible exception of *L. Bruchii*, and inhabits a considerable variety of hosts on which it is subject to many variations. In all cases the basal cell of the outer appendage proliferates one to several times distally from the inner side, producing a variable number of branches which are almost invariably once branched above their basal cells in a characteristic fashion, and which are very variably developed as to length, number, curvature etc. The size, development, and color of the receptacle and perithecium also vary very greatly; but viewing the series of variations as a whole, I have been unable to discover sufficient grounds for even varietal separation, and have concluded to assemble all the forms having this type of appendage under a single name, although I have separated one type, *L. cristatella*, which appears to be sufficiently constant, and in which the branches of the outer appendage are always simple. Although the position of growth in this last mentioned form does not appear to affect the character of individua's, it produces a marked effect in the case of most of the variations of *L. Homophoetae*, in which individuals growing on the under surface of the host are usually characterized by a distinctly more luxuriant development, especially of the appendages, the branches of which are apt to be more numerous, longer, and stouter than in individuals which are found on the elytra, although the longest individuals seen, measuring 500 μ to the tip of the perithecium were taken from the extremities of the elytra. This is especially marked in the very abundant material obtained from *Systena 5-littera*. Individuals from this host growing on the inferior surface, develop a fan-like series of stout incurved branches; while on the elytra a nondescript type is found, with scanty slender and irregularly curved branches; and the same is true to a somewhat less marked degree in individuals obtained from corresponding positions on species of *Homophoeta*, although the forms on these hosts present minor differences. Of all

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the variations which have been examined that which occurs on *Asphaera nobilitata* in Trinidad is the most striking, and would undoubtedly be referred to a distinct species were there not variations which occur on other species of the genus which approach so closely to the type form that even a varietal separation seems undesirable. This Trinidad form is very large, and grows either on the elytra or at the base of the anterior legs of its host, its color is paler than that of individuals from other species of *Asphaera*. The perithecium is distinguished by an often well developed neck-like base formed from the lower tier of wall-cells, and is usually bent abruptly outward from the appendages. The latter are very stout and long. Larger specimens have the following measurements: perithecia 225 × 35 μ; appendages 480 μ; receptacle 280 × 40 μ; total length to tip of perithecium 400 μ. In this and several other forms the cells, of the receptacle, especially, contain a distinctly yellow or orange protoplasm, the color of which is apparently due to the peculiar yellow juices which fill the bodies of many of the Chrysomelidae.

The hosts on which forms referable to *L. Homophoetae* have been found are as follows:

(a) *Homophoeta* sp., No. 2475b and *H. aequinoctialis* Linn., No. 2475, Port of Spain, Trinidad: on *H. 6-gutta'a* Say, No. 1791, Brazil;

(b) *Systena* sp., No. 1640, Los Amates, Guatemala (Kellerman); on *S. basalis* Jac., Nos. 2489 and 2490, Hayti (Mann): on *S. 5-littera* Linn.; Nos. 2469 and 2470, Port of Spain, Trinidad; No. 2479, Suriname (Rorer); No. 1769, Teapa, Mexico (Biologia Coll.).

(c) *Psylliodes* sp., No. A+, Jamaica (Brues).

(d) *Disonycha* sp. No. 1843, Guatemala (Kellerman); on *D. recticollis* Jac., No. 1769, Costa Rica (Biologia Coll.) and No. 1642, Guatemala (Kellerman). A dark form with numerous short rather slender curved appendages hardly ever reaching to the tip of the perithecium, their inner margins conspicuously indented at the septa.

(e) *Oedionychus sublineata* Jac., Teapa, Mexico, No. 1772, (Biologia Coll.). A paler form, straight, with few straight appendages, the largest specimen measuring 500 μ to the tip of the perithecium.

(f) *Monocesta atricornis* Clk., No. 2221, Manaos, Amazon, (Mann). A small dark form with few very long scanty appendages, the receptacle small, the perithecium bent abruptly against the appendages which may reach 425 μ in length, although the total length to the tip of the perithecium is only about 120 μ.

(g) *Lactica scutellaris* Oliv., No. 1754, Balaclava, Jamaica, W. I. A form similar to that on (f) but with straight perithecium.

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(h) *Asphaera nobilitata* Fab., No. 2472, Port of Spain, Trinidad; *A. transversofasciata* Jac., Bugaba, Mexico, (Biologia Coll.), No. 1773; *A. elegantissima* Schf., No. 2218, Rio Madero, Amazon (Mann); *A. Siebersii* Ill., No. 2211, Para, Brazil (Mann).

Laboulbenia cristatella nov. sp.

Hyaline, becoming suffused with olivaceous brown. Receptacle small; cell I larger than cell II, which is somewhat roundish, its margins more or less convex; cells III + IV much smaller than cell II; cell V small and narrow, often partly free above; cell VI small, flattened, and oblique. Insertion-cell narrow, opaque, indistinguishable from the basal cell of the outer appendage. Basal cell of the outer appendage black below and externally, broad and quite hyaline above, bearing externally and distally a series of branches radially placed; the outer (lowest) branch furcate, its basal cell obliquely hyaline within, externally black, the two branchlets broadly blackened externally, except distally, the outer wholly so at the base; the remaining branches always simple, stout, usually tapering somewhat toward the base and apex, broadly blackened externally, except distally; usually curved toward and beyond the perithecium, forming a crest-like series which exceeds the latter in length: basal cell of the inner appendage very small, brown, bearing a short simple branch on either side both two-celled, the lower cell deeply suffused, the upper hyaline, separated from it by a dark septum and bearing usually a pair of relatively large long antheridia, sometimes only one. Perithecium not quite free, straight, becoming dark olive brown, very slightly inflated; the tip broad, well distinguished, blackened distally, especially on the inner side; the apex with broad, rounded, rather prominent lip-cells slightly oblique inward; the outer hyaline, the inner deeply blackened, except the narrowly hyaline papillate prominent edge. Perithecia 60–76 × 18–22 μ. Receptacle 50–64 × 22–28 μ. Appendages longest 125 μ. Total length to tip of perithecium 100–140 μ.

On inferior surface of *Haltica scutellata* Oliv. No. 2473, Port of Spain, Trinidad (Type). On *Asphaera Siebersii* Ill. No., 2233, Para, Brazil, (Mann). On elytra of *Lactica nigriceps* Boh., Para, Brazil, (Mann).

The above description is drawn from the Trinidad material, which is abundant and in good condition. The material from Brazil is in

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both instances somewhat smaller, the branches of the appendages mostly shorter, hardly reaching to the tip of the perithecium and usually swollen, rather than tapering at the tip. The species is very closely allied to *L. Homophoetae* from which it is distinguished by the simple branches of its outer appendage, by its small size and other minor differences. It may prove however, to be merely a variety of *L. Homophoetae*.

Laboulbenia funebris nov. sp.

Dull olivaceous, becoming deeply tinged with brown. Perithecium somewhat less than one half free, becoming almost opaque, except for a hyaline area on the inner side just below the deeply blackened extremity; the lip-cells hyaline about the pore, turned somewhat outward; the outer more prominent, rounded, tipped by minute papillae: the outer margin directly continuous with that of the receptacle, hardly convex, bent inward abruptly at the tip; the inner straight or slightly convex. Receptacle symmetrically broader from below upward; the basal cell hyaline below, subtriangular; the subbasal cell somewhat longer, separated from cells III + IV and VI, which is relatively large, by somewhat oblique septa; cell III + IV rather large, cell V very minute, translucent, all the upper portion of the receptacle deeply suffused, the cell-boundaries becoming hardly distinguishable, and concolorous with the perithecium. Insertion-cell broad and thick, wholly and deeply suffused; the appendage consisting of a single outer, simple, yellowish olive, rather stout tapering, but slightly divergent outer branch of six or eight cells, relatively stout; the basal cell of the inner appendage bearing two shorter, smaller, simple branches, of usually not more than three or four cells, the subbasal bearing distally a single antheridium. Perithecium 75–80 × 25 μ. Spores 45 × 5 μ. Appendage, longest 140–150 μ. Total length to tip of perithecium 125–160 μ, greatest width 35 μ.

On the elytra of species of *?Haltica*. Mus. Comp. Zool. No. 1790, (no locality); No. 1841, Guatemala, (Kellerman).

In general appearance this ordinary looking form is not unlike the smaller specimen of *L. rigida* figured in my first Monograph the appendages being very similar. It also resembles some forms of *L. vulgaris* and of *L. polyphaga* but cells III and IV are always replaced by a single cell.

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Ceraiomyces.

More than a dozen species of this type are now available for comparison and an examination of them indicates that the genus is so closely allied to *Laboulbenia* that it may possibly have to be united with it eventually; although for the present, at least, it seems more convenient to retain the name. The cell spoken of in the original description as the "stalk-cell of the appendage," which corresponds to cells III to V in *Laboulbenia*, is, in some of the species, closely united to the perithecium, as in the West Indian forms described below; and it is therefore necessary to modify the original diagnosis based on two species in which this cell was quite free. Since the type may be regarded as corresponding to a reduced form of *Laboulbenia*, it has seemed best to refer to this cell as "cell III" of the receptacle; it being understood, however, as has been formerly pointed out, that not only this cell, but the three corresponding cells in *Laboulbenia* above alluded to, belong strictly to the appendage. The latter, in the species described below, is, for the most part, greatly reduced; its cells becoming more or less obliterated, the antheridia alone being distinctly visible. In some young individuals, however, the fundamental structure is very like that of the most simple types in *Laboulbenia;* there being outer and inner cells which give rise to corresponding sets of branches, both of which always bear antheridia. It may be mentioned further that all of the following species are characterized by possessing a normal foot, and in no instance penetrate the host by means of a haustorium.

Ceraiomyces Epitricis nov. sp.

Basal and subbasal cells of the receptacle subequal, nearly hyaline, the former slightly suffused with brown just above the foot, the two together not quite as long as the perithecium; cell III clearly defined several times as long as broad, of nearly the same diameter throughout, concolorous with the perithecium. Appendage consisting of a small basal cell obliquely inserted on a dark basal septum and bearing on the inner side two large antheridia subtended by minute cells; and externally a single antheridium, less often two, subtended by a larger cell; the antheridia becoming slightly suffused with brown. Perithecium three fifths or more free above the insertion of the appendage, translucent brown, except the small clearly defined flattened stalk-cell;

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the basal cells very small, but clearly defined; the lower third united to cell III; the inner margin above it vertical, straight as far as the insertion of the trichogyne, above which the tip bends inward more or less distinctly; the outer margin abruptly bent opposite the insertion of the appendage, forming a considerable angle, so that the body appears to be bent inward; the portion of the margin above this angle running straight to the hardly distinguished tip; the somewhat compressed hyaline apex subtended externally by a distinct rounded hyaline prominence; the pore terminal; the lip-cells not prominent. Perithecium 80–90 × 32 μ. Basal and subbasal cells of receptacle 50–70 × 16 μ. Appendage, including antheridia, 30–32 μ. Cell III, 28 × 7 μ. Total length to tip of perithecium 140–160 μ.

On elytra of *Epitrix convexa* Jac., No. 2462, Port of Spain, Trinidad.

In general form this species most nearly resembles *C. Chaetocnemae* from which it is easily separated by the form of the perithecium and the peculiar external prominence which subtends its apex.

Ceraiomyces obesus nov. sp.

Cells I and II faintly brownish, distinguished by a constriction; the former more than twice as large as the latter, its lower half narrowed to the foot; cell III concolorous with the perithecium, united to it throughout, and extending nearly to its middle. Appendage arising from an oblique insertion, marked by a dark septum, hyaline, consisting of two or three very small hardly distinguishable cells bearing usually two inner and one outer antheridium. Perithecium somewhat more than half free above the insertion of the appendage, arising obliquely and externally from cell II, which it almost conceals when not viewed laterally, its outline almost symmetrically long-oval: uniform rich translucent brown, tapering to the rather broad, blunt, slightly oblique apex; the hyaline lip-edges bearing each a minute but distinct papilla, and subtended by darker shades. Perithecium 106–112 × 52–60 μ. Appendage including antheridia 18 μ. Receptacle, cells I–II, 52 × 18 μ; cell III, 36 × 10 μ. Total length to tip of perithecium 140–160 μ.

Near base of anterior legs of *Epitrix convexa* Jac., Port of Spain, Trinidad, No. 2464.

This species is distinguished by its relatively very large, nearly symmetrically oval perithecium, the axis of which is at a considerable angle to that of the receptacle, when viewed laterally.

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Ceraiomyces minisculus nov. sp.

External margin, from tip of foot to apex, almost an arc of a circle, the corresponding opposite margin almost straight except for the slight protrusion of cell III. Basal cell of the receptacle triangular, about as broad as long, yellowish; the subbasal cell flattened distally, oblique below the base of the perithecium with which it is concolorous and from which it is hardly distinguishable; cell III longer than the portion of the receptacle below it, very narrow and hardly distinguishable from the body of the perithecium. Cells of the appendage minute, only one being distinguishable, and bearing two relatively large brownish antheridia, the outer spinose externally below its neck. Perithecium somewhat less than half free above the insertion of the appendage, relatively large, stout, very deep brown, barely translucent, the apex blunt, the hyaline lip-edges subtended by blackish shades. Perithecium 70 × 26 μ. Cells I–II, 18 μ long by 21 μ broad; cell III, 30–32 × 4 μ. Appendage, including antheridia, 22 μ. Total length to tip of perithecium 90–95 μ.

On antennae of *Chaetocnema nana* Jac., No. 1755, Balaclava, Jamaica, W. I.

This minute species is most nearly related to *C. obesus* from which it differs in the structure of its receptacle, and the form and position of its perithecium. The appendage is so reduced that little remains beside the antheridia.

Ceraiomyces dislocatus nov. sp.

Cells I and II nearly hyaline or becoming faintly yellowish, the former about five times as long as it is broad, abruptly enlarged above the small pointed foot, distally bent abruptly inward; the septum separating it from cell II thus more or less oblique and sublateral, marking a constriction, and giving to this part of the receptacle a more or less distinctly geniculate habit; cell II roughly isodiametric, bulging slightly externally, small, separated obliquely from the base of the perithecium; cell III relatively broad and distinct, concolorous with the perithecium. Appendage seated on a dark, somewhat oblique septum, hyaline; its two or three cells very small and indistinguishable at maturity; bearing two inner and one or rarely two outer slender faintly brownish antheridia. Perithecium more than half free above the insertion of the appendage, deeply suffused with

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blackish brown, darker below the apex, the outer margin broken by a slight basal, a median and a subterminal rounded elevation; the latter, together with a corresponding elevation on the inner side, rather clearly distinguishing the tip which is bent distinctly toward the appendage; the anterior (inner) and left lip-cells more prominent, somewhat displaced by a slight twist, separated by furrows, terminating in minute flattish hyaline papillae; the other two lip-cells shorter and rounded, so that the apex is asymmetrical. Perithecium 80–90 × 35 μ. Cell I 50–85 × 15 μ; cell II about 17 × 18 μ; cell III 24–28 × 7 μ. Appendage, to tips of antheridia, about 25 μ. Total length to tip of perithecium 125–175 μ.

On the mid-inferior surface of the abdomen of *Chaetocnema minuta* Mels., No. 2460; Port of Spain, Trinidad.

A species very clearly distinguished by its geniculate receptacle, and distinctive perithecium.

Ceraiomyces Trinidadensis nov. sp.

Cells I and II hyaline; the former abruptly bent, more than twice as long as the subtriangular subbasal cell which is obliquely separated from the base of the perithecium and from cell III; which is clearly defined, concolorous with the perithecium, bulging slightly below the nearly horizontal, relatively broad insertion of the appendage. Appendage relatively large, the two cells of the outer branch hyaline, distinct, subequal, bearing usually two large antheridia which equal them in length. Perithecium translucent, blackish olive-brown, rather narrow, nearly symmetrical in outline, its upper half, or somewhat less, free above the insertion of the appendage; tapering distally to the broad subhyaline extremity, which is rounded and slightly sulcate and but slightly asymmetrical; the stalk- and basal cells hardly distinguishable. Perithecium 80–88 × 28 μ. Cell I, 28–35 × 18 μ; cell II, 18 μ; cell III, 28 × 10 μ. Appendage, including antheridia, 35 μ. Total length to tip of perithecium about 125 μ.

On the legs of *Epitrix convexa* Jac.; No. 2459, Port of Spain, Trinidad.

Though this species is not distinguished by any single striking peculiarity, it differs distinctly from any of the others by the form of its perithecium which ends in a broad blunt hyaline apex. It is most nearly related to *C. Chaetocnemae*, a larger more slender form.

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Ceraiomyces Chaetocnemae nov. sp.

Straight or but slightly curved, relatively long and slender. Cells I and II hyaline, becoming tinged with pale reddish yellow; cell I often somewhat elongate, usually somewhat more than twice as long as the subbasal cell, from which it is separated by a nearly horizontal septum; cell III concolorous with the perithecium, well distinguished. Insertion of the appendage somewhat oblique, blackened, the latter bearing two inner antheridia and one outer, the necks of which are subtended by a conspicuous spine. Perithecium slightly less than two thirds free above the insertion of the appendage; becoming dark blackish olivaceous; the stalk- and basal cells hardly distinguishable, rather long and narrow, tapering distally; the tip becoming distinguished by slight subtending elevations; the lips tending to turn slightly outward, two of them bearing minute flattened papillae which project above the otherwise rather bluntly rounded, slightly sulcate apex. Perithecium 90–116 × 38–42 μ. Spore 52 × 4 μ. Cells I–II, 60–122 × 22–25 μ. Cell III, 38–42 × 8–10 μ. Appendage 30 μ. Total length to tip of perithecium 160–250 μ.

On the elytra of *Chaetocnema* sp., No. 2248, Amazon, Mann. on *C. minuta* Mels. No. 2460, 2461, Port of Spain, Trinidad: on *Epitrix lucidula* Har., No. 2457, and *E. convexa* Jac., No. 2458, Port of Spain.

Although varying considerably in size, this species is much larger than any of the others which occur on Chrysomelidae, with the exception of *C. Nisotrae*. It appears to be quite rare and usually not more than one or two individuals are found together on a single host. A form apparently not separable from this species was also found in Port of Spain on a single individual of *Scolochrus* sp.

Ceraiomyces Nisotrae nov. sp.

Comparatively large, short and stout. Receptacle dirty brownish yellow throughout, obscurely punctate above the basal cell, which is short and abruptly bent, its upper half abruptly twice as broad; cell II hardly longer than broad and but slightly larger than the stalk-cell (cell VI of Laboulbenia), which lies just above it; cell III hardly extending above the perithecial cavity, and lying opposite the stalk- and basal cells, which are clearly defined and, like it, dirty yellowish brown. Insertion of the appendage slightly oblique, its basal cell almost wholly suffused with dark brown; basal cell of the outer branch

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usually bearing two branchlets, one two-, the other one-celled, each terminated by a very long slender antheridium; the inner branch single as a rule, consisting of a rather elongate cell terminated by an antheridium; the antheridia seated on blackened septa, below which a brown suffusion extends downward. Perithecium relatively large, distally somewhat broader; free except its basal and stalk-cells, deeply suffused with dark brown, faintly translucent; the external margin straight, or usually slightly concave; the inner convex and curved inward abruptly to form the tip, which is usually twisted somewhat less than one quarter; so that the outer lip, which is modified to form a brown, rounded projection, subtended on either side by curved ear-like processes, is usually almost lateral in position; the rest of the apex almost symmetrically rounded, somewhat inflated, sub-hyaline, contrasting with the basal portion of the tip which is concolorous with the body of the perithecium. Perithecium 100–125 $\times$ 35–40 μ. Spores 40 $\times$ 4 μ. Cells I–II about 50 $\times$ 22–24 μ. Cell III, 24–28 $\times$ 8–10 μ. Total length to tip of perithecium, average 175 μ, longest 200 μ.

Appressed along a ridge parallel to the outer margin, usually of the left elytron, of *Nisotra* sp., No. 2481, Kamerun, West Africa (Schwab): on *N. Chapuisi* Jac., Madagascar, M. C. Z., No. 2504.

The differences which separate this species from all other forms are so great that they need hardly be pointed out, the peculiar outgrowths of the lip-cells being in themselves sufficient to distinguish it. The Madagascar material, although inhabiting a smaller and very different species of the host-genus, corresponds in all essentials with specimens from Kamerun.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE DEMAGNETIZING FACTORS OF CYLINDRICAL RODS IN HIGH, UNIFORM FIELDS.

By B. Osgood Peirce.

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THE DEMAGNETIZING FACTORS OF CYLINDRICAL RODS IN HIGH, UNIFORM FIELDS.

By B. Osgood Peirce.†

Presented by E. H. Hall, March 11, 1914. Received March 5, 1914.

Any one who has been compelled to measure the permeability of iron under high excitations, by experiments upon relatively short rods exposed to strong magnetizing fields, must have had occasion to notice that, under these circumstances, the corrections for the effects of the self-demagnetizing forces are often very small. In the case of an ellipsoidal specimen, it is possible to predict the phenomena which one encounters in practice,[1] and although it is not easy to make long ellipsoidal test rods with accuracy, there is a sufficiently close analogy between the two cases to make the general conclusions reached for ellipsoids applicable to the cylindrical rods usually employed in the laboratory.[2]

It is well known that if an ellipsoid of soft, magnetizable material and of fixed, constant susceptibility, k, were placed in a uniform magnetic field, H, it would become uniformly magnetized by induction and that the components of the field within it would be

$$H_x/(1+2\pi abckK_0), \quad H_y/(1+2\pi abckL_0), \quad H/(1+2\pi abckM_0),$$

where a, b, c, are the lengths of the semiaxes,

$$K_0 = \int_0^\infty \frac{ds}{(s+a)^{3/2}\cdot(s+b)^{1/2}\cdot(s+c)^{1/2}}, \tag{1}$$

and L_0, M_0, have corresponding values.

† Deceased January 14, 1914.

1 Rayleigh, Phil. Mag., **22**, 1886; Maxwell, Vol., II; Webster, Elect. and Mag., §§ 192, 196; Peirce, N. P. F.; Yosida, Tokyo, Proc., **3**, 8, 1906; Mallik, Phil. Mag., **14**, 1907; **15**, 1908; Daniele, N. Cimento, **2**, 6, 1911; H. E. J. G. duBois, Deutsch. Phys. Gesell., Verh. **15**, 8, 1913.

2 Ewing, Phil. Mag., **176**, 1885; Tanakadaté, Phil. Mag., **26**, 1888; H. E. J. G. duBois, Phil. Mag., **29**, 1890. Wied. Ann., **46**, 1892; Rossler, Elektrotechnische Zeitschrift, **14**, 1893; Mann, Dissertation Berlin, 1895. Phys. Rev., **3**, 1896; Ascoli e Lori, Rendic. R. Acad. d. Lincei, **3**: 2, 1894; **6**: 2, 1897; J. L. W. Gill, Phil. Mag., **46**, 1898; Morin, Écl. Électr. **15**, 1898; Holborn, Sitzungsbericht d. Akad. Wiss. zu Berlin, **1**, 1898; Benedicks, Wied. Ann., **6**, 1901; Bihang Svenska V.-Akad. Handlinger, **27**, 1902; H. E. J. G. duBois, Wied. Ann., **7**, 1902; Gumlich, Elektrotechnische Zeitschrift, **22**, 1901; **30**, 1909; Peirce, These Proceedings, **44**, 1908; Am. Journal of Science, **28**, 1909; Shuddemagen, These Proceedings, **43**, 1907; Peirce, Ibid., **49**, 1913.

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If two of the semiaxes (b, c) were equal, and if the third axis had the direction of the outside field, H, which had been chosen also for the direction of the x axis, the field in the ellipsoid would agree with H in direction, and we might write,

$$K_0 = \frac{2}{e^2 \cdot a^3}\left(\frac{1}{2e} \cdot log\frac{1+e}{1-e} - 1\right), \tag{2}$$

where $e^2 = (a^2-b^2)/a^2 = (a^2-c^2)/a^2$. The ratio of the intensity of the field, H', within the ellipsoid to that of the exciting field, H, would be

$$\frac{1}{(1+2\pi abckK_0)} \tag{3}$$

Table I shows the numerical values of a^3K_0 and of $1 + 2\pi abckK_0$ for several different values of the ratio $m = a/b$.

TABLE I.

m	$a^3 \cdot K_0$	$1+2\pi abckK_0$
30	6.198	$1 + 0.0433k$
40	6.771	$1 + 0.0266k$
50	7.215	$1 + 0.0181k$
60	7.575	$1 + 0.0132k$
80	8.150	$1 + 0.0080k$
100	8.597	$1 + 0.0054k$
140	9.270	$1 + 0.0030k$
200	9.983	$1 + 0.0016k$

Even though it be not possible to realize these conditions exactly in practice, and a relatively slight departure from a truly ellipsoidal form in the testpiece may alter the conclusions appreciably, yet the measurements of several observers who have used such nearly ellipsoidal rods as they were able to procure, show a fairly close agreement with this theory, and we shall find it instructive to examine the numerical results obtained by applying it in the cases of one or two kinds of soft iron and steel to be bought in the market.

Some time ago, Mr. J. Coulson and I examined with great care a long rod of soft Bessemer steel in a uniformly wound solenoid of 20904 turns, 4.85 meters long, as well as shorter specimens of the same material between the poles of a massive soft iron yoke. Table II gives very approximately for this steel and for various values of H, the values of the susceptibility; of the fractional increase in the

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induction, B, due to an increase of one unit in the exciting field, H, in the metal; and of the fractional change in B due to a change in H of one percent of its own value. These quantities are denoted by k, X, Z, respectively.

TABLE II.

H	k	X	Z
30	38.3	0.00558	0.0017
40	30.1	0.00386	0.0016
50	24.9	0.00277	0.0014
60	21.3	0.00227	0.0014
80	16.5	0.00155	0.0013
100	13.6	0.00128	0.0013
120	11.0	0.00108	0.0013
160	9.0	0.00082	0.0013
200	7.4	0.00067	0.0013
300	5.2	0.00046	0.0013
400	4.1	0.00032	0.0013
500	3.3	0.00024	0.0012
800	2.2	0.00014	0.0011
1000	1.7	0.00011	0.0011
1500	1.1	0.00005	0.0008
2000	0.9	0.00004	0.0009
2500	0.7	0.00004	0.0010
(5000)	(0.34)	(0.00004)	(0.0019)

From the numbers in the last column of Table I, and those in the second column of Table II, it is easy to compute the ratio of the intensities of the magnetizing field (H') in the metal, and the external exciting field (H), for different values of m. The results of this procedure for $a/b = 30$, a/b, $= 50$, and $a/b = 100$, appear in Table III.

TABLE III.

H'	H'/H, *if* $a/b = 30$,	H'/H, *if* $a/b = 50$,	H'/H, *if* $a/b = 100$
30	0.377	0.591	0.828
40	0.435	0.648	0.861
50	0.482	0.689	0.882
60	0.521	0.718	0.897
80	0.584	0.770	0.918
100	0.630	0.803	0.932

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TABLE III — *Continued.*

H'	H'/H, *if* $a/b = 30$,	H'/H, *if* $a/b = 50$,	H'/H, *if* $a/b = 100$
120	0.666	0.826	0.941
160	0.720	0.862	0.955
200	0.758	0.880	0.962
300	0.816	0.914	0.973
400	0.850	0.933	0.978
500	0.876	0.943	0.982
800	0.913	0.962	0.988
1000	0.932	0.970	0.991
1500	0.955	0.981	0.994
2000	0.963	0.984	0.995
2500	0.970	0.987	0.996
(5000)	(0.985)	(0.994)	(0.998)

A comparison between the figures given in this table and the values of Z in Table II shows that, in the case of an ellipsoidal rod only 30 diameters long, the density of the flux of magnetic induction through the metal when the exciting field is as high as 2500 gausses, is not so much as one third of one per cent less than the corresponding flux density for an infinitely long rod. When the ellipsoid is 50 diameters long, the flux density for 2500 gausses does not differ by so much as one eighth of one per cent from the flux density in an infinitely long specimen under the same excitation, and only an extremely good determination of B is correct within this fraction when the value of B is above 25000.

It is interesting to note that if, under excitations above, say, 2500 gausses, we may assume the intensity of magnetization, $I = kH$, to be constant, so that $B - H$ is constant; dB/dH is unity and $\left(\frac{H}{B} \cdot \frac{dB}{dH}\right)$ increases somewhat with H.

In the case of the Bessemer steel mentioned here, where $I_\infty = 1694$, Z is 0.00190, for $H = 5000$, while Z is 0.00484, if H is 20000.

Table IV gives results obtained by Mr. Coulson and myself from a long series of tests upon a special brand of Norway Iron unusually permeable at high excitations.

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TABLE IV.

H.	k.	Z.	$(H'/H)_{30}$	$(H'/H)_{50}$
400	4.4	0.0010	0.841	0.922
500	3.5	0.0009	0.869	0.940
600	2.9	0.0006	0.889	0.950
700	2.4	0.0005	0.909	0.957
800	2.2	0.0004	0.913	0.961
900	1.9	0.0004	0.926	0.965
1000	1.8	0.0005	0.931	0.968
1500	1.2	0.0006	0.951	0.978
2000	0.9	0.0008	0.962	0.984
2500	0.7	0.0010	0.970	0.987

In the case of this iron, the flux density, under an excitation of 2500 gausses, would be less than for an infinitely long rod, by about one third of one per cent, for $a/b = 30$, and, by about one eighth of one per cent, for $a/b = 50$.

It has long been known that, although cylindrical rods of soft iron do not become uniformly magnetized when they are exposed longitudinally to uniform magnetizing fields, yet they behave in many other respects much like the ellipsoidal pieces which are more easily subjected to analysis.

A glance at the B vs. H curves obtained thirty years ago by Ewing from an iron wire 0.158 cms. in diameter and originally 47.5 cms. long, shows that under very high excitations the flux through the central cross section of a comparatively short piece of this wire would have been much the same as the corresponding flux through a specimen several hundred diameters long, but the demagnetizing factor in the case of such a rod seems to be a function of both the diameter and the length and not a function of the ratio of the two alone, and although at low excitations the numbers given by DuBois and by Shuddemagen are most useful, it is not easy to compute with certainty just how long a specimen must be in order that the flux through its meridian section may be assumed to be unaffected by the nearness of the ends of the rod. It happens that Mr. John Coulson and I, who have been studying the maximum value of I in different kinds of iron, have had occasion to determine the magnitude of the influence of the ends of some rods which we have been using in fields of 2500 gausses and upwards, and we have found that we might have used much shorter test pieces and a much less massive solenoid than we have

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employed in our work. Some of our observations are recorded briefly here in the hope that they may prove useful to other experimenters.

Figure 1 shows the general arrangement of the apparatus used in making the measurements described in this paper. Figure 2 indicates the forms of three of the several standards of mutual inductances which we used for calibrating the galvanometers. For a complete discussion of the apparatus used throughout this work reference is made to a preceding paper,[3] since the apparatus used for both investigations was essentially the same.

Our method of work was not new in any particular, but the necessity of using heavy currents and, therefore, of taking care of the heat equivalent of many kilowatts in our circuit, and the use of ballistic galvanometers with periods so long that a reversal of current in the highly inductive circuit could be accomplished before the galvanometer coil had moved appreciably from its position of rest, introduced many difficulties which could be overcome only after much anxious experimentation.

The large solenoid with which most of our work at high excitations was done, was made of about 300 kilograms of triply covered No. 10 copper wire wound uniformly, with great care, by Mr. George W. Thompson, the mechanician of the Jefferson Laboratory, upon a massive brass spool, 186.2 centimeters long, in inside measurements. There are two coils, one of 8117 turns in 14 layers, and the other, of slightly different wire, of 5872 turns in 10 layers. The field intensity in the centre of the solenoid when a current of one ampere passes through the first coil is 54.71 gausses, while at a point fifty centimeters from the centre on the axis, the intensity is 54.60 gausses. A current of one ampere sent through both coils in series creates a field of 94.19 gausses at the centre of the coil. This solenoid was repeatedly tested for leakage between the turns by means of a very carefully made test coil without iron, but we were never able to detect any evidence of fault.

For low excitations, we sometimes used a somewhat longer solenoid wound with No. 14 wire.

The test pieces were first packed in fine iron filings in a pipe closed at the ends by caps. This was placed horizontal and perpendicular to the meridian upon supports in a furnace where it was exposed to several hundred gas jets driven by a power compressor. The dimen-

[3] Peirce, These Proceedings. **49**, 1913.

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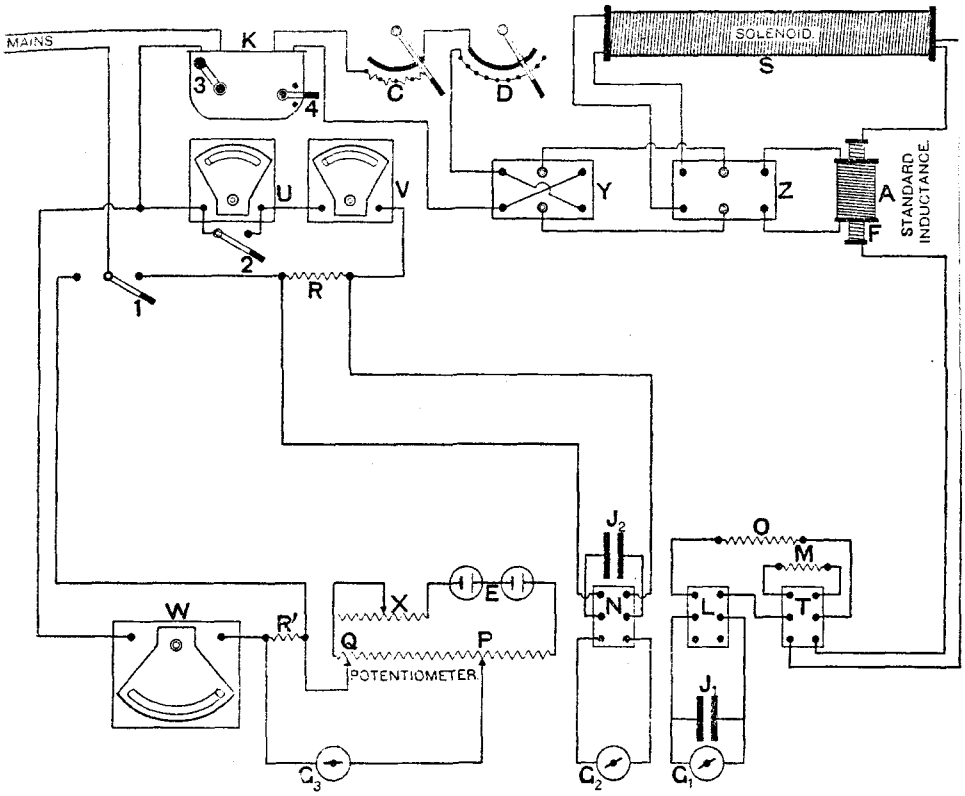

FIGURE 1. Shows diagrammatically the general arrangement of some of the apparatus used in making the observations described in this paper.

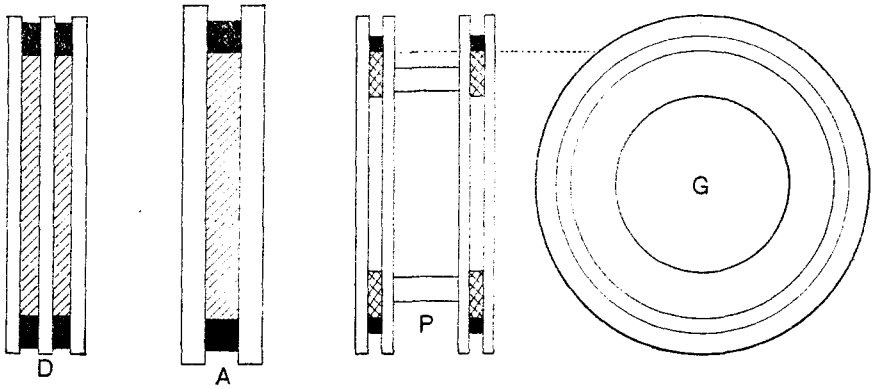

FIGURE 2. Three standards of mutual inductance.

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sions of the test pieces were obtained by aid of a set of gauges which had been tested against two comparators, one by Zeiss. The test rods received first a very thin coat of varnish and then the test coil of triply silk-covered copper wire which was in turn varnished and then heated in a stream of hot air until the whole was thoroughly dried. Usually two test coils were wound upon each rod and their indications compared, lest an injury or imperfection in one might escape notice.

Our first experiments were not so carefully carried out as the later ones, but they pointed to the same general conclusions. I need mention only two of them. In the first, we tested a rather short piece of cold-rolled shafting 1.269 cms. in diameter. This had originally a length of 79 diameters, but was cut shorter by steps to 63, 47, 32, 24, and 16 diameters, respectively. In an exciting field of 1280 gausses, the fluxes through the central section of the specimen seemed to be as 176, 175, 175, 175, 175, and 175.

The second experiment was made upon a rod of Norway Iron 1.110 cms. in diameter and successively 150, 130, 110, 70, and 30 diameters long. In this case the value obtained for the flux through the central section at the last step differed by less than one eighth of one per cent from the average value for the other steps, and as it happened this small difference was in excess. The truth is that all the values agreed within the small accidental error to be expected in the work. The magnetizing field had an intensity of 2700 gausses.

We were at first puzzled by a phenomenon which sometimes affected our results by a small fraction of one per cent. After a long day's work when the originally long rod under examination had been cut down, by a succession of steps, to a short one, and the resistance of the circuit had become a little greater than at the outset on account of the heat set free in it, we often found the same flux at very high excitations through the central section of our specimen which a slightly greater current had caused in the longer piece in the morning. This was not caused by a rise of temperature in the test piece because a vigorous flow of water of practically constant temperature was sent through the tube of the solenoid all day. It was not due to a change of sensitiveness in the ballistic galvanometer, as frequent calibrations showed.

It seems to be true that if a rod of soft iron of any length be repeatedly magnetized in a very strong field, in the one direction and in the other, alternately, a very slightly weaker field will eventually suffice, after it also has been many times reversed, to magnetize the iron as strongly as the original one.

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We thought it possible that we might avoid this difficulty by using at the same time a long piece and a short piece, cut freshly from a single rod, but experiment seemed to show that we could not expect the same permeability in two pieces obtained in this way and that the difference might easily be as much as one per cent at some excitations. We believe, however, that we have reduced the error in our conclusions due to the effect just mentioned to a very small fraction.

All our work, which lasted more than two months, pointed to the same results, and I need mention only a few representative experiments. They convinced us that in our determinations of the saturation values of the magnetization in different kinds of iron and steel, we might safely use much shorter test pieces than those which we had employed before.

Experiment *A* was made upon an annealed piece of Bessemer rod 0.635 cms. in diameter. The original length was 158 diameters, but this was cut down successively to 61, 31, and 18 diameters. At an excitation of 2600 gausses where *B* was 23530, the average of the fluxes of magnetic induction through the central sections of the various pieces, differed by less than one sixth of one per cent from any one of the individual values, and from the last value by a wholly inappreciable fraction.

Experiment *B* was made upon a Bessemer rod 0.795 cms. in diameter. The lengths of the pieces were 150, 100, 60, 40, and 20 cms. For *H* 1320, the flux through the central section of the shortest piece seemed to be about one fifth of one per cent lower than the average of the corresponding fluxes for the other pieces. At 1850 gausses and 2700 gausses, however, there was no such difference. For *H* 2570, *B* was 23700. Figure 3 shows some *B*–*H* curves for the pieces at low excitations where the effects of the presence of the ends of the rods are very apparent.

In Experiment *C*, we treated a somewhat stouter Bessemer rod. Its diameter was 0.879 cms. Its length was originally 164 diameters, but this was shortened by steps to 124, 100, 75, 45, and 24 diameters. At 2500 gausses the value of the flux through the centre of a piece only 19 cms., or 24 diameters, long, did not differ by so much as one tenth of one per cent from the mean of the fluxes for all the lengths under this excitation, where *B* was 23550. At an excitation of 1710 gausses, *B* was 22700, and it was not possible to prove that the shortest piece had a less flux than the others. Some of the results of our observations for this experiment are tabulated in Table V. *H* is the value that the field would have inside the solenoid if the iron were

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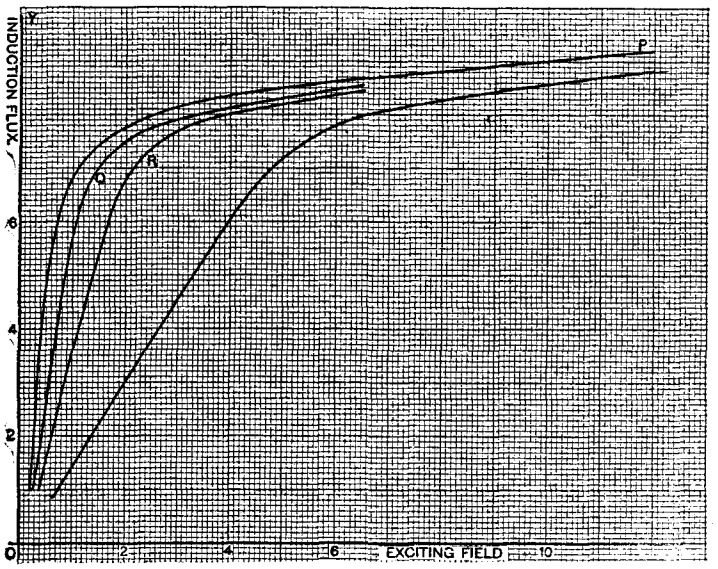

FIGURE 3 represents the B–H curves for four lengths of the same material, (experiment B). The curves OP, OQ, OR, and OS correspond to specimens of lengths 150, 60, 40, and 20 cms. respectively, plotted on an arbitrary scale. One unit on the Y axes corresponds to a B of 6000 gausses; and one unit on the X axes to a field H of 12.17 gausses.

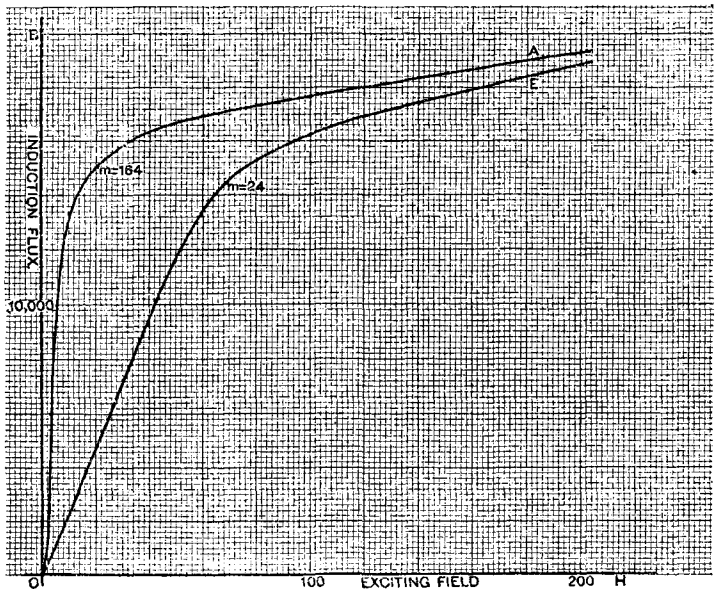

FIGURE 4 shows the B–H curves for two of the specimens used in experiment C. These curves OA and OE are plotted from Table V, for $m = 164$ and $m = 24$.

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TABLE V.

$m = 164$		$m = 124$		$m = 75$		$m = 45$		$m = 24$	
H	B	H	B	H	B	H	B	H	B
1.00	790								
2.70	2980								
3.92	6020								
5.35	8930	5.27	7250	5.27	4600	5.23	1593	5.23	1000
5.90	9770	7.67	10640	7.67	7250	7.61	4240	7.63	1600
7.72	11330	8.60	11600	8.40	8400	9.80	5600	9.60	2000
10.50	13330	9.60	12200	9.80	10000	11.8	7000	11.56	2480
17.00	13760	11.56	13320	11.56	11030	13.6	8400	13.5	3000
14.80	14440	14.00	14090	13.50	12200	14.9	9410	15.0	3400
18.90	15050	16.92	14620	16.85	13820	17.2	10800	16.8	3780
22.30	15380	20.00	15000	21.97	14980	21.8	12590	21.9	5000
24.75	15550	24.10	15420	26.0	15400	30.0	14840	32.8	7800
43.7	16430	42.6	16430	43.1	16490	42.6	16050	42.6	10120
85.1	17400	78.0	16920	52.0	16720	52.0	16400	60.0	13460
110.5	17890	106.0	17800	70.0	17140	70.0	16960	85.0	15700
127.0	18200	122.0	18100	113.9	17960	100.0	17600	100.0	16400
138.0	18400	136.0	18360	136.0.	18380	136.0	18280	118.0	17000
149.0	18600							140.0	17600
173.0	19000							156.0	18000
209.5	19240							174.0	18360
298.0	20200							190.0	18600
441.0	20890							203.8	18810
603.0	21450							413.	20710
824.0	21660							571.	21350
1066.	21970							1014.	21950
1235.	22240							1190.	22230
1794.	22770							1700.	22680
2600.	23590	2540	23570	2440	23600	2481.	23590	2492.	23550

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absent, B the induction flux through the central cross section of the rod under investigation, and m denotes the ratio of the length of the rod to its diameter. Figure 4 gives two B–H curves for this material.

In Experiment D, we studied the behavior of a rod of fine annealed tool steel 0.794 cms. in diameter for which the final value of the magnetization vector, I, is about 1540. This was furnished with a test coil of 21 turns wound about its centre and was tested first when it had a length of 91.3 cms. and again when it had been shortened to 21.8 cms. At excitations of 1300, 1800, and 2650 gausses the fluxes did not differ from each other by more than one fifth of one per cent of either. At 900 gausses, however, the shorter rod had a flux quite one per cent below that of the longer one. At 2700 gausses, the value of B was about 21960.

A rod of Special Magnet Steel, 0.805 cms. in diameter and originally 124 diameters long, gave upon being shortened to about 30 diameters, one half of one per cent less flux in a field of 1260 gausses, but in fields of about 1750 and 2600 gausses the fluxes in the case of the short piece were indistinguishable from the corresponding fluxes for the longer one. This was Experiment E.

It seems to be true, therefore, that in properly constructed solenoids with uniform fields above 2500 gausses, we may safely use rods only 25 or 30 diameters long with the expectation of finding that the fluxes through their meridian sections are the same, within the limits of ordinary careful laboratory practice, as if the pieces were infinitely long. At low excitations, however, the corrections for the ends of pieces as short as this would, of course, be very large. The magnetizing solenoid should be about 25 diameters of its own coil longer than the test piece. For ordinary work, where an accuracy of one half of one per cent will suffice, test pieces only 15 diameters long will often serve at excitations above 2500 gausses.

I wish to express my great obligation to the Trustees of the Bache Fund of the National Academy of Sciences for the loan of some of the apparatus used in making the observations mentioned in this paper.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

ON ELECTRIC CONDUCTION AND THERMOELECTRIC ACTION IN METALS.

Edwin H. Hall.

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ON ELECTRIC CONDUCTION AND THERMOELECTRIC ACTION IN METALS.

By Edwin H. Hall.

Presented by E. H. Hall, March 11, 1914. Received June 12, 1914.

Introduction.

Various considerations that I need not at the beginning set forth at length, but some of which will appear later in this paper, have led me to inquire whether we may not have, in the phenomena of electric conduction and of thermoelectric action in metals, the coöperation of electrons in two conditions. One condition, (*A*), I have conceived of as that of electrons passing from atom to atom of the metal so quickly, perhaps during actual contacts of the atoms, as not to become subject to the laws of gas pressure, the other condition, (*B*), I have thought of as that of electrons long enough free between the atoms to act according to the gas laws, though, in deference to the argument from radiation, I assume these free electrons to have a smaller kinetic energy of translation than gas molecules would have in a cavity within the metal.

Sir J. J. Thomson suggested in his *Corpuscular Theory of Matter*[1] an action somewhat like that which I imagine for the (*A*) electrons, though he seems presently to have abandoned this idea as unnecessary.[2] But since Thomson discussed the matter new experimental evidence, especially from the behavior of metals at low temperatures, has seemed to lead naturally back to the conception of electrons transmitted directly from atom to atom in metallic conduction. If we think of the atoms as being, at very low temperatures, in actual

1 Pp. 49 and 50. "It is easy to see, however, that a current could be carried through the metal by corpuscles which went straight out of one atom and lodged at their first impact in another; such corpuscles would not be free in the sense in which the word was previously used and would have no opportunities of getting into temperature equilibrium with their surroundings." etc.

2 See Tunzelmann's *Electrical Theory*, p. 301.

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contact with each other much of the time, we have suggested at once an explanation of the vanishing electric resistance and the vanishing specific heat observed in pure metals as they approach the absolute zero of temperature. With atoms pressed very close together we should expect some of the electrons, what Rutherford would call the peripheral ones, to be movable from one atom to another with comparative ease, and with the same conditions we should expect the degrees of freedom of the atoms to be fewer,[3] and the thermal capacity of the metal less, than at higher temperatures. A piece of metal in this condition, immersed in helium gas and subject to the impact of the gas molecules, may be compared to a brick building bombarded by tennis balls. If the individual bricks were free, they would absorb kinetic energy from the balls; but agglomerated in one huge mass they repel the attacks and remain almost unmoved.

On the other hand, thermo-electric phenomena appear to require the presence of free electrons within metals. In looking for a theory of electric conduction in solids we should not, even at the start, forget the fact that circuits exist in which the electric current is maintained at the expense of heat energy solely. Now, in all cases in which the transformation of heat into work is really understood, it is effected by means of change of dimensions, expansion and contraction of the working substance in which the heat resides and operates as molecular or atomic energy. In a thermo-electric current the electricity is the factor which undergoes a cyclic change; the metals are in a fixed state, though one of non-uniform temperature, and they neither expand nor contract after this fixed state is reached. It would seem, then, that the electricity must expand and contract in its cyclic course and serve as the vehicle and transformer of heat energy.

Hence my attempt to discover what a combination of the two kinds of electron action might be expected to do in a metal unequally heated. In the course of this undertaking, which has extended over some months, I have been led to change somewhat from time to time my point of view and the particular assumptions of which I have made use. For example, starting with the idea that the free electrons have a heat capacity which is constant, though much less than that of ordinary gas molecules, I later determined to try the experiment of taking this thermal capacity as a variable, increasing with rise of temperature but still, at ordinary temperatures, below that of gas

3 See Jeans, *Phil. Mag.*, Vol. 17 (1909), p. 794, where the possibility that the atoms of a metal may have very little thermal capacity when locked together is discussed.

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molecules.[4] This innovation seems to be justified by its success in meeting certain requirements of the situation.

The conclusion to which my reflections, carried on with the help of some mathematical machinery, have led me is, that the free electrons, though present and essential for thermoelectric action, are of relatively small importance in mere electric conduction.

Fundamental Assumptions.

I shall assume that n, the number of free electrons per cu. cm. of the metal at any temperature, T absolute, can be expressed as $n = k_n T^\nu$, and that R, of the free-electron gas-equation $pv = RT$, "reckoned for a single electron,"[5] can be expressed as $R = k_r T^\rho$, k_n, ν, k_r, and ρ being constants. These two assumptions give

$$Rn = k_r T^\rho \times k_n T^\nu = kT^q, \tag{1}$$

where k and q are constants.

For a single molecule of an ordinary gas the value of R is about 137×10^{-18}. I assume that R for an electron has a very much smaller value than this at low temperatures.

Whether any metal really satisfies the conditions indicated by (1) through any great range of temperature may well be doubted; but,

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[4] The "law of equipartion of energy," a law more familiar, perhaps, in the breach than in the observance, doubtless requires that electrons acting as gas particles among other gas particles of a different class shall attain the same mean translatory energy as the latter, provided the two classes of particles remain distinct from each other in their encounters. But if electrons collide with metal atoms containing or made up of electrons, and if during a collision it frequently or usually happens that the electron enters an atom and stays there, displacing another electron, there seems to be no reason for supposing that the mean translatory energy of the free electrons will equal that of the atoms.

[5] Let us, for one *gram* of electrons, write $pv = R'T$. Then, taking m as the mass per electron and c as the "velocity of mean square," we have

$$p = \tfrac{1}{3} mnc^2 = R'T \div v = R'T\, mn.$$

From these relations we get

$$p = n(R'm)T = nRT,$$

an expression which will be used frequently hereafter, and

$$c \propto (R\,T)^{\frac{1}{2}}.$$

This proportionality must replace in this paper the simpler relation, $c \propto T^{\frac{1}{2}}$, which holds for an ordinary gas. This substitution is highly important. For example, if ρ of equation (1) is 1, so that $R \propto T$, we have $c \propto T$, and so $\frac{1}{2}mc^2 \propto T^2$; that is, the thermal capacity of the electron is $\propto T$, and its heat-energy content $\propto T^2$, if $\rho = 1$.

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with freedom to make ρ and ν any constants whatever for any given metal, we should be able to represent the facts with considerable accuracy through moderate ranges of temperature, and we have under these conditions a field of possibilities which seems to be worth exploring.

Electric Conduction.

According to the ideas set forth in the preceding pages the specific conductivity of a metal is the sum of two parts, one due to the (A) electrons, the other to the (B), or "gas-kinetic," electrons. The first of these parts I shall represent as

$$K_a = F_1(\delta, T),$$

where F_1 is some function of δ, the mean distance between the centres of adjacent atoms of the metal, and of T, the absolute temperature of the metal.

I shall assume that previous discussions [6] of free electron theory have been correct in taking that part of the conductivity which depends on the free electrons as proportional to n (in equation (1)) and to τ, the mean time between collisions of a free electron with the atoms. Evidently τ is equal to the free mean path, between such collisions, divided by the mean heat-velocity of the electrons. The mean free path is some function of δ,—$F_2(\delta)$, let us say, which increases with rise of temperature under ordinary conditions, while the mean velocity [7] is proportional to $(RT)^{\frac{1}{2}}$; so that $\tau \propto F_2(\delta) \div (RT)^{\frac{1}{2}}$; with $R = k_r T^\rho$, as in equation (1). Accordingly we get, as the part of the specific conductivity which depends on (B),

$$K_b = k_b T^\nu F_2(\delta) \div T^{\frac{1}{2}(\rho+1)} = k_b F_2(\delta) T^{(\nu-\frac{1}{2}\rho-\frac{1}{2})}. \qquad (2)^b$$

For the total specific conductivity we now have

$$K = F_1(\delta, T) + k_b F_2(\delta) T^{(\nu-\frac{1}{2}\rho-\frac{1}{2})}. \qquad (2)$$

When a metal is heated under ordinary conditions,— that is, at constant pressure with increase of volume, we have as the temperature coefficient of K

$$\alpha_p = \frac{dK}{K dT} = \frac{1}{K}\left(\frac{\partial F_1}{\partial \delta}\right)_T \frac{d\delta}{dT} + \frac{1}{K}\left(\frac{\partial F_1}{\partial T}\right)_\delta + \frac{k_b}{K} \cdot \frac{dF_2}{d\delta} \cdot \frac{d\delta}{dT} \cdot T^{(\nu-\frac{1}{2}\rho-\frac{1}{2})} + \frac{k_b}{K} \cdot F_2(\delta)(\nu-\tfrac{1}{2}\rho-\tfrac{1}{2}) T^{(\nu-\frac{1}{2}\rho-\frac{3}{2})}. \qquad (3)$$

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6 For example, see p. 303 of Tunzelmann's *Electrical Theory.*
7 See footnote, p. 69.

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This quantity, as a whole, must have a negative value, if it is to accord with the experimentally known facts; but as to the signs of its separate terms there may be some question. It seems plain that the first term is negative and that the third term is positive. The second term, if there were no evidence to the contrary, I should take as positive, and at first I did so take it, believing that I had experimental ground, as well as *a priori* ground, for this belief. The experimental ground proved not to be good footing, and I am now inclined to the opinion that this term is negative, for reasons which will presently appear. The fourth term evidently has the sign of its factor $(\nu - \frac{1}{2}\ \rho - \frac{1}{2})$. Now according to my discussion of the Thomson effect (see equations (15)–(21)) this factor is probably positive in many metals, and, as the third term of α_p is obviously positive, we reach the conclusion that, in these metals at least, the negative value of the temperature coefficient α_p cannot be accounted for if the electric conduction within these metals is solely, or even mainly, by means of the (B) electrons. We are thus led to attach especial importance to the first term in the value of K, as given in equation (2), and to the first two terms in the values of α_p, as given in equation (3).

In seeking further light on the term $\frac{1}{K}\frac{\partial F_1}{\partial T}$, the sign of which has thus far been left in doubt, we naturally turn to such experiments as show the effect of increased pressure on the electric conductivity of metals. We have at command the data for calculating, in the case of several metals, approximately what the temperature coefficient of the conductivity would be if heating occurred with such increase of pressure as to keep the volume of the metal constant. If we call this coefficient α_v, and if we assume that we can find an expression for it by merely dropping from the value of α_p the two terms which contain the factor $d\delta \div dT$, we have

$$\alpha_v = \frac{1}{K}\frac{\partial F_1}{\partial T} + \frac{k_b}{K} \cdot F_2(\delta)\ (\nu - \tfrac{1}{2}\ \rho - \tfrac{1}{2})\ T^{(\nu - \frac{1}{2}\ \rho - \frac{1}{2})}. \qquad (3a)$$

In liquid mercury, according to the experiments of Barus,[8] the value of α_v is positive; but in the solid pure metals, so far as I know, it is negative and, though numerically less [9] than α_p, not very much less. If, then, equation $(3a)$ is a correct expression for α_v, it appears that the term $\frac{1}{K}\frac{\partial F_1}{\partial T}$ is negative, since the second term, as we have seen, is probably positive.

8 Bulletin of the U. S. Geol. Survey, No. 92 (1892), p. 75.
9 See the Appendix to this paper for discussion of this matter.

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The Thermodynamic Point of View.

The greater part of my argument from here on will lie in a thermodynamic treatment of thermoelectric action, and in the course of it I shall make free use of the fact, which I have pointed out in previous [10] papers, that the ordinary thermoelectric diagram, representing "thermoelectric heights" as functions of temperature, is in reality a temperature-entropy diagram. Following ordinary engineering practice, I shall take the temperature coördinate as vertical and the entropy coördinate as extending horizontally toward the right.

Putting aside for the present the consideration of part (A) of the electric current, I shall discuss part (B) at some length as if it existed alone, returning to the treatment of (A) later.

Several years ago I discussed the analogy which exists between the cycle described by water in the circuit of a heating system, or in the circuit of a steam engine, and the cycle described by electricity in a thermoelectric circuit. The motion of the water in each of the cases referred to is, of course, due to heat, but heat alone would not maintain circulation. The application or expenditure of the heat must be managed or maneuvered by agents which do none of the net work of the cycle. In the case of the steam engine this control is given by a system of valves or checks which permit movement in one direction but not in the other. In the heating system, valves and checks may be dispensed with, gravity exercising what in chemistry would be called the catalytic function of maintaining the desired action, circulation, at the expense of heat. In the thermoelectric circuit we must look for some agency to perform a like service. Thus, if we have a detached piece of copper with one end at temperature T and the other at a lower temperature T', a state of equilibrium exists within it such that there is no electric flow along the metal; it is the same with a detached piece of iron wire having its ends at the same temperatures, T and T'; but, if we join the warm end of the copper to the warm end of the iron and the cold end of the copper to the cold end of the iron, we find that a current of electricity flows from copper to iron at one junction and from iron to copper at the other junction. It is quite evident that, if we had to do with electric potential only, in the ordinary sense of the term, either there would be no flow on bringing the wires into circuit or there would be flow in the same direction at both

10 For example, These Proceedings, **46**, 649 (1911).

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junctions. Let us therefore consider carefully what may be the nature of the equilibrium which exists in a detached piece of wire having a temperature gradient.

The Boltzmann Aërostatic Equation.

I shall make use of the idea that each metal exercises a specific "intrinsic" attraction for electrons. Helmholtz long ago assumed such an attraction [11] for electricity in discussing the "double layer" at the surface between different substances. Several years since I made the suggestion that this attraction might be a function of the temperature of the metal. More recently O. W. Richardson [12] has made use of the Boltzmann formula

$$\frac{n_1}{n_2} = \epsilon^{-\frac{W}{RT}} \tag{4}$$

in dealing with the forces exerted upon electrons by electrical charges or by the attracting metal atoms. In this formula "n_1 is the concentration of the electrons at a point A and n_2 that at a point B," and "W is the work done in taking an electron from A to B and R is the gas constant in the equation $pv = R\theta[= RT]$, reckoned for a single electron."

I feel greatly indebted to Professor Richardson for his discussion of this matter, and when I took up the question now before us I expected to use his ideas and conclusions, except in so far as they might be modified by my assumption that R for an electron is less than R for a gas molecule. I have, however, upon close examination of the matter, been forced to the conclusion that, in applying formula (4) to a discussion of the Thomson effect, involving of course a difference of temperature between the two points A and B, he has fallen into error.

Boltzmann gives the equivalent of the formula in question under the heading *Aërostatik*,[13] and it is very easily derived, as follows, for an atmosphere of uniform temperature in equilibrium:

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11 This at least is my interpretation of certain passages in his papers; for example, the following sentence from an article in the *Monatsbericht d. k. Akad. d. Wiss. zu Berlin, Nov.* 3, 1881, S. 951: "Soll in einen Leiter, dessen Potential (elektrostatisch gemessen) p und dessen galvanische Constante k ist, ein neues Quantum Elektricität dE eingeführt werden, so ist dazu die Arbeit $(p-k)\ dE$ nothig."

12 Phil. Mag., Vol. 23 (1912), pp. 263–278.

13 Gastheorie, Vol. I, § 19.

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Let l = height above the earth's surface,
ρ = density of the gas,
p = pressure " " "
g = gravity acceleration,
R' = gas constant for unit mass,
T = absolute temperature.

Then $-\frac{dp}{dl} = g\rho = g\frac{p}{R'T}$, whence

$$\frac{dp}{p} = -\frac{g}{R'T}dl, \text{ and so } log\ p = -\frac{g}{R'T}l + log\ k,$$

or $p = k\epsilon^{-\frac{gl}{R'T}}$, where k = pressure at the earth's surface.

Then for any two points A and B, with pressures p_1 and p_2 respectively, and molecular concentrations n_1 and n_2 respectively, we have

$$\frac{p_2}{p_1} = \frac{n_2}{n_1} = \epsilon^{-\frac{g(l_2 - l_1)}{R'T}} = \epsilon^{-\frac{W'}{R'T}}.$$

In this equation W' is the work of lifting unit mass from l_1 to l_2 against the pull of gravity. If we wish to deal with a single molecule of mass m, we can write

$$\frac{p_2}{p_1} = \frac{n_2}{n_1} = \epsilon^{-\frac{W'm}{R'mT}} = \epsilon^{-\frac{W}{RT}},$$

which is the Boltzman equation as used by Richardson.

Thermoelectric Equilibrium in a Detached Wire

(*with consideration of* (*B*) *electrons only*).

In dealing with the free electrons in a metal unequally heated we have a case somewhat like the one just discussed but considerably more complicated. In place of g we must now put $f,'$ the resultant of attractions and repulsions per gram of free electrons, this resultant being called positive when it is directed along the path of diminishing l, and we must consider f' as a variable. In place of R' for a gram of gas we must put R' for a gram of free electrons, and take R' as a variable according to equation (1). Moreover T is now to be taken as a variable along l. We shall write

$$\beta = dT \div dl,$$

and shall treat β as a variable.

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Now in place of the equation $\frac{dp}{p} = -\frac{g}{R'T}dl$ we have

$$\frac{dp}{p} = -\frac{f'}{R'T}dl = -\frac{f'}{R'\beta}\cdot\frac{dT}{T}.$$

But $p = R'Tnm$, and so

$$\frac{dp}{p} = \frac{dR'}{R'} + \frac{dn}{n} + \frac{dT}{T} = -\frac{f'}{R'\beta}\cdot\frac{dT}{T},$$

whence

$$\frac{dR'}{R'} + \frac{dn}{n} = -\left(\frac{f'}{R'\beta} + 1\right)\frac{dT}{T},$$

or, if we multiply both R' and f' by m, and so get R and f for a single electron,

$$\frac{dR}{R} + \frac{dn}{n} = -\left(\frac{f}{R\beta} + 1\right)\frac{dT}{T}. \qquad (5)$$

When we observe that f, β, and R are all variables, the integration of this equation appears at first to present difficulties. But two of our fundamental assumptions, expressed in equation (1), give $Rn = kT^q$, where k and q are constants, and the only way to make this equation agree with equation (5) is to have the factor $\left(\frac{f}{R\beta} + 1\right)$ a constant. Thus we get by the integration of (5)

$$Rn = kT^{-\left(\frac{f}{R\beta}+1\right)} = kT^q, \qquad (6)$$

and so

$$\frac{n_2}{n_1} = \left(\frac{T_2}{T_1}\right)^q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In dealing with our detached piece of wire I shall use F to indicate *virtual* potential for a single electron,— that is, the total potential due to attractions and repulsions of electric charges *together with the attractions of the metal atoms*,[14] as exerted on a single electron. If the distance l is measured from the cold end of the wire, we have

$$f = \frac{dF}{dl}, \text{ and } \beta = \frac{dT}{dl}, \text{ so that } \frac{f}{\beta} = \frac{dF}{dT}. \qquad (8)$$

We have seen that equation (6) is a necessary consequence of equations (1) and (5). As (1) expresses merely certain fundamental

[14] If the assumption of an attraction of the metal for the electrons, as a function of temperature, were omitted, the word *virtual* as applied to the potential would be omitted.

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assumptions, our only question here is whether (5) is necessarily true. Examination shows that it involves the assumption, not explicitly made thus far, that the free electrons in unequally heated metal *tend* toward equality of gas-pressure, that the difference dp, between two isothermal planes differing by dT, must be balanced, if we are to have equilibrium, by the force f, applied to every free electron between the planes.

This assumption, which implies that the free electrons tend to the condition

$$n_1 R_1 T_1 = n_2 R_2 T_2, \tag{9}$$

where the subscripts (1) and (2) refer to any two parts of the metal, is by no means a matter of course when we are dealing with a gas permeating narrow passages where differences of temperature exist.

It is well known that, if a thin partition pierced only by a very small hole separates two bodies of an ordinary gas, one at temperature T_1, the other at temperature T_2, the condition of equilibrium between the two bodies of gas is not equality of pressure, but $p_1 \div p_2 = T_1^{\frac{1}{2}} \div T_2^{\frac{1}{2}}$, or, since $p_1 : p_2 :: n_1 T_1 : n_2 T_2$,

$$n_1 T_1^{\frac{1}{2}} = n_2 T_2^{\frac{1}{2}}.$$

That is, there is a certain tendency of the gas from the cold chamber to the warm chamber, which must in the end be balanced by superior pressure in the warm chamber.[15]

Going to the case of electrons, for which we assume R to be a variable, we have as the law of equilibrium under thermal effusion alone

$$n_1 (R_1 T_1)^{\frac{1}{2}} = n_2 (R_2 T_2)^{\frac{1}{2}}, \tag{9'}$$ [16]

or, since $p = nRT$, $p \propto (RT)^{\frac{1}{2}}$.

---

15 Maxwell, toward the end of his memoir on *Stresses in Rarefied Gases*, says, "The passage of gases through porous plates, as was shown by Graham, is of an entirely different kind from the passage of gases through capillary tubes, and is more nearly analogous to the flow of a gas through a small hole in the thin plate.

"When the diameter of the hole and the thickness of the plate are both small compared with the length of the free path of a molecule, then, as Sir William Thomson has shown, any molecule which comes up to the hole on either side will be in very little danger of encountering another molecule before it has got fairly through to the other side.

"The finer the pores of a porous plate, and the rarer the gas which effuses through it, the more nearly does the passage of a gas through the plate correspond to what we have called effusion." etc.

16 For, another way of stating the condition of equilibrium in the case of thermal effusion is, that the *momentum* of the particles per cu. cm. shall be the same at one place as at another. If we take c as the "velocity of mean square," this condition gives $n_1 c_1 = n_2 c_2$, and we have already seen, in the footnote on p. 69, that $c \propto (RT)^{\frac{1}{2}}$.

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The differential expression of this law is

$$dp = \frac{p}{2}\frac{dR}{R} + \frac{p}{2}\frac{dT}{T} \tag{10}$$

That is, the difference of pressure which the free electrons in a stratum of thickness dl can bear without drift, and without the help of electrical force, is

$$\frac{p}{2}\frac{dR}{R} + \frac{p}{2}\frac{dR}{T}.$$

If the difference of pressure on the two faces of the stratum is greater than this quantity, we must, in order to have equilibrium, balance the excess of dp by the electrical force $-fndl$ acting up the temperature incline. Thus we get

$$dp = \frac{p}{2}\frac{dR}{R} + \frac{p}{2}\frac{dT}{T} - fndl,$$

whence, as

$$n = p \div RT, \text{ and } \beta = dT \div dl,$$

we have

$$\frac{dp}{p} + \frac{f}{R\beta}\cdot\frac{dT}{T} = \tfrac{1}{2}\frac{dR}{R} + \tfrac{1}{2}\frac{dT}{T}.$$

As $p = nRT$, we get from this equation, by substituting for p and dp,

$$\tfrac{1}{2}\frac{dR}{R} + \frac{dn}{n} = -\left(\frac{f}{R\beta} + \tfrac{1}{2}\right)\frac{dT}{T}. \tag{5'}$$

Now the assumptions which give equation (1) give also the equation

$$R^{\frac{1}{2}}n = k_r^{\frac{1}{2}}T^{\frac{\rho}{2}} \times k_n T^{\nu} = k'T^{q'}, \tag{1'}$$

and in order to make (5') and (1') agree we must, as in the case of equation (5), treat $(f \div R\beta)$ as a constant. Accordingly we have

$$R^{\frac{1}{2}}n = k_r' \, T^{-\left(\frac{f}{R\beta} + \frac{1}{2}\right)} = k'T^{q'}, \tag{6'}$$

where k' and q' are constants.

Not knowing which of the two conditions, (9) or (9'), represents the gas-pressure tendency of the free electrons the more nearly, I shall try each of the two corresponding equations, (6) and (6'), in turn. But before doing this, I must call attention to the fact that neither form of this tendency can make any great change in what we may call the natural values of n_1 and n_2,— that is, the values which would be found in two detached pieces of the same metal, one at T_1, the other at T_2. For, if the number of the free electrons is not very small

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indeed compared with the number of the atoms, the transfer of a very slight proportion of these electrons from one part to another of a detached wire would make a notable difference in the state of electric charge and affect greatly the value of F and f.

It is to be noted, also, that the occurrence of a great or small electric concentration, n, at any place in a metal does not imply a proportional state of *charge* there; for the electrons, in merely getting free from the atoms of the metal, do not change the general condition of charge in their neighborhood. The positive charges remaining on the deserted atoms adjacent are still effective.

(1) *If the gas pressure tendency is toward* $n_1R_1T_1 = n_2R_2T_2$: —

In this case equation (6) holds, and we can form at once a table of values of $(f \div \beta)$ corresponding to chosen values of q.

TABLE I.

CASES IN WHICH EQUATION (6) HOLDS.

q	Rn	$(f \div \beta)$	(Virtual volts per degree C.) [17]
0	constant	$-R$	$-\ 86\times10^{-6}r$
-1	$\propto T^{-1}$	0	0
-2	" T^{-2}	$+R$	$+\ 86\times10^{-6}r$
$+\frac{1}{2}$	" $T^{\frac{1}{2}}$	$-1\frac{1}{2}R$	-129 " "
1	" T	$-2R$	-172 " "
$1\frac{1}{2}$	" $T^{1\frac{1}{2}}$	$-2\frac{1}{2}R$	-215 " "
2	" T^2	$-3R$	-258 " "

[17] The values given in the last column are found as follows:

Charge of electron = approx. 159×10^{-22} electro-mag. units.

Force, in dynes, acting on 1 electron in a potential gradient of 1 volt per cm.
$= 159\times10^{-22}\times10^{8} = 159\times10^{-14}$;

Value of R for 1 molecule of an ordinary gas $= 137\times10^{-18}$;
" " " " 1 electron $= 137\times10^{-18}\times r$,

r being some quantity less than 1. Farther on some more definite estimate of the value of r will be attempted.

$$\frac{(\text{Volts per cm.})}{(\text{Degrees " "})} = \frac{f \div (159\times10^{-14})}{\beta} = \text{etc.}$$

The variation of electric charge which one might conceivably find along the wire, by means, for example, of a sufficiently sensitive pith-ball electroscope, might be very different from the "virtual potential gradient"; for it is to be remembered that f may be due in part to the specific attraction of the metal for the electrons, this attraction acting as a function of temperature, and it is probable that such action would not be perceptible at any workable distance outside the metal.

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(2) *If the thermal effusion law,* $n_1(R_1T_1)^{\frac{1}{2}} = n_2(R_2T_2)^{\frac{1}{2}}$, *holds:* — In this case we use equation (6′) and can find values of $(f \div \beta)$ corresponding to chosen values of q'.

TABLE II.

CASES IN WHICH EQUATION (6′) HOLDS.

q'	$R^{\frac{1}{2}}n$	$(f \div \beta)$	(Virtual volts per degree C.)
0	constant	$-\frac{1}{2}R$	$-43\times10^{-6}r$
$-\frac{1}{2}$	$\propto T^{-\frac{1}{2}}$	0	0
-1	" T^{-1}	$+\frac{1}{2}R$	$+43\times10^{-6}r$
-2	" T^{-2}	$+1\frac{1}{2}R$	129 " "
$+\frac{1}{2}$	" $T^{\frac{1}{2}}$	$-R$	-86 " "
1	" T^{1}	$-1\frac{1}{2}R$	-129 " "
$1\frac{1}{2}$	" $T^{1\frac{1}{2}}$	$-2R$	-172 " "
2	" T^{2}	$-2\frac{1}{2}R$	-215 " "

It may be that an equation intermediate between (6) and (6′) would come nearer to the truth than either (6) or (6′). For it seems probable that thermal effusion of the electrons has some effect in the interatomic spaces, though less effect than it would have in the case of a thin partition pierced by a narrow hole. I shall, however, for the present go on with equation (6), with occasional references to equation 6′) and its consequences.

The Thomson Effect: — Taking equation (6) as the condition for electrical equilibrium in a detached wire having a temperature gradient, I shall now find the Thomson effect coefficient, σ, in this metal. This coefficient I shall so define that σdT will mean the mechanical equivalent of the heat absorbed during the passage of the electromagnetic unit quantity of electricity, $\frac{1}{e}$ electrons, if e is the numerical value of the electron charge, from a part of the metal at temperature T to a part at temperature $T + dT$, under the conditions of electrical equilibrium. That is, I am dealing with a reversible process and ignore for the present the consideration of *resistance*-heat, which, according to custom, we may assume to be as small as we please to make it. My only innovation here is the change of sign of σ,— for example, from + to − in copper and from − to + in iron, because the current of electricity is now thought of as a movement of the negative electrons. In my equations e is to be taken as a positive quantity.

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We have to consider the passage of electrons through the region lying between two parallel plane surfaces, each of which is isothermal and also equipotential, the temperatures being T and $T + dT$, respectively. We have to take account of three changes in the energy condition of the electrons during that passage, change of kinetic energy due to the change of temperature, change of potential energy of surrounding electron gas pressure, change of potential energy F due to attractions and repulsions. If R were constant, the first two changes together would equal the mass of the electrons multiplied by the rise of temperature and by the specific heat, C_p, of the electron gas, at constant pressure. We may indeed, if we please, use the terms C_v and C_p; but, as neither of the specific heats is a constant when R is variable, we should find them of no great service. It is to be noted, however, that, even when R is variable, we have, c being velocity of mean square of the electrons and energy being reckoned in ergs,

$$\textit{kinetic energy per gram} = \tfrac{1}{2}\, vmnc^2,$$

and

$$pv \text{ “ “ “ } = \frac{RT}{m} = \tfrac{1}{3}\, mnc^2 v,$$

whence

$$k.e. \text{ “ “ “ } = \tfrac{3}{2}\,\frac{RT}{m}, \tag{11}$$

$$k.e. \textit{ per electron } = \tfrac{3}{2}\, RT, \tag{12}$$

$$pv \textit{ energy per electron} = RT. \tag{13}$$

When, therefore, we have the electromagnetic unit quantity of electricity in the form of $\frac{1}{e}$ electrons entering the region in question at the T surface and issuing at the $T + dT$ surface, we have

$$\textit{gain of k.e. energy} = \frac{3}{2e}\left[(RT)_{T+dT} - (RT)_T\right] = \frac{3}{2e}\left(R + T\frac{dR}{dT}\right)dT,$$

$$\textit{gain of pv energy} = \frac{1}{e}\left(R + T\frac{dR}{dT}\right)dT,$$

$$\textit{gain of F energy (see eq. (8))} = \frac{dF}{e} = \frac{1}{e}(f \div \beta)\, dT.$$

The total gain of energy by the electrons, which must equal the heat energy absorbed from the metal between the two planes T and $T + dT$, is

$$\sigma dT = \frac{5}{2e}\left(R + T\frac{dR}{dT}\right)dT + \frac{1}{e}\,(f \div \beta)dT, \tag{14}$$

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whence, by use of equations (1) and (6), we get

$$\sigma = \frac{5}{2e}(1+\rho)R - \frac{1}{e}(1+q)R = \frac{k_r}{e}\left(\frac{3}{2}(1+\rho) - \nu\right)T^\rho. \quad (15)$$

If, on the other hand, we go through the argument regarding σ from the point of view of thermal effusion, as expressed in (6′), where $(f \div \beta) = -R(\frac{1}{2} + q')$, and $q' = \nu + \frac{1}{2}p$, we get

$$\sigma = \frac{1}{e}\Big(2(1+\rho) - \nu\Big)R = \frac{k_r}{e}\Big(2(1+\rho) - \nu\Big)T^\rho. \quad (15')$$

Now on the thermo-electric diagram, as it is usually made, we find three types of lines representing the various metals, all these lines being straight. The character of these representative lines and their inclinations toward each other remain unchanged when we make the diagram in the form shown by Fig. 1, with the temperature axis vertical and the entropy axis horizontal. If we mark the temperature interval dT on line (1)–(1) of this diagram, the area under this portion of the line, between the two verticals extending to the S axis, will represent the amount of heat energy absorbed by the unit quantity of electricity, $\frac{1}{e}$ electrons, in passing along the metal (1) through a rise of temperature dT. This amount of heat is TdS and is a positive quantity for the line (1)–(1). Evidently we have $\sigma dT = TdS$, and, if the (1)–(1) is really a straight line, $dT \propto dS$, and so $\sigma \propto T$. That is, in this case, the ρ of (15) or (15′) must equal 1, and this must be true for every metal which can properly be represented on our diagram by a straight line.

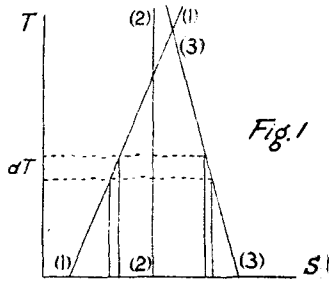

Fig. 1

For the line (2)–(2) dS is zero and σ also is zero. For the line (3)–(3) the area under dT represents the heat taken from the metal by $\frac{1}{e}$ electrons in going along metal (3) through a fall of temperature dT, and we have $\sigma(-dT) = TdS$, so that σ is negative, though it is still $\propto T$, if (3)–(3) is a straight line.

Dealing, then, with metals represented by such lines[18] as (1)–(1)

[18] If a line is convex upward, as from my own observations I believe the line for iron to be, the indication is that ρ is numerically > 1.

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and (3)–(3), and assuming that we have to do with free electrons, electrons (B), only, we should have to conclude that ρ is 1. To determine the algebraic sign of σ we must look to the relative magnitude of ρ and ν. According to equation (15), ρ being 1, we have

$$\sigma>0, \text{ if } \nu<3;\ \sigma = 0, \text{ if } \nu = 3;\ \sigma<0, \text{ if } \nu>3.$$

From (15′) we should get

$$\sigma>0, \text{ if } \nu<4;\ \sigma = 0, \text{ if } \nu = 4;\ \sigma<0, \text{ if } \nu>4.$$

Turning back now to equation (2), and assuming that we have to do with (B) electrons only, we get as the specific conductivity

$$K_b = k_b F_2(\delta) T^{(\nu - \frac{1}{2}(\rho + 1))},$$

where F_2 (δ) is a factor which increases with rise of temperature. Accordingly, K_b must increase with rise of temperature, unless $\nu<\frac{1}{2}(\rho+1)$,—that is, unless $\nu<1$, if $\rho=1$. But we have just seen that, in order for a metal to have σ negative and proportional to T, ν must be greater than 3, and so $\nu - \frac{1}{2}(\rho + 1)>2$, which will make K_b proportional to some power of T higher than the second. Metals commonly represented by straight lines on the thermoelectric diagram, with σ positive for the conventional conception of current and therefore negative for the electron conception of current, are copper, silver, zinc and antimony. There may be inaccuracy in taking the lines for these metals as straight, but it is unlikely that this inaccuracy is great enough to account for the absurdity here indicated regarding conductivity. It seems difficult to avoid the conclusion that the conception of free electrons acting alone is insufficient to account for the phenomena of both electric conduction and thermo-electric action.

Accordingly we must presently return to the consideration of those electrons which take part in the action (A).

The antagonism between ν and ρ which is to be observed in equation (15) and (15′) is rational. Great concentration of free electrons at the hot end of a wire, corresponding to a large value of ν, must tend by gas-pressure of the electrons to give a large value of F at the cold end, so that the term $(f \div \beta)$ in the value of σ (see equation (14)) will be negative. Rapid increase of R, on the other hand, with rise of temperature, corresponding to a large value of ρ, involves large thermal capacity of the electrons and tends to keep σ positive.

The increment of F for any rise of temperature dT along the wire is readily found from equation (8) to be

$$dF = -(1 + q)RdT = -(1 + \rho + \nu)RdT, \qquad (16)$$

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if equation (6) is followed, and

$$dF = -(\tfrac{1}{2} + q')RdT = -(\tfrac{1}{2}(1 + \rho) + \nu)RdT, \qquad (16')$$

if equation (6′) is followed.

Integrating from T' to T we get from (16)

$$(F - F') = -\left(1 + \frac{\nu}{1 + \rho}\right)(RT - R'T') \qquad (16_a)$$

and from (16′)

$$(F - F') = -\left(\frac{1}{2} + \frac{\nu}{1+\rho}\right)(RT - R'T') \qquad (16_a')$$

The difference of potential which might, conceivably, be detected by a sufficiently sensitive electroscope applied to the two ends of the wire in succession would not, probably, be equal to our $(F–F')$; for, according to our assumptions, the value of F is dependent upon the attraction of the atoms for the electrons as well as upon electric *charge*, and this action of the atoms would perhaps not be effective at any distance from the metal greater than the range of ordinary molecular attraction.

Equilibrium in a Single Unequally Heated Wire

(*with regard to both* (*A*) *and* (*B*) *electrons*).

The part played by the (A) electrons in electric conduction has been considered in connection with equation (2). We must now ask what they have to do with thermo-electric action or thermo-electric equilibrium.

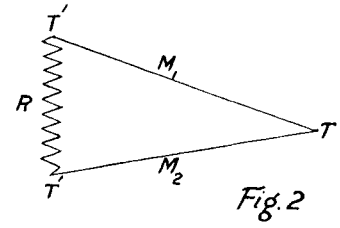

Fig. 2

It is to be observed that, if we take our single wire as part of a thermo-electric circuit like that indicated in Fig. 2, with a very large resistance, R, introduced at one junction, we shall have a current flowing around the circuit, though conditions as near as we please to those of equilibrium exist in each of the metals M_1 and M_2. That is, we may assume thermo-electric action, bringing into play the conductive function of the (A) electrons, while keeping all our equations, from (5) to (10) inclusive, just as we have derived and used them with respect to the (B) electrons alone.

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These equations do not involve the (A) electrons, and they must hold irrespective of these electrons, if we have, as we still suppose, conditions very close indeed to those of equilibrium. But as soon as we begin to consider actual flow, however slow it may be, we must take into account the (A) electrons and the amount of heat energy they absorb from the metal.

The Thomson Effect: — Let us consider, as before, what happens at and between two equipotential and isothermal surfaces, separated by a distance dl and a temperature interval dT. If we let K_a represent, as before, the part of the specific electric conductivity which depends on the (A) electrons, K_b that part which depends on the (B) electrons, and $K = K_a + K_b$, we shall have $(K_a \div K)$ as the (A) fraction of the current and $(K_b \div K)$ as the (B) fraction.

Let the ratio $(K_a \div K)$ be called $(1 - x)$, and $(K_b \div K)$ be called x, just as in a mixture of water and water vapor the proportion of the latter is commonly called x. In the passage of electrons from the bound state to the (B) state we have, in fact, a process similar to the evaporation of water, and this analogy is helpful in the present argument.

The total energy of $\frac{x}{e}$ electrons (B) at the temperature T is made up of the kinetic energy $\frac{x}{e} \times \frac{3}{2}RT$, the pv potential energy $\frac{x}{e}RT$, and the attraction-repulsion potential energy $\frac{x}{e}F$.

The total energy of $\frac{1-x}{e}$ electrons (A) at the same temperature is made up of kinetic energy, which I shall assume for the present to be the same[19] as that of an equal number of (B) electrons, namely $\frac{1-x}{e} \times \frac{3}{2}RT$; of attraction-repulsion potential energy, $\frac{1-x}{e}F$; and of another quantity of potential energy, $\frac{1-x}{e}\Phi$, due to the attraction of the particular metallic atoms to which the (A) electrons individually belong. Φ is a negative quantity; that is, $(-\Phi)$ is the amount of potential energy which an electron gains in being freed from an atom. Let $\Phi = k_\varphi T^\pi$, where k_φ and π are constants, k_φ being negative and π

19 The (B) electrons may be regarded as individuals which have dropped out of the (A) class by going astray from the short path leading from one atom to the next atom.

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probably so. Accordingly we get, as the total energy of our $\frac{1}{e}$ electrons at temperature T of the metal,

$$E = \frac{1-x}{e}\left(\frac{3}{2}RT + F + \Phi\right) + \frac{x}{e}\left(\frac{5}{2}RT + F\right) = \frac{1}{e}\left(\frac{3}{2} + x\right)RT + \frac{F}{e} + \frac{1}{e}(1-x)\,\Phi. \tag{17}$$

In looking for the amount of heat energy which is absorbed from the metal by our $\frac{1}{e}$ electrons in their passage from T to $T + dT$ through the metal, we must remember that x is a variable, increasing, according to my assumption, with rise of temperature. We have

$$\sigma dT = \frac{dE}{dT}\,dT,$$

whence

$$\sigma = \frac{1}{e}\left[\frac{3}{2}\frac{d}{dT}(RT) + \frac{d}{dT}(xRT) + \frac{dF}{dT} + \frac{d\Phi}{dT} - \frac{d(x\Phi)}{dT}\right]. \tag{18}$$

It may be well just here to inquire whether the (B) electrons, which we have found to be inadequate when taken alone, are needed at all,— whether the (A) electrons acting alone would serve our purpose. Let us accordingly assume, for the moment at least, that ν and x are each equal to zero. That part of F which is dependent on the free electrons will also be zero in this case, while the other part of F, the part depending on the attraction of the atoms as a function of temperature, will be included in Φ, provided $(-\Phi)$ is now defined as the gain of potential energy of an electron in being taken from an atom in the metal to a point outside the metal. Accordingly we get from (18)

$$\sigma = \frac{1}{e}\left[\frac{3k_r}{2}(1+\rho)\,T^{\rho} + k_{\varphi}\pi T^{(\pi-1)}\right] \tag{19}$$

The first term within the brackets is $+$, according to our assumption regarding R as a function of T. In the second term the factor k_{φ} is $-$, as we have seen before, but the factor π is also $-$, if, as seems probable, Φ diminishes numerically with rise of temperature. We seem, then, with (19) to have no provision for negative values of σ, such as, we know, occur in metals.

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Returning, then, to equation (18) and putting

$$x = k_x T^{\kappa}, \tag{20}$$

with k_x and κ as positive constants, we get

$$\sigma = \frac{1}{e}\Big[k_r\left(\tfrac{1}{2} + \frac{\rho}{2} - \nu\right) T^{\rho} + k_x k_r\left(1 + \kappa + \rho\right) T^{(\kappa+\rho)} + k_\varphi \pi T^{(\pi-1)} k_x k_\varphi \left(\kappa + \pi\right) T^{(\kappa+\pi-1)}\Big]. \tag{21}$$

If we take the point of view of thermal effusion, according to which $dF \div dT = -R\left(\frac{1}{2} + \frac{\rho}{2} + \nu\right)$, we get from (18)

$$\sigma = \frac{1}{e}\Big[k_r\left(1 + \rho - \nu\right) T^{\rho} + k_x k_r\left(1 + \kappa + \rho\right) T^{(\kappa+\rho)} + k_\varphi \pi T^{(\pi-1)} - k_x k_\varphi \left(\kappa + \pi\right) T^{(\kappa+\pi-1)}\Big]. \tag{21'}$$

Neither equation (21) nor equation (21′) would permit us to have σ strictly proportional to T; but we are not sure that σ is strictly proportional to T in any metal. If the terms beyond the first in the second member are small compared with this first term, we may have approximately straight lines on the thermo-electric diagram,— that is, have σ nearly proportional to T, if $\rho = 1$.

Moreover, the presence of $(-\nu)$ in the coefficient of this first term provides for possible negative values of σ, if the first term is really the dominating part of this quantity. Of the other terms, the second and third are positive, according to the assumptions already made, while the fourth may be negative, if π, which is supposed negative, can be numerically greater than κ. I shall assume, for the present at least, that k_x is very small, thus making the second and fourth terms very small. To make k_x small is to make x small, so that I am here assuming that the greater part of the electric conductivity is due to the (A) electrons. This assumption accords well with what we know in regard to the temperature coefficient of conductivity. For even with equation (21) we cannot get a negative value of σ proportional to T, a condition approached very closely by several metals, without making $\rho = 1$ and ν greater than 1, so that the K_b term in the value of the conductivity, equation (2), will still increase with rise of temperature. The K_a term in (2) must apparently be the prevailing term, at least so far as the temperature coefficient of conductivity is concerned, in some metals, if not in all.

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It is to be observed that the very important part played by ν in the equations (21) and (21′) does not imply that n, the number of free electrons per cu. cm. of the metal, is so large as to be of the same order of magnitude as the number of atoms per cu. cm. The value of $(dF \div dT)$, by way of which ν gets into the equations in question, is not dependent upon the absolute value of n at any place, but upon the ratio of the n of one temperature to the n of another temperature. Thus it is possible for a comparatively small number of free electrons to have a great effect upon σ, by way of the (A) electrons which are subject to the $(dF \div dT)$ established by the (B) electrons.

As to the term $k_\varphi \pi\ T^{(\pi-1)}$ in equation (21) and (21′), although we are hardly at liberty to assume k_φ to be small in comparison with k_π, π, which we suppose to be negative, may be small, and the factor $T^{(\pi-1)}$ is less than T^{-1}. It seems not unlikely, then, that this term is small compared with the T^ρ term.

If we are satisfied that equations (21) and (21′) accord in a general way with the known phenomena of the Thomson effect, we must next inquire whether either of these formulas will give a good quantitative account of the Thomson coefficient σ as found by experiment in certain metals. Let us consider *lead*, with $\sigma = 0$, very nearly; *cobalt*, with $\sigma = 2000$[20]; and antimony, with $\sigma = -1000$.[20] I take cobalt and antimony because they have the limiting values known to me.

Taking the first term in the value of σ as the dominating one, we see that, if we take $\rho = 1$, as we do in other cases, $\sigma = 0$, according to (21), if $\nu = \frac{1}{2} + \frac{1}{2}\ \rho = 1$ or, according to (21′), if $\nu = 1 + \rho = 2$. Neither 1 nor 2 seems a very improbable value for ν.

Substituting R for $k_r T^\rho$ in this first term we have, approximately, for cobalt,

$$\sigma = \frac{1}{e}\left(\frac{1}{2} + \frac{\rho}{2} - \nu\right)R = 2000 \qquad (22)$$

or

$$\sigma = \frac{1}{e}(1 + \rho - \nu)\,R = 2000 \qquad (22')$$

The value of R_g, reckoned for a single molecule of an ordinary gas, is about 137×10^{-18}. If we for a moment assume this as the value of the electron R in (22) and (22′) and take $e = 159 \times 10^{-22}$ in electromagnetic units, and $\rho = 1$, we get from (22), $\nu = 0.77$, and from (22′), $\nu = 1.77$.

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[20] These are approximately, the values given for the metals in question at 0° C. in the *Recuil de Constantes Physiques* (1913) of the Société Française de Physique.

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But it is unlikely that R for the electron is as great as R_g for a gas molecule at ordinary temperatures.

If we take $R = 0.5R_g = 68.5 \times 10^{-18}$, we get

from (22), $\nu = 0.54$, and from (22′), $\nu = 1.54$.

If we take $R = 0.1R_g = 13.7 \times 10^{-18}$, we get

from (22), $\nu = -1.32$, and from (22′), $\nu = -0.32$.

If we put $\nu = 0$, we get

from (22), $R = 32 \times 10^{-18}$, and from (22′), $R = 16 \times 10^{-18}$.

As negative values of ν seem improbable, the indication of this investigation is that in cobalt, at the temperature for which $\sigma = 2000$, R may well be as small as $0.25\ R_g$, but is probably greater than $0.1R_g$.

Turning now to antimony we have

$$\sigma = \frac{1}{e}\left(\frac{1}{2} + \frac{\rho}{2} - \nu\right)R = -1000 \qquad (23)$$

or

$$\sigma = \frac{1}{e}(1 + \rho - \nu)R = -1000 \qquad (23')$$

If we take $R = R_g$, we get

from (23), $\nu = 1.12$, and from (23′), $\nu = 2.12$.

If $R = 0.5R_g$, we have

from (23), $\nu = 1.23$, and from (23′), $\nu = 2.23$.

If $R = 0.1R_g$, we have

from (23), $\nu = 2.16$, and from (23′), $\nu = 3.16$.

Here also, since large values of ν seem rather improbable, the indication appears to be that R is greater than $0.1\ R_g$.

On the whole, thus far, it appears that the hypothesis of combined action of (A) electrons and (B) electrons, with the former playing much the greater part in electric conduction and the latter having a very important function in a metal not uniformly heated, gives a reasonable account of the phenomena of electric conduction,[21] including change of conductivity with change of temperature, and of the Thomson effect, in pure metals.

It is now time to consider how well the same hypothesis will apply to the phenomena at the junction of two metals, the Seebeck effect and the Peltier effect.

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21 It seems not unlikely that the change of resistance of metals in melting, in most cases an increase, is due mainly to the change of volume, which also in most cases is an increase. See Appendix for data and discussion.

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Conditions of Equilibrium at a Junction of Two Metals at the same Temperature.

Let the two metals be called M_1 and M_2, and let each be throughout at the temperature T. As the thermo-electric force of an isothermal circuit is zero, whatever the arrangement of the metals, we may assume the transition from M_1 to M_2 to be as gradual as we please to make it, by way of an intermediate stretch of an alloy of the metals, this alloy changing by any law from pure M_1 at one end to pure M_2 at the other end.

If we take the ordinary view of an alloy and think of it as a physical mixture of the component metals in small particles, much larger, however, than the atoms or molecules, we shall have the mean n and the mean R, in the successive cross-sections of the bridge reaching from M_1 to pure M_2, changing gradually, by the increasing proportion of M_2 particles, from n_1 and R_1 to n_2 and R_2. It seems reasonable, if not inevitable, to assume that for any given cross-sectional slice of this bridge we shall have

$$\frac{n_1 - n}{n_1 - n_2} = \frac{R_1 - R}{R_1 - R_2}, \text{ or } R = R_1 + (n - n_1) \cdot \frac{R_1 - R_2}{n_1 - n_2}, \tag{24}$$

and

$$\frac{dn}{n_1 - n_2} = \frac{dR}{R_1 - R_2}, \text{ or } dR = dn \cdot \frac{R_1 - R_2}{n_1 - n_2}. \tag{25}$$

For equilibrium of condition of the slice in question, the thickness of which we shall call dl, we must, if we disregard thermal effusion and assume that the free electrons, as a *gas*, tend to uniformity of pressure, have the difference between the electron gas-pressures at the two faces equal to the pull of the virtual potential gradient on all the free electrons in the slice. This consideration, with the reflection that p, the gas pressure of the electrons, is $n\,R\,T$, leads to the equation

$$dp = T\frac{d(nR)}{dl}\,dl = -\,n\frac{dF}{dl}\,dl, \tag{26}$$

whence

$$dF = -\,TR\frac{dn}{n} - TdR. \tag{27}$$

Substituting for R and dR from (24) and (25), we get

$$dF = -\,2T\frac{R_1 - R_2}{n_1 - n_2}\,dn - T\left(R_1 - n_1\frac{R_1 - R_2}{n_1 - n_2}\right)\frac{dn}{n}, \tag{28}$$

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whence

$$F_2 - F_1 = T\frac{R_2n_1 - R_1n_2}{n_1 - n_2}\log\frac{n_1}{n_2} + 2T(R_1 - R_2). \qquad (29)$$

If R_1 is equal to R_2, this reduces to the familiar form

$$F_2 - F_1 = RT\log\frac{n_1}{n_2}, \text{ or } \frac{n_2}{n_1} = \epsilon^{-\frac{(F_2 - F_1)}{RT}},$$

the Boltzmann equation which we have called (4).

The condition for gas-pressure equilibrium in case of thermal effusion, being $p = (RT)^{\frac{1}{2}} \times$ a constant, becomes now, in the isothermal bridge passing from M_1 to M_2, $p = R^{\frac{1}{2}} \times$ a constant. If there were no electrical complication, the condition for equilibrium in this bridge would be

$$\frac{d}{dl}\left(\frac{p}{R^{\frac{1}{2}}}\right) = \frac{1}{R^{\frac{1}{2}}}\frac{dp}{dl} - \frac{p}{2R^{\frac{3}{2}}}\frac{dR}{dl} = 0,$$

or

$$dp = \frac{p}{2}\frac{dR}{R}. \qquad \text{(See eq. (10))}$$

The actual condition for equilibrium is

$$dp - \frac{p}{2}\frac{dR}{R} = -n\frac{dF}{dl}dl,$$

whence, as $p = nRT$, we get

$$T\frac{d(nR)}{dl}dl = \frac{T}{2}ndR - n\frac{dF}{dl}dl, \qquad (26')$$

and so

$$dF = -TR\frac{dn}{n} - \frac{1}{2}TdR. \qquad (27')$$

Substituting for R and dR from (24) and (25) we get

$$dF = -\frac{3}{2}T\frac{R_1 - R_3}{n_1 - n_2}dn - T(R_1 - n_1)\frac{R_1 - R_2}{n_1 - n_2}\frac{dn}{n} \qquad (28')$$

whence

$$F_2 - F_1 = T\frac{R_2n_1 - R_1n_2}{n_1 - n_2}\log\frac{n_1}{n_2} + \frac{3}{2}T(R_1 - R_2). \qquad (29')$$

This differs from (29), which was obtained without the hypothesis of thermal effusion, only in having $\frac{3}{2}$, instead of 2, as the coefficient of

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the last term. If $R_1 = R_2$, this last term disappears and we have as from (29)

$$F_2 - F_1 = RT \log \frac{n_1}{n_2}.$$

The difference of virtual potential expressed by equation (29) applies to the (A) electrons as well as to the (B) electrons; for it is to be remembered that, even when we have been dealing with a temperature gradient, the rate of change of F has depended not at all upon x, though the Thomson effect was found to be a function of x, as the Peltier effect doubtless will prove to be.

The Peltier Effect: — Referring to equation (17) we see that, when $\frac{1}{e}$ electrons pass reversibly at temperature T from M_1 to M_2, the gain of total energy of these electrons, which must be equal to the amount of heat energy absorbed by the electrons from the metals at the junction, is

$$Q = E_2 - E_1 = \frac{3 + 2x_2}{2e} R_2 T + \frac{F_2}{e} + \frac{(1 - x_2)\Phi_2}{e} - \frac{3 + 2x_1}{2e} - \frac{F_1}{e} - \frac{(1 - x_1)\Phi_1}{e}. \tag{30}$$

Substituting for (F_2–F_1) from equation (29), we get

$$Q = \frac{T}{e}\left[(x_2 - \tfrac{1}{2})R_2 - (x_1 - \tfrac{1}{2})R_1\right] + \frac{T}{e} \cdot \frac{R_2 n_1 - R_1 n_2}{n_1 - n_2} \log \frac{n_1}{n_2} + \frac{1}{e}\left[(1 - x_2)\Phi_2 - (1 - x_1)\Phi_1\right]. \tag{31}$$

This is the Peltier effect heat at the junction of temperature T.

With thermal effusion equation (30) would hold unchanged, but (31) would become

$$Q = \frac{T}{e}(x_2 R_2 - x_1 R_1) + \frac{T}{e} \cdot \frac{R_2 n_1 - R_1 n_2}{n_1 - n_2} + \frac{1}{e}\left[(1 - x_2)\Phi_2 - (1 - x_1)\Phi_1\right] \tag{31'}$$

The Volta Effect: — It is evident that the equilibrium which we are discussing between two metals at their junction is of the *mobile* type, the superior gas-pressure of the electrons in one metal maintaining a movement of individual electrons from this metal to the other, while the superior "virtual potential" of the second metal has become great enough to maintain an equal movement of other individual electrons

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from this metal back to the first, the case being pretty closely analogous to the equilibrium between a liquid and its saturated vapor. A still closer analogy, perhaps, is found in the equilibrium of an isothermal atmosphere over some part of the earth. Individual particles in the upper strata are falling toward the earth under the pull of gravity, but their places are taken by equally numerous molecules projected upward by heat energy from the denser layers below.

The question now to be considered is, whether our "virtual" potential-difference, $\frac{1}{e}(F_2-F_1)$ for the electromagnetic unit charge, is the same thing as the Volta potential-difference, which quantity has been the subject of a vast amount of arguing and experimenting for more than a century. In approaching this question let us write

$$\frac{1}{e}(F_2-F_1) = \delta V = \delta_a V + \delta_c V, \tag{32}$$

where $\delta_a v$ is that part of $\frac{1}{e}(F_2-F_1)$ which depends on the difference of specific attraction of the two metals for the electrons, and $\delta_c V$ is the part which is due to the difference of electric *charge* of the two metals.

To give my conception of the Volta potential difference I shall describe an experiment which, if we could work with perfectly clean metals in a vacuum or in a gas absolutely inert, would be easy to carry out, but which, failing this exceedingly difficult condition, must be regarded as imaginary.

In Figure 3 let M_2 be a plate of metal connected with two quadrants

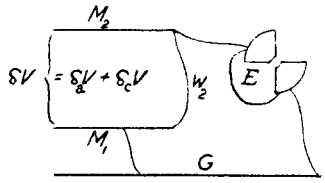

Fig 3

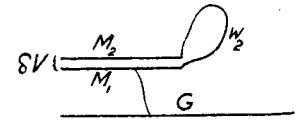

Fig. 4.

of an electrometer, E, and with a distant plate, M_1, of another metal, both M_1 and the quadrants being grounded. For simplicity let us suppose that the quadrants and all connecting wires are of metal (2).

There is, accordingly, no difference of potential between M_2 and the quadrants in Figure 3, but the potential of M_2 exceeds the potential of M_1 by the amount, positive or negative, which we call δV.

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In Figure 4, M_2, disconnected from the electrometer but still connected with the grounded M_1 by the wire w_2, is brought very near to M_1, so that the two act as the plates of a condenser. The difference of potential between them is still δV, but M_2 now acquires a considerable charge, Q. The two plates are next disconnected, and M_2 is then removed to its first position, out of range of the condenser action of M_1, and is connected with the quadrants, now insulated from the ground, as in Figure 5. The charge Q distributes itself over M_2 and the quadrants, but, as the capacity of this combination is much less than that of M_2 when near M_1, the potential now exceeds that of M_1 by the amount $\triangle V = \delta_a V + N \delta_c V$, the value of N being greater the less the distance between M_2 and M_1 in Figure 4.

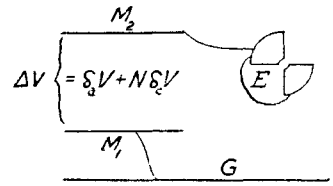

Fig. 5.

If we assume, as we shall do for convenience here, that the capacity of the quadrants is negligible compared with the capacity of M_2 by itself, we take the capacity of M_2 when near M_1, as in Figure 4, to be N times the capacity of M_2 by itself.

After grounding M_2 and the quadrants for a moment, thus reducing their excess of potential over M_1 to the original δV, we bring M_2 again to the same position as in Figure 4 and again connect it with M_1 by means of wire of metal (2); but now through a part of this wire flows a

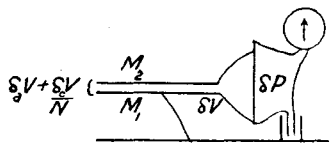

Fig. 6.

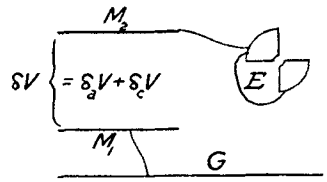

Fig. 7.

current, so that by a potentiometer arrangement, indicated in Figure 6, we can make the M_1 end of the wire exceed the M_2 end of it by a difference of potential δP. If we make δP such that the difference of potential between M_2 and M_1 in Figure 6 is $\delta_a V + (\delta_c V \div N)$, the *charge* on M_2 will be precisely what it was after M_2 and E were grounded from the condition shown in Figure 5; that is, when M_2 is separated from M_1 in Figure 6 and is afterward brought to the position and con-

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nection of Figure 7, its potential excess above M_1 will be δV, just as it was before the last approach of the two plates.

Now in Figure 6 the M_1 end of the connecting wire, being of metal (2), is at potential δV above M_1. Accordingly we have

whence

$$\delta V - \delta P = \delta_a V + \delta_c V - \delta P = \delta_a V + (\delta_c V \div N),$$

$$\delta_c V = \frac{N}{N-1} \cdot \delta P \tag{33}$$

As δP is easily measured, and as N can be made large, we have here indicated a definite, even if not at present entirely practicable, method of measuring δV. This is the Volta potential difference, chemical action excluded.

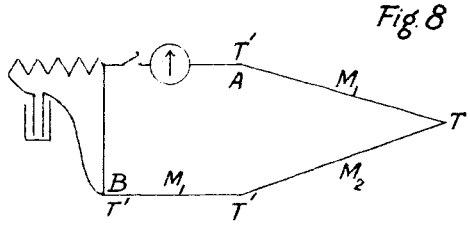
It is evident that the experiments just described would not give the value of $\delta_a V$ or even show whether such a difference of potential, due to differential attraction of the metals for the electrons, really exists. If it does not exist, if $\delta_a V = 0$, the Volta potential difference is our $\frac{1}{e}(F_2 - F_1)$.

But, according to equation (30), $\frac{1}{e}(F_2 - F_1)$ does not account for the whole of the Peltier effect heat, so that, even if $\delta_a V = 0$, the Volta effect is not so related to the Peltier effect that the value of one can be inferred from the value of the other.

Thermo-electromotive-force of a Complete Circuit.

If we have a complete circuit of M_1 and M_2, we may think of its total, or net, electromotive force as measured by means of a potentiometer, and we may with advantage have this potentiometer applied as in Figure 8, having contact with M_1 only, an isothermal stretch of this metal being joined to M_2 at the T' end. The measurement here suggested

Fig. 8



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is strictly analogous to measurement of the e. m. f. of a galvanic cell in open circuit.

Our preceding discussion suggests two expressions for this net e.m.f. of the circuit. On the one hand, since we begin and end with the same metal, M_1, at the same temperature, T', the difference of F between these two terminals must be independent of the specific attraction of M_1 for the electrons, and must therefore represent the charge-difference of potential, the difference of potential in the ordinary sense, between the terminals. Accordingly we have for the open circuit of Fig. 8,

$$\begin{aligned} E = \frac{1}{e}\int_A^B dF = \frac{T}{e}\Big[& \frac{R_2 n_1 - R_1 n_2}{n_1 - n_2} \log \frac{n_1}{n_2} + 2(R_1 - R_2) \\ & + \Big(1 + \frac{\nu_1}{1+\rho_1}\Big) R_1 - \Big(1 + \frac{\nu_2}{1+\rho_2}\Big) R_2 \Big] \\ - \frac{T'}{e}\Big[& \frac{R'_2 n'_1 - R'_1 n'_2}{n'_1 - n'_2} \log \frac{n'_1}{n'_2} + 2\,(R'_1 - R'_2) \\ & - \Big(1 + \frac{\nu_1}{1+\rho_1}\Big) R'_1 + \Big(1 + \frac{\nu_2}{1+\rho_2}\Big) R'_2 . \Big] \end{aligned} \tag{34}$$

Equation (34′), from (16′) and (29′), for the case of thermal effusion, would differ from (34) only in having, as the first term within the brackets containing ν, $\frac{1}{2}$ instead of 1, and in having $\frac{3}{2}$ instead of 2 as the coefficient of the $(R_1–R_2)$ and $(R_1'–R_2')$.

The other way of getting an expression for E is to integrate dQ from A to B in Figure 8, assuming now that a current, due to the thermo-electromotive-force, is flowing in the circuit, with sufficient resistance between A and B, at the low temperature part of the cycle, to absorb there practically all the work done by this current, the resistance of the rest of the circuit being negligible in comparison. But, if we perform this integration, we find that the expression obtained for E reduces to precisely what we already have in (34) or in (34′), as of course it should.

If R were a constant and the same for both metals, and if ν_1 were equal to ν_2, we should get from each of these equations

$$E = \frac{1}{e}(T - T')\, R \log \frac{n_1}{n_2}. \tag{35}$$

That is, E would be simply proportional to $(T–T')$; but in order that this relation may hold we must have the representative lines of the two metals, on the temperature-entropy diagram, the same distance

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apart at every temperature, which is equivalent to saying that the Thomson effect coefficients, σ_1 and σ_2, of the two metals must be equal at every temperature.

Examination of equations (15) and (15′) shows directly that the conditions just imagined, with respect to R and ν, would make σ the same for both metals. This agreement is important only in so far as it gives evidence that the argument carried through in this paper is self-consistent.

Inspection of equation (34), which expresses the net or total e.m.f. of the circuit, answers in some measure the question raised on p. 72; namely, what conditions of the thermo-electric circuit furnish the mechanism for the production of electric energy at the expense of heat. It is to be noted that this equation makes no reference to the possible tendency of either metal to draw electrons to itself from the other metal, or the possible tendency of hot metal to draw electrons from cold metal, or *vice versa*. The absence of any such reference is rational; for the net amount of work done on any electron in passing completely around the circuit must be independent of any such attraction. It is to be observed also that equation (34) does not contain x, which indicates the relative importance of the (B) electrons and the (A) electrons in conduction. This omission results from my assumption that the (A) electrons have the same value of R, at any given temperature, that the (B) electrons have. Further consideration may show this assumption to be unjustifiable, in which case more complicated formulae must replace some of the equations of this paper.

On the other hand, it may appear later that I have made too sharp a distinction between the (A) electrons and the (B) electrons.

Although the theory set forth in this paper has been formed with no regard to the transverse electromagnetic effects observed in a conductor carrying a current across a magnetic field, it nevertheless, taken in connection with the idea that the free electrons may have something like the Maxwellian distribution of heat velocities, seems to offer some hope of light in these dark places. I must, however, defer discussion of this matter.

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Appendix.

Sir J. J. Thomson,[22] after discussing at length various aspects and consequences of the free electron theory of electric conduction, comes "to the conclusion that the mechanism by which we have supposed the electric current to be conveyed through a conductor is at most only a part and not the whole of the process of metallic 'conduction.'" He continues, "One reason for this conclusion is the large changes which take place in the electrical resistance of some metals on fusion, changes which do not seem to be accompanied by any corresponding change in thermoelectric quality. Thus the conductivities of tin, zinc, and lead at their melting points are, when the metals are in the solid state, about twice what they are in the liquid."

He then goes on to say that the free electron theory, as previously developed by himself and others, would predict for these substances "a Peltier effect between the solid and the liquid metal about half the magnitude of that between bismuth and antimony, and thus, as these effects go, exceedingly large. Now Fitzgerald, Minarelli and Obermeyer, as quoted by G. Wiedemann, '*Elektricität*,' II, p. 289, could detect no sudden change in thermo-electric circuits with these metals when they passed from the solid to the liquid state."

This passage from Thomson, only vaguely remembered, had little if anything to do with the course of my speculations and argument as developed in the paper just concluded; but at the end, being strongly of the opinion that the part played by the free electrons in metallic conduction is a very subordinate one, and that the spaces between the atoms are the great obstacles to electric flow, I naturally turned with curiosity to the question whether the change of resistance in the fusion of metals can be accounted for by the mere change of volume, and so of mean distance between adjacent atoms, which occurs in this change of physical state.

Examination of the available data in this field of inquiry speedily showed one important fact; that, of the twelve metals for which we have trustworthy information concerning change of volume and change of resistance in fusion, ten, cadmium, caesium, copper, lead, mercury, potassium, sodium, thallium, tin, and zinc, increase in both volume and resistivity during fusion, while two, bismuth and gallium, decrease in both properties. Antimony, which decreases in resistance during fusion, may increase slightly in volume during the same change; but this is still in doubt. Northrup, who has been doing excellent

22 *Corpuscular Theory of Matter*, p. 75.

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work,[23] in studying the effect of fusion on the resistance of metals, declares the behavior of solid antimony at temperatures considerably below the melting point to be erratic, and he gives no estimate of the change of volume during melting.[24] Gold, which undoubtedly increases in resistance on becoming liquid, is very generally believed to expand at the same time, but diligent search and inquiry have failed to bring me quantitative information in regard to this change of volume.

In trying to push my investigation further and find whether, among metals in general, the observed changes of volume are of the right magnitude, as well as of the right sign, to account for the change of resistance, I meet with serious difficulties. There is a great dearth of satisfactory data, especially in regard to changes of volume during fusion, and moreover the theory of conduction which I have been setting forth, though it would predict increase of resistance with increase of volume, other things being equal, has nothing obvious to say as to the effect of fusion, without regard to change of volume, on resistance. Nevertheless, I believe that the data here brought together, and the results of an experiment which I have tried with these data, are worth putting into print. The data appear in the first three columns of the following table *X*. The results of my handling of these data make the last two columns, headed t_r and t_v respectively. Of these, t_r is the estimated temperature at which the solid, if it could remain such at atmospheric pressure through the implied heating or cooling, would have the resistivity which the liquid has at the melting point; t_v is the estimated temperature at which the solid, if it could remain such at atmospheric pressure, would have the volume which the liquid has at the melting point.

Change of Resistivity and Volume in Fusion.

t_m = melt. temp. of the metal.
R_s = resistivity " " " solid at t_m.
R_l = " " " " liquid at t_m.
V_s = sp. volume " " " solid at t_m.
V_l = " " " " " liquid at t_m.
t_r = estimated temp. at which solid wd. have res. $= R_l$
t_v = " " " " " " " sp. vol. $= V_l$.

23 See, for example, vol. 175 of the *Journal of the Franklin Institute*.

24 Pascal and Jouniaux, C. R., Feb. 9, 1914, give 6.55 as the density of liquid antimony at its melting point, 631° C. They hope to determine the density of the solid at the same temperature. The value given by Landolt and Börnstein for the solid at 20°, taken with the value they give for the coefficient of expansion, would make the density of the solid at 631° about 6.50.

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TABLE X.

Metal	t_m	$R_l \div R_s$	$V_l \div V_s$	t_r	t_v
Bismuth	270°	0.48	0.967	30°	−565°
Cadmium	320°	1.97	1.047	600°	832°
Caesium	27°	1.69	1.023	112°	107°
Copper	1083°	2.09	1.027	1760°	1240°
Gallium	30°	0.50?	0.98?		
Gold	1063°	2.28	>1?	1950°	1790?[25]
Lead	327°	1.92	1.032	675°	716°
Mercury	−39°	4.22	1.036	735°	236°
Potassium	60°	1.61	1.024	174°	160°
Rubidium	38°	1.37	1.023	109°	126°
Sodium	98°	1.51	1.015	210°	171°
Thallium	300°	1.90	1.043	780°	768°
Tin	232°	2.15	1.027	700°	638°
Zinc	420°	2.19	1.04?	800°	1220°

In some cases t_r and t_v were obtained by calculation, in others by a graphical process which is illustrated in Figures *A* and *B*. It is evident that the extrapolations indicated by the dotted extensions of the curves in these Figures are extremely hypothetical, and there was, perhaps, no good *a priori* ground for expecting that t_r and t_v would prove to be even of the same order of magnitude.

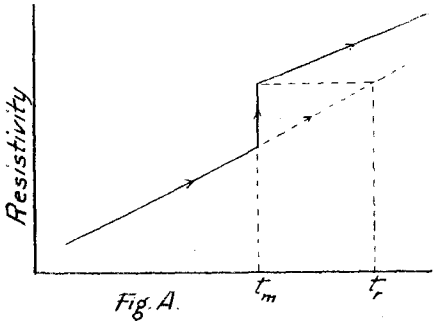

Fig. A.

[25] If $V_l \div V_s = 1.03$.

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That they are, in many cases, pretty nearly the same appears to be a fact of some significance, though this significance may be at present obscure. In explanation of and apology for the very marked disagreement of these two hypothetical temperatures in the case of bismuth, and in illustration of the degree of precision needed in the data used, I will say that, if $(V_l \div V_s)$ for bismuth were taken as 0.98, t_v would be $-235°$; and if the ratio were taken as 0.99, t_v would be 17°. Evidently the precise determination of $(V_l \div V_s)$ in a crystalline substance like bismuth is very difficult.

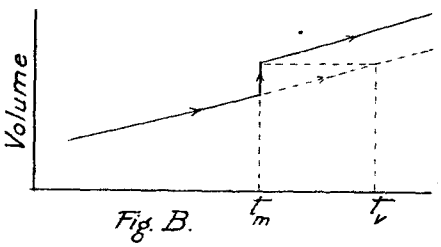

Fig. B.

In drawing curves like those of Figures *A* and *B*, I sometimes ignored the course of the curve for the solid just below the melting point, for the reason that for some metals there was a rapid change in the curvature here, as if the melting were already incipient.

Although one can hardly study the table here given without being convinced that the change of volume in fusion is very closely connected with the change of resistance, and is in some large measure the cause of it, examination of the known phenomena of conduction in the solid state shows that we cannot account for changes of resistance by consideration of changes of volume only. For example, the following Table Y, the second, third and fourth columns of which are taken from an article by E. Grüneisen,[26] shows by comparison of the fourth and fifth columns that increase of volume is not a very important factor in the increase of resistance which accompanies rise of temperature at constant pressure in the metals here exhibited.

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26 Bericht. d. Deutschen Physikalischen Gesellschaft, Heft **6**, S. 198 (1913).

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TABLE Y.

FOR 0° C.

Pure metals	1 $\frac{1}{v}\left(\frac{dv}{dT}\right)_p$	2 $-\frac{1}{v}\left(\frac{dv}{dp}\right)_T$ *cm.²/kg*	3 $\frac{1}{w}\left(\frac{dw}{dT}\right)_p$	4 $-\frac{1}{w}\left(\frac{dw}{dp}\right)_T$ *cm.²/kg*	5 $\frac{1}{w}\left(\frac{dw}{dT}\right)_v$
Al.	70×10^{-6}	133×10^{-8}	40×10^{-4}	43×10^{-7}	37.7×10^{-4}
Ni.	40 "	56 "	60 "	16 "	59 "
Cu.	45 "	75 "	43 "	22 "	42 "
Ag.	54 "	90 "	40 "	39 "	37.7 "
Cd.	90 "	210 "	40 "	100 "	35.7 "
Pt.	27 "	40 "	39 "	20 "	37.6 "
Au.	40 "	59 "	40 "	30 "	38 "
Pb.	88 "	210 "	42 "	150 "	35.7 "

The method by which the numbers of column 5 are found may be shown by an example. We have for aluminium

(1) Coef. of cubical expansion = 70×10^{-6}.

(2) " " compressibility = 133×10^{-8}.

$\therefore$ No. of atm. (cm^2/kg) to keep v const. during 1° heating

$$= 70 \div 1.33 = 52.5.$$

(3) Compression coef. of resistivity = 43×10^{-7}.

$\therefore$ Decrease ($-dw/w$) of resistivity caused by 52.5 atm.

$$= 43 \times 10^{-7} \times 52.5 = 226 \times 10^{-6}.$$

(4) Temp. coef. (p const.) of resistivity = 40×10^{-4}.

(5) Temp. coef. (v const.) of resistivity

$$= 40 \times 10^{-4} - 226 \times 10^{-6} = 37.7 \times 10^{-4}.$$

From the data given by Barus[27] the following statement for liquid mercury is derived:

27 Bulletin of the U. S. Geol. Survey, No. 92, 74 (1892).

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(1)	(2)	(3)	(4)	(5)
180×10^{-6}	300×10^{-8}	8×10^{-4}	300×10^{-7}	-10×10^{-4}

The negative sign of the value of (5) indicates, as Barus points out, that in liquid mercury "the immediate electrical effect of rise of temperature, ... is a decrement of specific resistance."

Whether a like statement would hold true of other metals in the liquid state we have, so far as I am aware, insufficient data to determine.

Summary.

1. It is pointed out that thermal capacity of the electrons seems to be a necessary condition for thermoelectric action.

2. It is assumed that, through a considerable range of temperature, the number of free electrons per cu. cm. of a metal is represented by the formula $n = k_n T^{\nu}$, where k_n and ν are constants, T being absolute temperature, and that R, of the equation $pv = RT$, "reckoned for a single electron," is represented by the formula $R = k_r T^{\rho}$, where k_r and ρ are constants, R being less, at ordinary temperatures, for an electron than for a gas molecule.

3. Electric conduction is supposed to be maintained in part by free electrons, (B), acting very much like gas molecules in the interatomic spaces of the metal, and in part by other electrons, (A), which pass directly from atom to atom, perhaps during collisions, without taking part in the gas-pressure action of the (B) electrons.

4. Formulas are obtained for the specific conductivity and the Thomson effect coefficient of a metal, as these properties would be if dependent entirely on the free electrons; and it is found that these two expressions are either incompatible with each other or in disaccord with observed facts. Argument leads to the conclusion that the free electrons are necessary for the phenomena of thermo-electric action but play an unimportant part in electric conduction.

5. Expressions are found for the Peltier effect and for the difference of "virtual" potential at the junction of two metals, virtual potential being due in part to electric charge and in part to the specific attraction of metals for the electrons. In this connection the Volta effect is discussed.

6. Inspection of the formula obtained for the net e.m.f. of a thermoelectric circuit shows this total to be dependent on the T's, n's, ν's and ρ's of the system and not upon any specific attraction of metals for the electrons.

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7. An Appendix gives data relative to change of volume and change of electric resistance in the melting of metals. It is a general, if not an invariable, rule that change of resistance in melting is of the same sign as change of volume. An attempt is made at a quantitative comparison of these two changes.

8. Attention is called to the fact, first pointed out by Barus, that in liquid mercury rise of temperature without change of volume would bring a *decrease* of resistance.

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ON THE THEORY OF THE RECTILINEAR OSCILLATOR.

By Edwin Bidwell Wilson.

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ON THE THEORY OF THE RECTILINEAR OSCILLATOR.[1]

By Edwin Bidwell Wilson.

Introduction.

Planck's rectilinear oscillator is of great importance in theoretical physics. To grant this it is not necessary to assert a belief in the physical existence of the oscillator, much less to assert Planck's belief in such existence. The Carnot engine may not exist; but the concept has proved so fruitful as to be fundamental in the general field of thermodynamics. The (perfect) semipermeable membrane may be a figment of the imagination; but, in special fields in thermodynamics, reasoning based on the membrane is of crucial importance. And in that branch of thermodynamics which deals with radiant energy, the oscillator has held the position of cornerstone. Moreover, as in the case of the Carnot engine and semipermeable membrane, there does exist in physics a (more or less imperfect) prototype of the oscillator — the radiator or resonator of Hertz and his followers in radiotelegraphy.

In addition to the thermodynamic importance there is an interest attaching to the oscillator as the simplest model of the atom. Hypothetical models of the atom vary greatly; some are founded on statical configurations, others on dynamic equilibrium, some call for a number of corpuscles per atom comparable with the atomic weight, others for a number far greater than this, some represent the positive charge as widely spread out with the negative corpuscles imbedded in it, others concentrate the positive charge at a point. It is not necessary to take the oscillator very seriously as a model of an atom; all that is useful is to observe that by its means energy is absorbed and emitted and that thus it performs the functions of matter in establishing a temperature equilibrium in a closed field of radiant energy.

The object of this article is to make some comments on the equation which defines the motion of the oscillator. These comments are obvious at first sight to any one who examines the differential equations used in the theory of the oscillator, but as I cannot find them in the literature, I venture to print them at this late day.

1. **The differential equations.** Let us consider first some of the

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[1] Read to the Academy at its Stated Meeting, March 11, 1914. Received June 16, 1914.

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prime facts or hypotheses underlying the analysis. These suppositions are as follows:

1°. The oscillator executes in the main a simple harmonic motion. This leads to the statement of the (approximate) equation

$$\frac{d^2x}{dt^2} + k^2x = 0, \tag{1}$$

where x is the coordinate defining the state of the system.

The basic reason for this assumption is that it is desirable for the oscillator in its function of emitter of energy to be as simple as possible, that is, to emit monochromatic radiation of a definite frequency; for our customary analysis resolves heterogeneous radiation into monochromatic elements.

(If we wish to assume that the oscillator consists of a widely spread uniform positive charge at rest and of a concentrated negative charge vibrating to and fro through the center of the positive charge, we may obtain a model of the oscillator which satisfies the condition that the force on the moving particle shall be proportional to the displacement; for if a particle moves within a uniform sphere and if the elemental attraction follows the Newtonian law, the resultant force is proportional to the distance of the particle from the center of the sphere.)

2°. The motion of the oscillator is not affected by frictional resistance. This leads to (or corresponds with) the omission of a possible corrective term proportional to the velocity dx/dt in (1).

Perhaps the best reason for omitting frictional terms is that there seems to be no necessity for complicating the mathematical equations or physical hypotheses by admitting friction. In the Hertz radiator there is frictional dissipation of energy due to the resistance of the circuit; in the idealised oscillator the resistance would naturally be omitted.

3°. The motion is damped by the emission of energy. If the form of (1) appropriate to the principle of energy be used, that is, the integrated form, the undamped motion would be characterized by the equation

$$\left(\frac{dx}{dt}\right)^2 + k^2x^2 = C \qquad \text{or} \qquad \frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + k^2x^2\right] = 0,$$

and the damping would then be inserted by a change to the form

$$\left(\frac{dx}{dt}\right)^2 + k^2x^2 = C - 2\int_0^t F\,dt \quad \text{or} \quad \frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + k^2x^2\right] = -2F, \tag{2}$$

where F is the rate of radiation.

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The modification of the equation is thus based directly on the principle of energy. The form in which F is thrown depends somewhat on the point of view.

If the oscillator is considered as a Hertz dipol and the ordinary solution for the electromagnetic field set up by this radiator at large distances from its center is obtained, we find by the application of Poynting's theorem that a certain amount of energy is radiated off per period and, in particular, that the amount is proportional to the square of the acceleration. Then the equation, written for a complete integrated period T is

$$\int_{t_0}^{t_0+T} \left\{ \frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + k^2x^2 \right] + 2\mu \left(\frac{d^2x}{dt^2}\right)^2 \right\} dt = 0.$$

An integration by parts transforms the third term. Whence

$$\int_{t_0}^{t_0+T} \left\{ \frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + k^2x^2 \right] - 2\mu \frac{dx}{dt}\frac{d^3x}{dt^3} \right\} dt + 2\mu \left[\frac{dx}{dt}\frac{d^2x}{dt^2}\right]_{t_0}^{t_0+T} = 0. \tag{3}$$

Now if t_0 be an epoch for which either the velocity or the acceleration vanishes, the last term drops out. Under these conditions

$$\int_{t_0}^{t_0+T} \left\{ \frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + k^2x^2 \right] - 2\mu \frac{dx}{dt}\frac{d^3x}{dt^3} \right\} dt = 0.$$

The conclusion is then drawn that

$$\frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + k^2x^2 \right] - 2\mu \frac{dx}{dt}\frac{d^3x}{dt^3} = 0. \tag{4}$$

Exceptions can be taken to this conclusion whether the argument is as above or as given in slightly different form by Planck.[2] It by no means follows that, because the integral of a function over a complete period from t_0+nT to $t_0+(n+1)T$ vanishes, the integrand must therefore be identically null.

What we may infer is that

$$\frac{d}{dt}\left[\left(\frac{dx}{dt}\right)^2 + k^2x^2 \right] - 2\mu \frac{dx}{dt}\frac{d^3x}{dt^3} = a(t)$$

where $a(t_0+(n+1)T) - a(t_0+nT) = 0$. There is nothing at all to

2 Planck, Vorlesungen über die Theorie der Wärmestrahlung, first edition, 110 (1906). Abraham, Theorie der Elektrizität, second edition, 70 (1908). Rüdenberg, Annalen der Physik, **25**, 346–466 (1908). (In his second edition, Planck introduces the quantum hypothesis early and omits this derivation of the equation of the oscillator.)

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suggest that the function $a(t)$ has a negligible effect upon the motion. It therefore follows that (4) or the equation

$$\frac{d^2x}{dt^2} + k^2x - \mu \frac{d^3x}{dt^3} = 0, \tag{5}$$

obtained therefrom by discarding the factor dx/dt, is at best only approximate. Concerning the degree of approximation we may perhaps be willing to admit that the error is always extremely small; but we should be very chary about admitting that the error is necessarily small relative to the terms retained.

That (5) is only an approximate result is clearly brought out by Planck in his derivation, but the clearness is somewhat obscured by a footnote referring to Abraham's derivation in a manner which implies, though it does not state, that the latter proof is exact. Abraham gives in the first instance essentially the proof which we have given above. He goes on, however, to show by a discussion of the physical dimensions of the terms that the equation (5) is the only possible one. What he really proves is that under certain hypotheses (5) *is the only possible linear equation of the form*

$$\frac{d^2x}{dt^2} + k^2x = \sum_i c_i \frac{d^iv}{dt^i}.$$

The physical dimensions are equally well satisfied by the non-linear equation

$$\frac{dx}{dt}\left[\frac{d^2x}{dt^2} + k^2x\right] + \mu\left(\frac{d^2x}{dt^2}\right)^2 = 0, \tag{6}$$

which follows naturally and immediately from (2), that is, from the principle of energy, provided it is assumed that the rate of radiation is proportional *at each instant* to the square of the acceleration, instead of merely proportional thereto when integrated over a complete period, and which would arise from the equation in integrated form, provided one were willing to apply to it directly the reasoning which is applied after an integration by parts.

Lorentz has also derived (5) by a method which throws emphasis on the approximate nature of the result.[3] That he regards the equation with some suspicion may be inferred from his statement of a later date[4]: Eine Gleichung "die übrigens den Mathematikern noch manche Frage darböte."

3 See, for example, his Theory of Electrons, 251, (1909).

4 Physikalische Zeitschrift, **11**, 1250 (1910) It was this remark which so strengthened my own suspicions as to lead me to take up the question some time ago.

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The calculation of the instantaneous rate of radiation of energy from a moving electron, as contrasted with the integrated rate hitherto used, has been carried out by a number of authors in different ways. The rate is almost universally found proportional to the square of the acceleration when the square of the velocity is negligible relative to the square of the velocity of light.[5] The mathematical statement of this result is contained in (6), and this equation therefore might more properly be taken as representative of the motion of the oscillator than the equation (5),— provided, of course, that we are willing to use the electron model of the oscillator.

A comparison of (5) and (6) will throw some light on the legitimacy of the approximations used in deriving (5). The terms to be compared are

$$-\mu \frac{dx}{dt}\frac{d^3x}{dt^2} \quad \text{and} \quad \mu\left(\frac{d^2x}{dt^2}\right)^2.$$

On the supposition that the oscillator executes in the main a simple harmonic motion, the signs of the two terms are the same. But the phases of the magnitudes are very different; for the first term is a maximum as the particle passes through the position of equilibrium and vanishes in the positions of extreme elongation, whereas the second term is a maximum in the extreme positions and vanishes in the position of equilibrium. The average magnitudes of the terms are the same, but it is difficult to see how two terms of equal average magnitude could be more different than these two in their effect upon the equation of motion.

We shall now discuss the two forms (5) and (6), the first rather briefly, because it has been tolerably well treated, the second more in detail, because it seems to have been neglected.

2. **Discussion of (5).** As we have to deal with cubic equations and their (approximate) solutions, it is necessary to determine the order of magnitude of the coefficients and it is desirable to put the equations into their simplest form. If m is the mass of the vibrating particle, e the charge in electrostatic units, c the velocity of light, and if we adopt the customary coefficient for the radiation term, we have

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5 For two different lines of deduction see J. J. Thomson, Electricity and Matter, chap. 3 (1904), and Wilson and Lewis, The Space-time Manifold of Relativity, These Proceedings, **48**, 389–507 (1912), with especial reference to p. 480. A. Macdonald in his Electric Waves, 75 (1902), finds a steady rate of radiation from the oscillator.

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$$m\left(\frac{d^2x}{dt^2}+k^2x\right)-\frac{2}{3}\frac{e^2}{c^3}\frac{d^3x}{dt^3}=0$$

or

$$m\frac{dx}{dt}\left[\frac{d^2x}{dt^2}+k^2x\right]+\frac{2}{3}\frac{e^2}{c^3}\left(\frac{d^2x}{dt^2}\right)^2=0.$$

As numerical values we may assume

$$e=4.7\ 10^{-10},\qquad m=9\ 10^{-28},\qquad c=3\ 10^{10}.$$

Hence

$$\frac{2}{3}\frac{e^2}{mc^3}=6\ 10^{-24}.$$

Substitute $t=a\tau$ and choose $a^2k^2=1$. Then

$$\frac{d^2x}{d\tau^2}+x-6\,k\,10^{-24}\frac{d^3x}{d\tau^3}=0$$

and

$$\frac{dx}{d\tau}\left[\frac{d^2x}{d\tau^2}+x\right]+6\,k\,10^{-24}\left(\frac{d^2x}{d\tau^2}\right)^2=0.$$

Now $k=2\pi c/\lambda$, where λ is the wave length. The observed range of λ in the spectrum is from 10^{-5} to $3\ 10^{-2}$ with λ about $5.5\ 10^{-5}$ in the center of the visible spectrum. The range for the coefficient of the last term in the equations is therefore from about 10^{-7} in the ultra-violet to about $4\ 10^{-11}$ in the infra-red with $2\ 10^{-8}$ for the value near the middle of the range of visibility.

The numerical forms of the equations are therefore

$$\frac{d^2x}{d\tau^2}+x-\kappa\frac{d^3x}{d\tau^3}=0 \tag{5'}$$

and

$$\frac{dx}{d\tau}\left[\frac{d^2x}{d\tau^2}+x\right]+\kappa\left(\frac{d^2x}{d\tau^2}\right)^2=0 \tag{6'}$$

with

$$10^{-7}>\kappa>4\ 10^{-11}\qquad\text{or}\qquad\kappa=2\ 10^{-8},$$

the latter being taken as a sort of standard value.

(It is not necessary to make the special assumptions involved in the electron model of the oscillator if we desire merely to determine the order of magnitude of κ. If (5') be replaced by

$$\frac{d^2x}{d\tau^2}+\kappa\frac{dx}{d\tau}+x=0, \tag{7}$$

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which is nearly equivalent to it, since x and d^2x/dt^2 are nearly equal, the amplitude of the motion is seen to suffer a reduction in the ratio

$$e^{-\frac{1}{2}\kappa 2\pi n} : 1$$

during n complete oscillations. We may now use experiments on the distinctness of interference fringes for different path-lengths to obtain an estimate for κ, and we find that $2\ 10^{-8}$ is an entirely reasonable value. See Abraham, loc. cit.)

The complete solution of (5′) will be of the form

$$x = C_1 e^{\alpha\tau} + e^{-\beta\tau}(C_2 \cos\gamma\tau + C_3 \sin\gamma\tau),$$

where $\alpha, -\beta \pm \gamma i$ are the roots of the cubic

$$r^2 + r - \kappa r^3 = 0.$$

The root α is positive and of great magnitude, approximately $1/\kappa$, say, $\frac{1}{2}\ 10^8$. As Planck says, this root has no physical significance.[6] This means that for every determination of the constants C_1, C_2, C_3, the value of C_1 must vanish and the solution of the equation reduce to

$$x = e^{-\beta\tau}(C_2 \cos\gamma\tau + C_3 \sin\gamma\tau).$$

It is certainly unfortunate that we are forced to discard so arbitrarily one root of an equation which we have labored so carefully to establish. Moreover, as the solution of the equation now depends only on two constants, the elimination of these constants by differentiation will give an equation of the second order which accounts satisfactorily for the phenomenon as far as its mathematical side is concerned. This equation, which is almost identical with (7), contains a friction term, it is true, and is unsatisfactory physically because of this fact. But there is no real reason for considering it as less satisfactory than (5′) which has a physically meaningless root and which moreover, while ostensibly allowing for radiation on a physical basis, actually requires the rate of radiation to be maximum where there is a general agreement that it should be minimum, and minimum where it should be maximum.

3. Discussion of (6). We turn next to the equation (6′) and, for the sake of brevity, write it as

$$\kappa f^2 + vf + vx = 0, \tag{8}$$

f and v designating respectively acceleration and velocity when the

[6] If we make clear the assumption of the quasi-stationary state as a preliminary condition in our approximations which lead to the equation, the large root is sure to be inadmissible by virtue of our assumption.

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unit of time is the fraction $\lambda/(2\pi c)$ of the second. This equation differs very minutely from (5′) or (7), but there is no reason to believe that the motion defined by the equation should resemble very closely the motion defined by those. As the integration of the non-linear equation of the second order is not simple, we shall first examine the equation to find certain elementary qualities of the solution.

A. When the velocity vanishes, the acceleration also vanishes.

B. When the acceleration vanishes, either the velocity or the displacement vanishes.

C. In the neutral position $x = 0$, either the acceleration vanishes or the acceleration and velocity satisfy the relation $\kappa f + v = 0$.

D. Arbitrarily assumed (initial) values of the acceleration and position determine the velocity, and similarly values of the acceleration and velocity determine the position, but assumed values of the velocity and position allow the acceleration either of two values

$$f = -\frac{v}{2\kappa} \pm \frac{1}{2\kappa}\sqrt{v^2 - 4\kappa v x}.$$

E. Values of v and x which make $v^2 - 4\kappa v x$ negative are not admissible. Hence if x is positive, v must either exceed $4\kappa x$ or be negative; and if x is negative, v must be positive or less than $4\kappa x$ (algebraically).

F. The case of x positive and v greater than $4\kappa x$ leads to a *point d'arrêt*. For under these conditions f is negative, no matter which sign is taken before the radical, and the velocity must be decreasing as x increases; when v becomes equal to $4\kappa x$, the acceleration becomes imaginary, and so does the motion. The same arises under the case of x negative and v less than $4\kappa x$.

G. The case x positive and v negative also leads to a point d'arrêt. For under these conditions if the negative sign is taken with the radical, then f is negative and the magnitude of v is increasing as x diminishes toward zero, the particle will reach the origin with a definite velocity and pass through to the negative values of x where it falls under the case *F;* and if the positive sign is taken with the radical, so that f is positive, the particle is slowing down, and we have to consider two possibilities, first where the particle reaches the origin before its velocity vanishes and where the argument just given will then again be applicable, second where the particle comes to rest before reaching the origin only to start on again with a negative f and fall under the first supposition of this case *G.* A similar discussion may be given for the hypothesis x negative and v positive. (There

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is a special case in which the particle just comes to rest at the origin, remaining there in a sort of unstable equilibrium.)

The consequences of this discussion of the second type of equation — the equation (6) founded on the radiation formula generally adopted — may be formulated in the unsatisfactory statement that, except for two special possibilities, the motion runs into a point d'arrêt where at a certain time and in a certain position, with a definite velocity and acceleration neither of which vanishes, the motion suddenly becomes imaginary. This is very disconcerting; it is at direct variance with our accepted notions of what happens in mechanical or electrodynamic systems.

It is also a consequence that the oscillator governed by equation (6) cannot oscillate.

Unfamiliar as this result may sound to those who have a feeling that a slight change in the differential equation introduces only a slight change in the motion, we must nevertheless admit the result and develop the contrary feeling that a slight change may entirely alter the type of motion. This fact has been brought out by Borel in a paper just printed [7] wherein it is shown that the equation $v^2 + x^2 = a^2 + \lambda t$, no matter how small the constant λ, does not define an oscillatory motion, nor does the motion approach the oscillatory type as λ approaches zero. Any one might admit that, in the course of a long time, λt would become important and the type of motion be changed; but this is not the true explanation, for, no matter how small the constant λ, the motion departs from the simple harmonic type from the time $t = \pi/2$, that is, after executing a quarter oscillation.

4. **The integrals of (6).** We shall now turn to the integration of the equation

$$\frac{dx}{d\tau}\left(\frac{d^2x}{d\tau^2} + x\right) + \kappa\left(\frac{d^2x}{d\tau^2}\right)^2 = 0. \qquad (6')$$

The first thing we shall treat is motion from a position of rest. In

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7 Borel, Annali di Matematica (**3**), **21**, 225, (1913). The equation of Borel corresponds to a constant radiation of energy. The solutions of the equation do not exhibit the point d'arrêt, but are continuous. It has seemed to me, notwithstanding the publication of Borel's paper, and perhaps even on account of it, that the printing of my own results would be of interest. The comments on Borel's paper, which he prints as a footnote to a brief account of his results in his new book, Introduction Géométrique à Quelques Théories Physiques, 107 (1914) appear less applicable to the results of our present work.

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this case v and f are both zero. Let us therefore assume a solution for x as a series

$$x = a(1 + \alpha\tau^3 + \beta\tau^4 + \gamma\tau^5 + \ldots)$$

in τ. The equations for the determination of the coefficients are

$$36\alpha^2\kappa + 3\alpha = 0, \qquad 144\alpha\beta\kappa + 18\alpha^2 + 4\beta = 0,$$
$$(144\beta^2 + 240\alpha\gamma)\kappa + 60\alpha\beta + 5\gamma = 0, \ldots.$$

Hence

$$\alpha = 0, \qquad \beta = 0, \qquad \gamma = 0, \ldots.$$

or

$$\alpha = -\frac{1}{12\kappa}, \qquad \beta = \frac{1}{64\kappa^2}, \qquad \gamma = -\frac{11}{3840\kappa^3}, \ldots.$$

Thus one solution for x is $x = a$, the particle remains at rest in the initial position, and the other solution is

$$x = a\left(1 - \frac{\tau^3}{12\kappa} + \frac{\tau^4}{64\kappa^2} - \frac{11\tau^5}{3840\kappa^3} + \ldots.\right), \tag{9}$$

the particle moves with an acceleration toward the origin, as was foreseen under G above.

If this solution in series is valid, that is, if the series converges, it is at any rate of no value for purposes of calculation unless τ is exceedingly small, that is, for intervals of time which are very small compared with the period of monochromatic light ($\tau = 2\pi$). The value of the series is first to show how the motion starts and second to show that there is a possibility of motion, that is, that there is a solution of the equation other than the solution $x = a$ with the given initial conditions. In discussing this question we shall follow the method given by Picard.[8]

Let the equation (6′) of the second order be written as two simultaneous equations of the first order, namely,

$$\kappa\left(\frac{dv}{d\tau}\right)^2 + v\left(\frac{dv}{d\tau} + x\right) = 0, \qquad \frac{dx}{d\tau} - v = 0. \tag{10}$$

These may be expressed in solved form as

$$\frac{dx}{d\tau} = v, \qquad \frac{dv}{d\tau} = -\frac{v}{2\kappa} \pm \frac{1}{2\kappa}\sqrt{v^2 - 4\kappa v x}.$$

The fundamental theorem on the existence of integrals fails for those

[8] E. Picard, Traité d'Analyse, **3**, chap. 3.

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values of the variables for which two values of the derivatives become equal. In this case the failure is when

$$v^2 - 4\kappa v x = 0.$$

This equation factors, and there are therefore two surfaces

$$v = 0, \qquad v = 4\kappa x$$

in the space (v, x, τ) which are singular in the sense that the fundamental existence theorem fails.

Of these surfaces one, namely, $v = 0$, satisfies the differential equation (6′) and is therefore singular also in the sense that it is the envelope of particular solutions. It is possible to put the particular solutions in evidence. For let x be positive so that $v \leqq 0$. Introduce a new variable $v = -z^2$ and let the sign of z be so chosen that z increases with τ. We have then the equation

$$-2z\frac{dz}{d\tau} = -\frac{z^2}{2\kappa} \pm \sqrt{z^4 + 4\kappa z^2 x}.$$

In cancelling the variable z the double sign goes out. For since f is decreasing with τ (see G above) and since

$$f = \frac{dv}{d\tau} = -2z\frac{dz}{d\tau},$$

it follows that dz/dt is positive. We therefore have to integrate the equations

$$\frac{dz}{d\tau} = \frac{z}{4\kappa} + \frac{1}{4\kappa}\sqrt{4\kappa x + z^2}, \qquad \frac{dx}{d\tau} = -z^2,$$

with the initial conditions $z = 0$, $x = a < 0$, to obtain the particular solutions which are tangent to the singular solution $v = 0$. For these equations the existence theorems are applicable. As a solution exists, it must be (9) which we found by undetermined coefficients.

The other singular solution $v = 4\kappa x$ is not singular in the sense that it is enveloped by particular solutions but in the sense that it is a cusp locus for particular solutions (see Picard, loc. cit.). Without following Picard through the individual steps of the work we may write down the result. If v_0 and x_0 are two values satisfying the relation $v = 4\kappa x$ at $\tau = \tau_0$, the solution may be expanded into the form

$$x = x_0 + \alpha(\tau_0 - \tau) + \beta(\tau_0 - \tau)^{\frac{3}{2}} + \gamma(\tau_0 - \tau)^2 + \delta(\tau_0 - \tau)^{\frac{5}{2}} + \ldots$$
$$v = v_0 + \alpha'(\tau_0 - \tau) + \beta'(\tau_0 - \tau)^{\frac{3}{2}} + \gamma'(\tau_0 - \tau)^2 + \delta'(\tau_0 - \tau)^{\frac{5}{2}} + \ldots$$

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Now let us take the case $x_0 > 0$, $f_0 = -v_0/2\kappa$. The constant β is zero, and

$$x = x_0 - v_0(\tau_0 - \tau) - \frac{v_0}{4\kappa}(\tau_0 - \tau)^2 + \delta(\tau_0 - \tau)^{\frac{5}{2}} + \ldots\ldots$$

$$v = v_0 + \frac{v_0}{2\kappa}(\tau_0 - \tau) + \frac{5}{2}\delta(\tau_0 - \tau)^{\frac{3}{2}} + \ldots$$

The value of δ, which does not vanish, may readily be found, if desired.

This solution brings out the fact very clearly that the motion runs into a point d'arrêt. The curve, to be sure, has a cusp, but the expansion is in powers of $\sqrt{\tau_0 - \tau}$, which means that the time cannot exceed τ_0. If we could imagine that time satisfies the same possibilities of reversal of direction as space, we could interpret this cuspidal motion, but as time can flow only in one direction, the motion must be interpreted as having a point d'arrêt. This does not mean that the two parts of the geometric locus cannot be interpreted, but that they must be representative of two different motions terminating in the same point d'arrêt.

We may indeed trace the motion back from the point d'arrêt along the two branches. We have

$$f = -\frac{v}{2\kappa} \pm \frac{1}{2\kappa}\sqrt{v^2 - 4\kappa v x}.$$

The acceleration is negative for both signs before the radical; x has increased up to x_0 at $\tau = \tau_0$ and v has decreased to v_0. In following the motion backward through time we find that x decreases and v

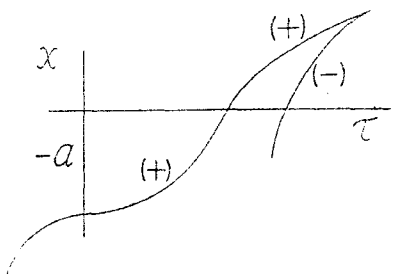

FIGURE 1.

increases. For the motion with the positive sign before the radical the retardation is less than for the motion with the negative sign. Hence in going backward, the former motion gives larger values of x

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and smaller values of v than the latter motion for the same value of $\tau_0 - \tau$. When the (+) motion passed the origin, the acceleration was zero; when the (−) motion passed the origin, the acceleration was negative and twice as large as at the cusp. The (+) motion may be traced back to a point where the particle comes to rest for a certain negative value of x; this corresponds to the solution (9), and the motion may be traced still further back, the velocity and retardation increasing all the while. The (−) motion may likewise be traced back through the origin $x = 0$, the velocity and retardation increasing.

The curve which shows roughly the relation between x and τ is given in Figure 1. If we call the curve which corresponds to the positive sign $x = a\Phi_{(+)}(\tau)$ and the curve which corresponds to the negative sign $x = a\Phi_{(-)}(\tau)$, we may at once write the complete solution of the differential equation (6′), namely,

$$x = C_1\Phi_{(+)}(\tau - C_2), \qquad x = C_1\Phi_{(-)}(\tau - C_2),$$

with the singular solutions $x = C$. The first solution could have been written at once from (9); the particle is at rest at $x = C_1$ when $\tau = C_2$.

The way in which the constants of integration enter into the solution (which could be foreseen from the differential equation itself inasmuch as the equation admits the two-parameter group $x' = x + a$, $\tau' = \tau + a$) shows that the motion, though not periodic, has definite intervals of time independent of the constants of integration associated with it — the time from rest to the origin and the times (two possibilities) from the origin to the point d'arrêt.[9]

5. Numerical Calculations. To ascertain these periods of time we must integrate the equation (6′). Let

$$a = a\,e^{-\int u d\tau}, \qquad \frac{dx}{d\tau} = -xu, \qquad \frac{d^2x}{d\tau^2} = x\left(u^2 - \frac{du}{d\tau}\right). \tag{11}$$

The equation in u is

$$\kappa\left(u^2 - \frac{du}{d\tau}\right)^2 - u\left(u^2 - \frac{du}{d\tau}\right) - u = 0$$

or

$$\frac{du}{d\tau} = u^2 - \frac{u}{2\kappa} \pm \frac{1}{2\kappa}\sqrt{u^2 + 4\kappa u}.$$

Suppose $x = a < 0$, $v = 0$, $f = 0$ are the initial conditions. Then $u = 0$. Moreover, as f becomes positive, $du/d\tau - u^2$ becomes

[9] These latter would not appear in the case Borel treated.

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positive. Hence the positive sign must be taken with the radical. When $x = 0$, u has become positively infinite. The time of fall to the origin is therefore

$$\int_0^T d\tau = T = \int_0^\infty \frac{du}{u^2 - \frac{u}{2\kappa} + \frac{1}{2\kappa}\sqrt{u^2 + 4\kappa u}}. \tag{12}$$

Introduce the substitution

$$y^2 = \frac{u}{u + 4\kappa}, \qquad u = \frac{4\kappa y^2}{1 - y^2}. \tag{13}$$

Then

$$T = \int_0^1 \frac{4\kappa\, dy}{8\kappa^2 y^3 + (1 - y^2)(1 - y)}.$$

The integration may be performed by partial fractions. It is necessary to factor the denominator approximately. One root is nearly equal to -1, the other two are imaginary. The real root may be developed in powers of κ^2. The first approximation gives

$$y = -1 + 2\kappa^2.$$

The denominator is factorable as

$$(1 + 8\kappa^2)\ (y + 1 - 2\kappa^2)\ [y^2 - 2(1 - 5\kappa^2)y + 1 - 6\kappa^2].$$

The reduction of the integrand to partial fractions gives

$$\kappa\,(1 - 2\kappa^2)\left[\frac{1}{y + 1 - 2\kappa^2} - \frac{y - (1 - 5\kappa^2) - (2 - 7\kappa^2)}{y^2 - 2\,(1 - 5\kappa^2)\,y + 1 - 6\kappa^2}\right].$$

For the integration

$$\tau = \kappa\,(1 - 2\kappa^2)\left[\tfrac{1}{2}\log\frac{(y + 1 - 2\kappa^2)^2}{y^2 - 2\,(1 - 5\kappa^2)\,y + 1 - 6\kappa^2} + \frac{2 - 7\kappa^2}{2\kappa}\tan^{-1}\frac{y - 1 + 5\kappa^2}{2\kappa}\right].$$

Substituting the limits 0 and 1, we have

$$\tau = \kappa\,(1 - 2\kappa^2)\left[\log\left(\frac{1 - \kappa^2}{\kappa}\,\frac{1 - 3\kappa^2}{1 - 2\kappa^2}\right) + \frac{2 - 7\kappa^2}{2\kappa}\left(\tan^{-1}\frac{5\kappa}{2} + \tan^{-1}\frac{1 - 5\kappa^2}{2\kappa}\right)\right].$$

The κ^2 is now everywhere negligible, and

$$T = \frac{\pi}{2} + \kappa\log\frac{1}{\kappa} + \frac{\kappa}{2}.$$

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As κ is in the neighborhood of $2\ 10^{-8}$, the difference between τ and $\pi/2$ is about $4\ 10^{-7}$.

We desire next to follow the motion through the origin and on to the point d'arrêt.[10] Construct the Riemann surface for the integrand in (12). The surface has branch points at $u = 0$ and $u = -4\kappa$, and a junction line between them. We have let u trace the real axis from 0 to ∞; now let u turn through a large circuit in the upper half-plane

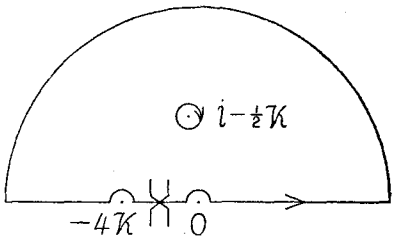

FIGURE 2.

and come down to the other end of the axis of reals. No time elapses during this process, for approximately $d\tau = du/u^2$. The variable x merely swings around the origin on an infinitesimal semi-circumference and is ready to start off on positive values as u comes in along the negative real axis. That the branch point $u = -4\kappa$ corresponds to the point d'arrêt for the motion is readily surmised, and may be proved from the defining equations (11).

The time from the origin to the point d'arrêt is therefore

$$T' = \int_{-\infty}^{-4\kappa} \frac{du}{u^2 - \frac{u}{2\kappa} - \frac{1}{2\kappa}\sqrt{u^2 + 4\kappa u}}.$$

But with the substitution (13) the integrand becomes again the same:

$$T' = \int_1^{\infty} \frac{4\kappa\ dy}{8\kappa^2 y^3 + (1 - y^2)(1 - y)}.$$

Hence

$$T' = \kappa(1 - 2\kappa^2)\left[-\log\frac{1 - \kappa^2}{\kappa} + \frac{2 - 7\kappa^2}{2\kappa}\left(\frac{\pi}{2} - \tan^{-1}\frac{5\kappa}{2}\right)\right].$$

10 The simplest method seems to be one dependent on the theory of functions of a complex variable; for it enables us easily to pass around the singularity at the origin.

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Again κ^2 is negligible and

$$T' = \frac{\pi}{2} - \kappa \log \frac{1}{\kappa} - \frac{5}{2}\kappa.$$

If we had desired merely the total time $T + T' = \pi - 2\kappa$ for the swing from rest to point d'arrêt, we could have obtained the result more directly by the theory of residues. The integral

$$T + T' = \int_0^{-4\kappa} \frac{du}{u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\sqrt{1 + \frac{4\kappa}{u}}},$$

taken around the path in Figure 2 which passes through infinity, is equivalent to the sum of two integrals

$$-\int_{-4\kappa}^{0} \frac{du}{u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\sqrt{1 + \frac{4\kappa}{u}}} + \int_{0} \frac{du}{u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\sqrt{1 + \frac{4\kappa}{u}}}, \qquad (14)$$

where the first is taken along the direct path on the upper side of the junction line and where the second is taken around that pole of the integrand which lies in the upper half of the upper sheet of the Riemann surface. As the calculation of the position of this pole is necessary for the work of the next section we shall now check the above value of $T + T'$.

The denominator of the integrand vanishes when

$$u^4 - \frac{u^3}{\kappa} + \frac{u^2}{4\kappa^2} - \frac{u^2}{4\kappa^2}\left(1 + \frac{4\kappa}{u}\right) = 0.$$

This gives one zero at the branch point $u = 0$; the integral to $u = 0$, however, is finite as the root is simple and the branch point has two sheets. The other roots are determined by the cubic

$$-\kappa u^3 + u^2 + 1 = 0 = -\kappa(u - \tfrac{1}{\kappa} - \kappa)(u^2 + \kappa u + 1).$$

To terms in κ the approximation gives the three roots

$$u = \frac{1}{\kappa} + \kappa, \qquad u = \pm i - \frac{\kappa}{2}.$$

The first lies in the under sheet and does not concern us. The root $u = i - \frac{1}{2}\kappa$ lies in the upper half of the upper sheet, and the residue of the corresponding pole has to be found. The expansion

$$u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\left(1 + \frac{2\kappa}{u} - \frac{2\kappa^2}{u^2}\right)$$

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of the denominator is legimate in the neighborhood of $u = i - \frac{1}{2}\kappa$. The denominator therefore becomes

$$u^2 + 1 - \frac{\kappa}{u} = \left(u - i + \frac{1}{2}\kappa\right)\left(u + i - \frac{1}{2}\frac{\kappa}{u} - \frac{\kappa i}{u}\right)$$

The residue is

$$\left.\frac{1}{u + i - \frac{1}{2}\kappa - \frac{\kappa i}{u}}\right]_{u = i - \frac{1}{2}\kappa} = \frac{1}{2i(1 + i\kappa)} = \frac{1}{2i}(1 - i\kappa).$$

The second integral in (14) is therefore $\pi(1 - i\kappa)$, which is $2\pi i$ times the residue.

To calculate the first integral in (14) we may rationalize:

$$-\int_{-4\kappa}^{0} \frac{u^2 - \frac{u}{2\kappa} - \frac{u}{2\kappa}\sqrt{1 + \frac{4\kappa}{u}}}{u^4 - \frac{u^3}{\kappa} - \frac{u}{\kappa}}\, du.$$

The first two terms of the denominator and the first of the numerator are of the order κ^2, or higher between -4κ and 0 and may be neglected. The result is

$$-\int_{-4\kappa}^{0} \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4\kappa}{u}}\, du =$$

$$-\frac{1}{2}\left[u + \sqrt{u(u + 4\kappa)} + 4\kappa \log\left(\sqrt{u + 4\kappa} + 5u\right)\right]_{-4\kappa}^{0} =$$

$$-2\kappa - 2\kappa \log(\pm i),$$

where the indeterminate sign and the indeterminate value of the logarithm must be properly chosen. This need not trouble us as the term is imaginary, and its only function is to cancel the imaginary part of $\pi(1 - i\kappa)$. The total value of $T + T'$ is thus seen to be $\pi - 2\kappa$ as previously determined.

We have next to find the value of $x = X$ at the point d'arrêt. This may be written as

$$X = a\,c \int_{0}^{-4\kappa} \frac{u\, du}{u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\sqrt{1 + \frac{4\kappa}{u}}},$$

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where the integral is again taken over the roundabout path of Figure 2. We may write

$$-\int_0^{-4\kappa} \frac{u\,du}{u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\sqrt{1+\frac{4\kappa}{u}}} = \int_{-4\kappa}^{0} \frac{u\,du}{u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\sqrt{1+\frac{4\kappa}{u}}} - \int_0 \frac{u\,du}{u^2 - \frac{u}{2\kappa} + \frac{u}{2\kappa}\sqrt{1+\frac{4\kappa}{u}}},$$

where the first integral on the right is taken along the junction line, and the second about $u = i - \frac{1}{2}\kappa$. The former is approximately

$$\int_{-4\kappa}^{0} u\left(\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{4\kappa}{u}}\right) du = \frac{1}{2}\int_{-4\kappa}^{0} u + \sqrt{u^2 + 4\kappa u}\, du,$$

which is of order κ^2 and vanishes to the present degree of approximation. The latter is

$$-2\pi i\left[\frac{u}{u + i - \frac{1}{2}\kappa - \frac{\kappa i}{u}}\right]_{u = i - \frac{1}{2}\kappa} = -2\pi i\,\frac{i - \frac{1}{2}\kappa}{2i\,(1 + i\kappa)}$$

$$= -\pi i - \frac{1}{2}\pi\kappa.$$

The value of X is therefore,

$$X = -a\,e^{-\frac{1}{2}\pi\kappa} = -a\,(1 - \tfrac{1}{2}\pi\kappa).$$

The point d'arrêt therefore lies slightly nearer the origin than the point of rest.

To obtain the amount of energy lost by radiation during the swing from the starting point of no velocity to the point d'arrêt we might evaluate the integral of $\kappa(d^2x/d\tau^2)^2$ as we have evaluated the integrals for the time and displacement. But it is simpler to figure the final kinetic energy and potential energy and to subtract their sum from the initial potential energy $\frac{1}{2}a^2$. The final velocity is $v = 4\kappa x$, with $x = X = -a(1 - \frac{1}{2}\pi\kappa)$. The kinetic energy $\frac{1}{2}v^2$ is therefore negligible to the order of our approximations. The potential energy is $\frac{1}{2}x^2 = \frac{1}{2}(a^2 - \pi\kappa)$ which is less than $\frac{1}{2}a^2$ by the amount $\frac{1}{2}\pi\kappa$. Thus the energy radiated during the swing is essentially the same as that radiated during one swing on the assumption that the motion is practically simple harmonic.

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We have now concluded a tolerably complete qualitative and quantitative discussion of the equation

$$m\frac{dx}{dt}\left(\frac{d^2x}{dt^2}+\kappa^2x\right)=-\frac{2}{3}\frac{e^2}{c^3}\left(\frac{d^2x}{dt^2}\right)^2, \tag{15}$$

which arises by the application of the principle of energy with the usual law of instantaneous radiation, and which would arise from the integrated form of the energy equation by the same reasoning as is applied, after a transformation by integration by parts, namely, that if an integral vanishes, its integrand also vanishes, to obtain the equation

$$m\left(\frac{d^2x}{dt^2}+\kappa^2x\right)=\frac{2}{3}\frac{e^2}{c^3}\frac{d^3x}{dt^3}. \tag{16}$$

It is probable that if the non-linear equation (15) had been easily integrable and had given a motion essentially simple harmonic and damped, the transformation to (16) would never have been made. And the fact that (15) is difficult of integration, and when actually integrated gives an aperiodic discontinuous motion does not seem a very satisfactory justification for making the transformation.

6. **Conclusions.** One of the first inferences of our work is that, whatever we may think of the merits of either of (15) or (16), the Abraham-Planck-Rüdenberg derivation of (16) is without much mathematical or physical validity. The derivation of Lorentz and Ritz[11] is not open to the same objections; for it proceeds directly from the retarded potentials to (16) through a series of approximations. The law of radiation which this equation states is that the rate of radiation is

$$m\frac{dx}{dt}\left(\frac{d^2x}{dt^2}+\kappa^2x\right)=\frac{2}{3}\frac{e^2}{c^3}\frac{dx}{dt}\frac{d^3x}{dt^3},$$

which for oscillatory motions is nearly proportional to the square of the velocity and is physically nearly equivalent to a friction proportional to the velocity. The average rate of radiation is, of course, that which corresponds to that found by Hertz.

Our work throws some light on the method of presenting the differential equation for the oscillator of radiotelegraphy. We may assume that the oscillations are damped simple harmonic; we know that the simplest mathematical representation of such motion is by an equa-

11 Lorentz, Theory of Electrons, loc. cit.; Walther Ritz, Gesammelte Werke, 401 ff.

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tion of the second order with constant coefficients, and we may determine the logarithmic decrement of this equation so that the average loss of energy shall be equal to that determined by the theory of Hertz.[12] This is frankly phenomenological and seeks not to go too far below the superficial appearances; but it is probably better than to delve so deeply that we must sacrifice either our skin or our conscience in getting back to the surface.

We have thus far confined our attention to the oscillator, but similar results may be obtained for other systems. For instance, J. J. Thomson[13] has proposed as a model of the atom a center repelling a corpuscle with a force varying as the inverse cube of the distance (and furthermore attracting as the inverse square if the corpuscle lies within a tube issuing from the center). To treat the motion under the repelling force he writes

$$m\frac{d^2x}{dt^2} = \frac{ce}{x^3},$$

integrates the equation and then calculates the radiation by

$$const. \int \left(\frac{d^2x}{dt^2}\right)^2 dt.$$

The motion is therefore obtained by ignoring the reaction of the radiation.

If, however, we write the equation for rate of change of energy at each instant, we have an equation of the type

$$\frac{dr}{dt}\left(\frac{d^2r}{dt^2} - \frac{1}{r^3}\right) = -\kappa\left(\frac{d^2r}{dt^2}\right)^2$$

or

$$f = \frac{-v}{2\kappa} \pm \frac{1}{2\kappa}\sqrt{v^2 + \frac{4\kappa v}{r^3}}.$$

For approach to the center, $v < 0, f > 0$; but the acceleration becomes imaginary when $-vr^3 = 4\kappa$, and the motion has a point d'arrêt. Motion away from the center does not have this peculiarity.

The real interest of our work lies, however, not in the critique of current "demonstrations," nor in a tour de force integration of a

12 See, for instance, J. Fleming, Principles of Electric Wave Telegraphy, chaps. 3, 5 (1906).

13 J. J. Thomson, On the Structure of the Atom, Phil. Mag. (6) **26**, pp. 792–799, Oct. 1913.

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certain differential equation, nor in pointing out that the integration of the equation leads to a physical point d'arrêt, but in the general inferences or alternative inferential possibilities of importance to physical science which the discovery of the point d'arrêt may suggest.

It is clear that we shall not have to abandon the statement of physical problems in terms of differential equations possessing mathematically continuous solutions in order to introduce into physics discontinuities even more startling than those introduced by the adherents of current theories of quanta. The statement of simple cases of rectilinear motion in equations of the second degree and second order has led to cusps in the space-time locus and, by virtue of the irreversibility of time, to physical discontinuities. (Such a conclusion could hardly be drawn from Borel's work, for his equation led to no point d'arrêt.)

If we regard the existence of the point d'arrêt as inconceivable, we must look about for a method of doing away with it. One possibility is to accept literally the method of calculation employed by J. J. Thomson as cited above, namely, to regard the motion of the electric charge as determined by an ordinary equation of mechanics, which omits altogether the reaction of the radiation, and to compute the radiation subsequently. This would mean that the radiant energy did not come out of the field or motion (potential and kinetic energy) of the system but was a phenomenon inherently arising when charges moved.[14] This would be a shock to those who have implicit confidence in the conservation of energy as now generally understood, but would not perhaps be so serious to those who take the position of Ritz[15]: La localisation de l'énergie doit donc être comptée au nombre des conceptions logiquement inutiles (et peut-être parfois nuisibles) de la théorie. . . Dans le cas le plus général du rayonnement électromagnetique, la conservation de l'énergie n'est plus une loi, mais une convention.

Another method of escape would be to deny the accuracy of the law that the radiation is proportional at each instant to the square of the acceleration. The law is indeed only an approximation to the general form given by Abraham and Heaviside (with the apparent assumption of the quasi-stationary state) and proved by Lewis and me (on the basis of the principle of relativity) to be rigorously true for the point electron moving with any velocity and acceleration,[16]— for with the

14 This may remotely suggest Bateman's double-barreled gun in his Corpuscular Radiation, Phil. Mag. (6) 26, pp. 579–585, Oct. 1913.

15 Gesammelte Werke, pp. 343, 345.

16 Wilson and Lewis, loc. cit., (5).

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usual definitions of field and energy the law becomes merely a geometric theorem in four dimensional non-Euclidean geometry. It is reasonable, however, to suppose that the use of the general form of the law could not remove the difficulty; for the point d'arrêt arises under conditions of relatively minimal velocity. Moreover, a qualitative discussion of the equation which arises under the general law indicates[17] that the difficulty is not removed.

It is tolerably clear that we cannot avoid the point d'arrêt and maintain simultaneously the point electron, ordinary mechanics, and ordinary electromagnetic theory. This conclusion may be welcome to the school that follows Bohr in abandoning both mechanics and electromagnetic theory[18] as hitherto understood even when transformed as suggested by the principle of relativity. And we certainly do not wish to imply that this abandon may not work out satisfactorily.

The conservative, however, will naturally try to work out of the present difficulty by abandoning the point electron, especially as electrons are generally supposed to have magnitude. It is a bit hard to see why the assumption of a very small finite size for the electron must fundamentally vitiate the reasoning which leads to the law of the square of the acceleration; but we shall not go into this question, because we are not acquainted with any derivation of this law which does not in some form practically assume the electron to be a point. Moreover, the work of Lorentz and Ritz, starting with a distributed electron and introducing approximations based on the assumption of small velocities and accelerations, establishes a reaction proportional to the rate of change of the acceleration and thus leads on mechanical principles to a law of radiation (at any rate to a loss of mechanical energy) proportional to the (scalar) product of the velocity and rate of change of acceleration. And it is interesting to note that this reaction and rate of change of energy is actually independent of the size or shape assumed for the electron, that is, may be assumed to hold for the point electron.

Howeverso apologetic one may be in regard to (16), that equation seems somewhat less in need of apologies than (15) or its derivation from (15) in integrated form.

Massachusetts Institute of Technology,
Boston, Mass, *May*, 1914.

17 Just what assumptions in non-Newtonian mechanics should be made to treat this problem it is perhaps difficult to state, and consequently the question is here left as a qualitative judgment of the author.

18 N. Bohr, Phil. Mag., (6) 26 (1913) a series of three articles On the Constitution of Atoms and Molecules, pp. 1, 476, 857.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

PLANCK'S RADIATION FORMULA AND THE CLASSICAL ELECTRODYNAMICS.

By David L. Webster.

Investigations on Light and Heat published with Aid from the Rumford Fund.

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PLANCK'S RADIATION FORMULA AND THE CLASSICAL ELECTRODYNAMICS.

By David L. Webster.

Presented by G. W. Pierce. Received, December 1, 1914.

Introduction. As Poincare [1] has pointed out, Planck's formula for black body radiation, or any other formula giving only a finite amount of energy per unit volume in the radiation field, involves the assumption of some discontinuities in the process of absorption and emission. Thus Planck assumes that while an oscillator may absorb energy continuously, it may radiate only when the energy is an integral multiple of $h\nu$, and that if it does then radiate, all its energy is radiated at once.

This assumption, being inconsistent with the classical electrodynamics, involves the abandonment of the explanations that the classical system gives of many phenomena. Its explanation of inertia, for example, depends directly on the theorem that every part of an accelerated electron will radiate electric forces proportional to its acceleration, and that these forces, acting on other parts of the electron, produce the force of inertia, proportional and opposite to the acceleration. This explanation must be abandoned if we make Planck's assumptions, which deny the existence of these forces in most cases.

Likewise, according to the retarded potential theorem, the classical explanations of all phenomena of the propagation of light through matter may be put entirely on the basis of continuous re-radiation from vibrating electrons, whose energy in many cases never reaches the value $h\nu$, because the amplitude of a forced vibration is so small unless the frequency is near that of resonance. Although light phenomena other than scattering are not ordinarily treated in this way, any method derived from the classical field equations must necessarily be equivalent to any other, and any assumptions that make one of these methods give wrong results must necessarily do the same for all. Therefore, if Planck's assumptions are true, all such explanations must be abandoned, and we must create a whole new theory of optics.

1 Journal de Physique, (5) **2**, p. 1 (1912).

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It might be supposed that we could remedy this defect by assuming the existence of two classes of oscillators, the first obeying the classical laws and the second obeying Planck's assumptions, and in that way account for both optical and thermal phenomena. This hypothesis, however, does not seem tenable.

For although the classical oscillators would do their best to account for other light phenomena, they would tend to give Rayleigh and Jeans's radiation law (which any system obeying the classical dynamics must give) rather than Planck's. Or, in terms of Planck's derivation, their entropy, computed from the thermodynamic probability, would be different from that of his oscillators because their distribution on the energy diagram would be different. Hence the radiation law would be different also, if any noticeable percentage of these oscillators were present. Moreover, Jeans's law, which they would tend to make the radiation obey, gives an intensity whose ratio to Planck's becomes infinite rapidly as λT diminishes. Thus an extremely small percentage of classical oscillators would give large deviations from Planck's law, while the Planck oscillators would not give the well known relations between absorption and dispersion, and other optical laws that are explained by the classical theory.

Moreover, unless the percentage of each class of oscillators for each frequency were exactly the same for every substance at each temperature the distribution of energy in the spectrum of a cavity would depend on the substances contained in its walls. Such a result is well known to be contrary to the second law of thermodynamics.

As Planck [2] says, the assumptions he makes are not the only ones that can lead to his law of radiation. The object of the present investigation is therefore to see whether the abandonment of the classical electrodynamics and its explanations of these and other phenomena is really necessary, and if it is not so, to find a mechanism giving both heat radiation and optical phenomena, and at the same time consistent with other phenomena as far as possible. The conclusion is that it is not necessary, and that the formula can equally well be derived from other assumptions, inconsistent with the classical mechanics only as applied to the internal structure of the electron, and consistent with the classical electrodynamics and its explanations of many phenomena; and, moreover, this explanation is based on a mechanism which has been shown by Parson [3] to be very useful in

2 "Heat Radiation," English translation by Masius, 154 (1914).
3 Not yet published.

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explaining chemical affinities, and which has other advantages over Planck's oscillator and most other models of the quantum.

Planck's Assumptions. The basis of the derivation of Planck's formula is Boltzmann's relation between entropy and probably, which Planck puts in the form $S = k \log W$ where S is the entropy and k is the gas constant reckoned for a single molecule, and W is the "probability," considered as the number of ways in which the system can be arranged so as to be indistinguishable from its present form.

In applying this law to a system of similar molecular oscillators, two arrangements are considered indistinguishable if they give equal numbers of oscillators in each of certain groups, any group, the n^{th}, being defined by the condition that the energy of every oscillator in it must lie between $(n-1)\,h\,\nu$, and $n\,h\,\nu$. For this rule to be a real criterion of indistinguishability, we must have the density of their representative points of the energy scale constant through each one of these intervals and changing abruptly on going from one interval to the next.

These assumptions, as Planck shows, give for the mean energy of the oscillators in the n^{th} group the value $\left(n - \frac{1}{2}\right) h\nu$. By setting the entropy equal to its maximum value consistent with a given total energy, he finds for the probability that a given oscillator will be in the n^{th} group the value $w_n = \alpha\gamma^n$ where $\alpha = \dfrac{2\,N\,h\,\nu}{2\,E - N\,h\,\nu}$ and $\gamma = \dfrac{2\,E - N\,h\,\nu}{2\,E + N\,h\,\nu}$, E being the total energy of N oscillators.[4]

Now to obtain a law of distribution of energy in the black body spectrum, it is necessary to have some law of emission and absorption for the oscillators. To keep in touch with the classical electrodynamics, he assumes that an oscillator absorbs energy as that system would require if one might neglect reradiation entirely; but to obtain the discontinuities required, he assumed, not the classical law of emission, but that the oscillator can emit only when the representative point reaches the boundary of one of these intervals, and that, if it does emit, it must emit its whole energy, $n\,h\,\nu$.

With these assumptions, he proves that the energy of such an oscillator, exposed to radiation of continuously distributed frequencies, will increase at a constant rate, thus insuring the constancy of the density of points in each interval; and he obtains the proper reduction of their density from one interval to the next by assuming that the

[4] Planck, l. c., § 139.

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probability η that the oscillator will emit when its energy reaches the value $n h \nu$ is given by the formula $\frac{1-\eta}{\eta} = pI$, where p is a constant, and I is the intensity of the electric vibration per unit interval of frequency. That is, the stronger the light, the more likely the oscillator is to go on accumulating energy. The value of p is obtained by setting the mean energy thus found approximately equal, for extremely large values of I, to the value it would have if the oscillator radiated as demanded by the classical theory. This is a new assumption, independent of the previous ones, and based on the experimental fact the Rayleigh's law of radiation is true for large values of λT.

The next step is to identify the distribution of points thus found with that obtained above from entropy considerations, and thus find I in terms of ν and T, and from I the energy density per unit frequency interval,

$$u = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Assumptions of the Present Theory. The starting point of the present theory is Parson's magneton, a ring of negative electricity of a diameter perhaps $\frac{1}{10}$ that of a hydrogen atom, revolving on its axis with a velocity of the order of that of light. This has been proposed by Parson as a substitute for the classical electron, and has given good results in the explanation of chemical affinities.

These magnetons are supposed by Parson to be free to move in a sphere of continuously distributed positive electricity, in which, as he shows, they have a strong tendency to group themselves in eights, thus giving the foundation of the periodic table of the elements. A detailed discussion of their groupings and the surprising way in which they explain not only the table, but also the exceptions to its rules, and many other chemical phenomena, will be found in his paper.

Inquiring into what may be expected of the vibrations of the magneton, we find a state of affairs somewhat more complex than in the classical electron theory. For we have not only the attraction of the positive electricity through which it moves, and the electrostatic repulsions of the other magnetons, both tending to make its equilibrium stable, but also the magnetic attractions of the others, tending to make it less stable, and perhaps still other repulsions by them, proportional to some inverse power of the distance higher than the second, and therefore having a strong tendency to promote stability. The combined effect must, of course, be a stable equilibrium.

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Moreover, as possible modes of vibration, we have not only a rigid displacement of the magneton as a whole, but also a disturbance of the flow of electricity around it, that may give electrical oscillations. These may be thought of as superposed on the continuous flow just as they would be in the case of a large ring of wire, heavily charged with negative electricity and at the same time carrying a current around the ring and performing electrical oscillations which may displace the centre of charge to any point in its plane. The displacement of the center of mass of the magneton during these oscillations might be anything, depending on how the charge was distributed and on what changes of thickness of the ring might be caused by the changes in distribution of its charge. We shall assume here, purely for convenience, that the center of charge and the center of mass move together. We shall also assume that any number of these waves may be superposed without disturbing one another, except by electrostatic action, as in the case of the ring of wire.

Let us now consider a magneton whose geometric center is displaced in the plane of the ring by a distance ξ', and whose center of charge and mass is displaced relative to the ring by a distance ξ'', or in all a distance $\xi = \xi' + \xi''$. We shall divide the intra-atomic forces acting on it into the following five classes:

(1) The resultant of the attraction of the positive electricity through which it moves and the repulsions of the other magnetons, equal to $-f\xi$, and acting on the electricity itself, rather than on the rest of the structure;

(2) The resultant of the magnetic attractions and all non-electrostatic repulsions between them, equal to $-f'\xi'$ and applied to the structure of the ring;

(3) The internal forces of the magneton, giving a force $-f''\xi''$ on the electricity and $+f''\xi''$ on the ring;

(4) The force of inertia, $-m\frac{d^2\xi}{dt^2}$ due to radiations caused by the acceleration of the electricity, and acting on it;

(5) The damping force $+\frac{2}{3}\frac{e^2}{c^3}\frac{d^3\xi}{dt^3}$ due to radiation, equal for simple harmonic motions depending on sin ωt to $-g\frac{d\xi}{dt}$ where $g = \frac{2}{3}\frac{e^2}{c^3}\omega^2$.

(6) Another damping force $-g''\frac{d\xi''}{dt}$ due to an assumed tendency

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of the internal mechanism of the magneton to transfer energy from the oscillation of the electricity on it to the steady current around it.

f, f', and f'' are assumed to be constant, and we shall assume the equilibrium of the ring under the action of (2) to be so stable that for visible and ultraviolet frequencies, the motion of the center of charge is due chiefly to the oscillation of the electricity on the ring, and very little to the motion of the ring as a whole. In the infra-red, the motion of the atom itself will, of course, change the whole character of the oscillation.

This plan does not agree entirely with Parson's assumptions, which make the magnetons interchange their positions in the atom rather freely; but I doubt if this is a serious difficulty, since even with these assumptions, a strong collision, with its accompanying distortion of the positive sphere, might easily cause considerable changes in their arrangement.

The damping force (6) is contrary to the laws of the classical mechanics as applied to the internal structure of the magneton. But this conflicts with no experimental facts, and, as we have seen, we must expect a violation of these laws somewhere in the absorbing and emitting system. The mechanism of this transfer cannot be ordinary electromagnetic induction, because simple considerations of symmetry show that the mutual inductance between these oscillations and the steady current is zero.

The energy transferred from the oscillations, according to these assumptions and others to be made below, will be found very small compared to the whole magnetic energy of the magneton, which is of the order of magnitude of mc^2. (This value will be discussed in more detail in a subsequent paper showing why we cannot expect the magnetic properties of the magneton to be detected by experiments on cathode rays or photo-electrons.) Therefore the increase in the velocity of the steady current that is given by this energy must be very small compared with the original velocity, and so will not interfere with the explanation of chemical phenomena by the magnetons.

Neglecting the electrostatic influence of neighboring molecules, we may say that all the above forces on the electricity must balance, and so must all those on the ring. This gives the equations,

$$(1) \qquad -f\xi - f''\xi'' - m\ddot{\xi} - g\dot{\xi} - g''\dot{\xi}'' + eE_\xi = 0$$

$$(2) \qquad -f'\xi' + f''\xi'' = 0.$$

Combining these, we obtain

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$$(3) \qquad \dot{m}\ddot{\xi} + \left\{ g + \frac{g'' f''}{f' + f''} \right\} \dot{\xi} + \left\{ f + \frac{f' f''}{f' + f''} \right\} \xi = e E_\xi$$

This equation, being of the standard form for forced harmonic motion with damping, shows at once that the classical theory of the propagation of light through matter is reproducible from this model. For, if the electric force is inclined obliquely to the plane of the magneton, the component in the direction perpendicular to this plane will produce practically no effect, on account of the immobility of the charge of the magneton in this direction. Other magnetons, being tipped in other directions, will supply the mobility that is lacking in the one we have considered.

In the infra-red, where the vibrations of atoms as a whole begin to be of importance, we must superpose the motion of the sort considered above on that of the atom itself. Since each atom has in general very little magnetic moment, it must have magnetons (especially in the groups of eight) turned in all directions. Therefore some of them will always be vibrating with a motion of the center of charge relative to the ring. Their rates of absorption, however, cannot be found from this equation because in a solid or liquid the vibrations of the atoms as a whole will be governed chiefly by collisions with other molecules, while in a liquid or gas the rotation of the molecules will change the direction of the axis of vibration continually and thus prevent the accumulation of large amplitudes. Therefore the rates of absorption are much less than this equation would indicate, as soon as we get to frequencies such influences become noticeable. That this occurs to some extent even in the visible spectrum at ordinary temperatures is indicated by the readiness with which absorption of most visible light generally produces heat rather than other effects, such as photo-electric currents or chemical changes, and also by the well known widening of absorption lines by pressure. The fact that such influences may result in a transfer of a certain amount of energy from the vibrations to molecular motions rather than to the steady current of the magnetons, is, as we shall see, entirely immaterial for the derivation of Planck's law, since this derivation depends on considerations of probability that are unchanged by this transfer.

Since each high frequency magneton exposed to radiation will execute a steady vibration, it must store up energy at a constant rate. Likewise at low frequencies, since the oscillations themselves are independent of the amount already stored, however much they may be affected by molecular motions, the rate of storing will, on the average,

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be constant. This result, which is attained in Planck's theory by the prohibition of all re-radiation or external influences, is quite necessary for the derivation of his law.

This accumulation of energy, moreover, must continue until the energy stored in the oscillator reaches some integral multiple of $h\nu$, so that even at the shortest wave lengths or lowest temperatures it must always be able to attain the value $h\nu$ at least. At longer wave lengths or higher temperatures, the oscillator will accumulate many quanta. It is, therefore, important to see how this energy will affect its properties.

If the oscillator is the classical electron,[5] the energy must all be stored in its vibration, so that for an amplitude ξ_0 to hold a quantum, we must have the energy

$$\frac{1}{2} m \omega_0^2 \xi_0^2 = h\nu = \frac{h\omega_0}{2\pi},$$

or

$$\xi_0 = \sqrt{\frac{h\lambda}{2\pi^2 m c}}$$

This value, being proportional to $\sqrt{\lambda}$, will be greatest for the longest waves for which the oscillator may safely be assumed to be an electron in a non-vibrating atom. Since the values of the Zeeman separations lead to this assumption certainly throughout the visible spectrum, we may apply this forumla for $\lambda = 8000$ Å finding

$$\xi_0 = 3.1 \text{ Å}.$$

Comparing this with the distance between carbon atoms in diamond, found by W. H. and W. L. Bragg [6] to be 1.52 Å, we find that the length of the whole vibration would have to be over four times the diameter of the atom.

This is another serious difficulty in the way of Planck's theory. For even if the oscillation consists of all the electrons in the atom moving together, so that the amplitude required is reduced in proportion to the square root of their number, the distance which the group may go before some of them get outside the atom is also greatly reduced. Moreover if the positive sphere is not absolutely uniform in density, the distance they can go before the frequency of vibration is changed is even less.

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5 Planck does not specify the charge or mass of the oscillator: this paragraph therefore starts with the most plausible assumption as to its nature.

6 Nature **91**, 557 (1913).

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If one could assume without conflict with measurements of the Zeeman effect that the atom itself takes an appreciable part in the motion at such frequencies, then the amplitude required for this energy would be less than what we have found. In this case, however, the electron would have to be able to distinguish carefully between the part of the atom's energy that it could include in the quantum and the part due to heat motions and vibrations with other frequencies.

There is, furthermore, another difficulty caused by the fact that the transfer of energy to heat motion of the molecules would increase rapidly with the amplitude of the oscillator, so that the rate of increase of the vibratory energy would diminish, rather than remain constant as it must for Planck's law.

These difficulties are all avoided by the magneton, because of its storing its energy in its steady current, and thus making the oscillation, as we have noted above, independent of the amount already stored. The smallness of the increase of the steady current that is required for this is evident from the fact that mc^2, which gives the order of magnitude of the magnetic energy, is 7.9×10^{-7} erg, while even at 1000Å, where the oscillator practically never holds more than one quantum, the quantum is only 2×10^{-11} erg.

Another point that is as necessary for the derivation of Planck's law as a satisfactory method of absorbing and storing the energy is a satisfactory law of emission. For this Planck assumes p. 153, "that the emission does not take place continuously, as does the absorption, but that it occurs only at certain definite times, suddenly, in pulses, and in particular we assume that the oscillator can emit only at the moment when its energy of vibration, U, is an integral multiple n of the quantum of energy, $\epsilon = h\nu$."

Just how a sudden pulse can have a definite frequency is difficult to imagine, and is not stated in his book.

The experimental facts, moreover, are against the assumption that the emission is absolutely instantaneous. For Fabry and Buisson,[7] have found that the path difference for interference observable in spectrum lines from the inert gases at low temperatures is often a very considerable fraction of a meter, and in the case of Krypton, with the tube immersed in liquid air, the path difference for wave length 5576 Å is 53 cm. or 950000 wave lengths. These path differences, moreover, are given within the limits of experimental error by Schönrock's formula derived from the kinetic theory, on the assumption that the light of each oscillator is really monochromatic and that the width of

7 C. R. **154**, 1224–7 (1912), or J. de Phys. **2**, 5, 442–64 (1912).

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the line is due only to the Doppler effect. Thus we may conclude that in such cases, at least, the time required for an emission is really quite considerable, and that Planck's assumption of practically instantaneous emission must be abandoned.

One way to evade this difficulty in the derivation of Planck's law is to assume that the radiation takes place at a constant rate until the whole energy, $n\,h\,\nu$, is emitted. This, however, is easier said than done; and one would be strongly tempted to look for some other radiation law, if Planck's had not been so well confirmed by direct experiments, not only those quoted in his book (p. 169) but also those published more recently by Coblentz,[8] and indirectly by the appearance of his constant h is the laws of so many other phenomena, such as specific heats and photo-electric effects.

This constant rate of radiation cannot be obtained by making the effect of the vibrating charge on the ether suddenly become some constant multiple of that which the classical theory would give, because it is well known that any such law would make the amplitude die out exponentially, rather than linearly, and approach a finite value, rather than zero. For the same reason, it cannot be obtained by having the absorbed energy stored as potential energy of an electron transferred from one equilibrium position in the atom to another, and re-emitted when the electron is jarred out of the latter position and falls back, with oscillations, into the former. Such models, moreover, would also be open to the objection to Planck's oscillator, that, to give a single quantum, they require too great an amplitude of vibration.

Another serious difficulty for such models of the oscillator is the fact that an essential point in the derivation of the law is the assumption that an oscillator may acquire an amount of energy that is *any integral multiple* of the quantum before it radiates. This applies, for example, to Bohr's atom, in which the transition from one equilibrium position to another makes the electron give out just one quantum, and the transitions between all other such positions will give different frequencies.

Still another point where such models are apt to be insufficient is in the explanation of photo-electric phenomena. For if, in the "lower" equilibrium, the electron is in a region of positive potential, and in the "higher" one it is either in such a region or removed to infinity, then, when it escapes from the higher position, it will either not leave the atom at all or else leave with an infinitesimal velocity. The only way

8 Bull. Bur. Stan., Jan. 15 (1914).

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for it to obtain the kinetic energy $h\nu - W_0$ indicated by experiments such as those of Richardson and Compton [9] is to have the higher position in a region of negative electrostatic potential, that is, to have the electron vibrate toward a mass of negative electricity without being thrown to one side of the path by the repulsion. This seems distinctly difficult to accomplish, and I do not know that it has ever been done.

To account for Planck's law and other experimental results, therefore, we are driven to the unpredictable and unsatisfactory assumption that when the energy stored in the magneton reaches any integral multiple of $h\nu$, the internal mechanism of the ring may start another oscillation, larger than the absorbing one, and that this emitting oscillation will maintain a constant amplitude, deriving its energy in some way from the steady current, until the excess energy stored in the magneton has been radiated, and its total energy is reduced to a standard amount. The probability η of starting to radiate at any particular multiple of $h\nu$ is the same as in Planck's theory.

To be sure that this mechanism is not, like Planck's, too big for the atom, we may calculate the amplitude of the emitting oscillation that is required to emit the energy faster than it is absorbed.

Rewriting equation (3) in the more condensed form

$$m\ddot{\xi} + b\dot{\xi} + \kappa\xi = eE_\xi = eE_0 \cos \omega t \tag{4}$$

where b and κ are abbreviations for the coefficients in (3), we may evaluate the rate of absorption for the frequency $\nu = \frac{\omega}{2\pi}$ as

$$R_\nu = e\overline{E_\xi \dot{\xi}}. \tag{5}$$

In using this equation (5) we are implicitly assuming that all the work done by the electric force goes into energy stored by the magneton, and we are therefore neglecting its extremely small re-radiation. Solving (4) for the case of a steady vibration, and substituting in (5), we obtain

$$R_\nu = \frac{\frac{1}{2} e^2 E_0^2 \omega_0^2 b}{m^2 (\omega_0^2 - \omega^2)^2 + b^2 \omega^2}$$

where $\omega_0 = \sqrt{\frac{\kappa}{m}}$

We may replace E_0^2 now by $Id\nu$ and integrate with respect to ν to obtain the rate of storing energy by the magneton. In this integra-

[9] Phil. Mag. **24**, 575 (1912).

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tion, ω may be replaced by ω_0 except in the expression $(\omega_0^2 - \omega^2)$ [10]. Performing the integration in this way, one obtains for the rate of absorption by vibrations in the ξ direction only, the value

$$\dot{U} = \frac{e^2}{4m} I \tag{6}$$

independent of ω_0 or b.

To see what amplitude is required to emit as fast as this, we may use the well known equation for the rate of emission,

$$\tfrac{1}{3} \frac{e^2}{c^3} \omega_0^4 \xi_0^2.$$

For the limiting case, let us set this equal to $\dot{U}$ and thus find the least permissible amplitude, ξ_0.

Thus

$$\xi_0^2 = \frac{3c^3 I}{4m\omega_0^4}$$

Inserting the value $\omega_0 = 2\pi\nu$ and

$$I = \frac{32\pi^2 h\nu^3}{3c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

we have

$$\xi_0 = \sqrt{\frac{h}{2\pi^2 m\nu}} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{6}$$

Evidently this amplitude will be greatest for the lowest frequencies for which equation (3) can be expected to hold with any accuracy, that is for frequencies situated somewhere in the visible range. It is, of course, impossible to set any precise limits here, but for the purpose of forming a rough idea of these magnitudes, let us calculate the amplitude for $\lambda = 6000$Å at temperatures of 300, 1000, and 2000° absolute.

For this case we have $\nu = 5 \times 10^{14}$ sec^{-1} and since $h = 6.415 \times 10^{-27}$ erg. sec, $m = 8.8 \times 10^{-28}$ gm and $k = 1.34 \times 10^{-16} \frac{\text{erg}}{\text{deg}}$ we have $h\nu = 3.2 \times 10^{-12}$ erg, while kT has the values 4.02×10^{-14}, 1.34×10^{-13}, and 2.68×10^{-13} erg respectively. Thus the values of ξ_0 at these temperatures are 3×10^{-17}, 2.5×10^{-5} and 6.5×10^{-2} Å respectively.

[10] See Lorentz "Theory of Electrons," note 62.

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If now the radius of the magneton is about 10^{-9} cm, or 0.1Å, the first two of these values are well within the limits of the possible amplitude of vibration of the center of charge, while the last is comparable with the radius of the magneton, and therefore suggests the possibility that the disturbance might result in the ejection of the magneton as a thermion. From the form of the expression for ξ_0 one would be led to infer that at lower ν's this would happen at lower temperatures. We must remember, however, that such speculations are very untrustworthy, because this formula (6) is developed on the hypothesis that the vibration is not disturbed by molecular motions — a hypothesis which we have seen to be incorrect even at the 6000 Å, which we have just considered. Hence we may infer that the amplitudes required here are probably within the possibilities of this mechanism, and that the results noted above point to a qualitative explanation of the emission of electrons by hot bodies.

A priori, this mechanism does not seem so good as some scheme for obtaining the quantum from the dimensions and charges of different parts of an atom. Its justification, however, lies in its ability to correlate experimental facts which are inconsistent with these other schemes.

First, by means of this mechanism, we may account for Planck's law of radiation, which, as we have seen, is not given by atomic models in which the atom can hold only a definite number of quanta.

Second, these assumptions do not, like Planck's, appear to be inconsistent with observed path differences in interference experiments, or known magnitudes of atoms.

Third, the magneton has been found by Parson [11] to be very efficient in explaining not only the periodic table of the elements, but even many of the exceptions to the laws indicated by this table.

Fourth, since it makes the emission of a quantum take the form of a very energetic oscillation, it gives a good chance for thermal emission of corpuscles, and, at low frequencies, for the interaction between colliding molecules that is necessary for the interchange of energy between heat and light, or in the absence of collisions for the flourescence of very rarified gases.[12] Moreover, since the amplitude is not dependent on certain equilibrium positions, as in most atomic models giving quanta, these assumptions can give an account of the photo-electric and photo-chemical phenomena at higher frequencies.

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11 l. c.

12 For a discussion of this point, see Phys. Rev. N. S. **4**, 177–94 (1914).

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Finally, this scheme gives Planck's formula without violating the classical laws determining the electromagnetic field in terms of the motions of electricity. Therefore it can also give an account of many optical phenomena, such as those of the propagation of light through matter, for which Planck's assumptions give no account at all.

Thus this theory correlates the phenomena of heat radiation, chemical affinities, thermions, photo-electric action, interference and many other optical effects, without the use of a mechanism that is inconsistent with known magnitudes. Consequently it may be pardoned for requiring these few arbitrary assumptions.

With regard to the internal mechanism that produces the emitting oscillations we can say very little, except that our intuition would naturally lead us to suppose that they could not exist. But we are here considering things of an order of magnitude so small that, while imagination is as valuable as ever, our intuition is very unreliable. For, after all, when we predict what a mechanism will or will not do, our predictions are not based ultimately on any ability of our own minds to guess the truth of a question, but rather on the accumulated results of our everyday experience, reenforced perhaps by that of our ancestors, and made more accurate by the results of premeditated experiments in the laboratory. This experience is all with things of visible size, and our intuitions tell us most naturally and emphatically what any mechanism would do if it obeyed the laws of visible things.

But why should this mechanism obey these laws? An enormous giant, capable of seeing only celestial objects and their motions, would predict everything in terms of inverse square laws of acceleration, and would be very much surprised, on increasing his power of vision, to find how rarely terrestrial objects obeyed such laws. Forces, especially those of friction, would then compel his attention; and they in turn would lay the foundation for another shock when he came to study the kinetic theory, with its frictionless attracting and repelling molecules, and their account of the origin of friction and other forces between larger bodies. Here he would have the sympathy of all students, whose intuition tells them that a roomful of ultramicroscopic bouncing billiard balls would bounce slower and slower until they settled in a hopeless heap on the floor. In another step, to the electron theory, the character of the forces changes again, and the phenomenon of inertia, the foundation of the dynamics of larger bodies, is explained as a result of electromagnetic forces. Is it, then, at all surprising if still more of these radical changes are in store for us in studying the ether and the internal mechanism of the magneton?

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We have been guided thus far by the necessity of correlating experimental facts. Let us therefore judge these assumptions by the number of experimental facts for which they account.

It would undoubtedly be possible to set up numerous models and sets of equations which would account, in a more or less complex way, for this law of emission, but it would not be profitable at present, as the laws would not be simple, and probably not very instructive. Therefore we shall not attempt to do so here.

The Derivation of Planck's Law. Planck's formula may now be derived by an argument almost like his, except as follows:

First, for the proof, Part IV, Chapter II, that the rate of absorption is constant with respect to time and proportional to the intensity of the light, we may substitute the fact that this is a well known result of an equation of the type of (3) when the intensity of the light is constant. The proportionality factor, as we have seen, is the same as in Planck's theory, though since it cancels out during the proof of the law of radiation, this fact is of no importance here.

Second, in Part III, Chapters III and IV, since the energy of a magneton can never get below the amount it has just after emission, the excess over this amount is what must be considered in the calculation of the entropy of the system.

Third, in the calculation of the distribution of energy among the oscillators, Part IV, Chapter III, we must divide them into two classes, those which are not emitting at the instant in question, and those that are; for the former, Planck's argument on the fraction of them in any energy interval at any time applies without change. Then since each one emits exactly as much energy as it absorbs, and at a constant rate, the distribution of the emitting ones, and therefore of both classes together, must be exactly like that of the absorbing ones. The transfer of energy to and from the molecular motions, being independent of the amount of energy in the steady current, cannot affect this distribution.

Thus we have Planck's law, derived by an argument almost identical with his, from assumptions which, although they are not so simple as his, are inconsistent with the classical dynamics only in the internal structure of the magneton. Being consistent with the classical electrodynamics, they correlate the phenomena of heat radiation with those of optics and electricity, at the same time giving an account of the laws of photo-electric and photo-chemical phenomena. Finally, being based on Parson's magneton, they correlate all these phenomena with those of chemical combinations.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE INFLUENCE OF THE MAGNETIC CHARACTERISTICS OF THE IRON CORE OF AN INDUCTION COIL UPON THE MANNER OF ESTABLISHMENT OF A STEADY CURRENT IN THE PRIMARY CIRCUIT.

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By B. Osgood Peirce.†

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Introduction.— Nearly sixty years ago, Helmholtz proved experimentally that the predictions of the mathematical theory which had been constructed to explain the courses of currents in inductively connected circuits, were fulfilled when the inductances were fixed constants, and shortly thereafter Du Bois Reymond put the equations for the currents in the two circuits of an induction coil with air core, into the forms in which they appear in modern textbooks. This simple theory, however, was soon found to be inapplicable when iron cores were used, and we now know that we cannot predict the march under given applied electromotive forces, of the currents in a number of coils wound on an iron core, without a knowledge of the magnetic history of the iron as well as of its magnetic properties, and that a general mathematical theory — if it were possible to form one — would be very complex. Notwithstanding this, it is often necessary to study the magnetic characteristics of a piece of iron by making it the core of a simple induction coil or transformer, and then determining experimentally the effect of its presence upon the manner of growth of the current in each circuit when a given electromotive force is applied to the primary. When the mass of the core is large, it is often important that the observer shall have at the outset a general knowledge of the manner in which the currents will change under a given set of initial conditions; this paper discusses briefly a few typical cases.

Magnets Employed in the Experiments.— In some of the experiments recorded in this paper, I have made use of an electromagnet (J, Fig. 1), the core of which, built up of varnished sheets of iron 0.38 millimeters thick, has a cross section of upwards of 150 square centi-

† Deceased.

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meters. The building-up curve of a current in the coil of this magnet is at the beginning slightly modified by the presence of eddy currents in the iron, but the effect is small. When this magnet is put a good many times successively through a cycle with a maximum excitation that will cause a flux density of about 14000, the throws of a ballistic galvanometer used to measure the total flux changes may differ from each other as much as 1% for different rounds, but the averages of a set of measurements made in the first case when the current is put on suddenly and in the second case when it is brought gradually in perhaps half a minute to its full value are practically indistinguishable.

Through the kindness of Dr. George Ashley Campbell I have been allowed to use also a number of toroids belonging to the American Telegraph and Telephone Co., the cores of which are made of wire only $\frac{1}{10}$ of a millimeter in diameter. A hysteresis diagram for any of these coils obtained by a step-by-step ballistic method, agrees exactly, within the limits of my measurements, with a similar diagram obtained by computation from an oscillograph record of a current curve in the coil. As we shall see this fact makes it possible, when

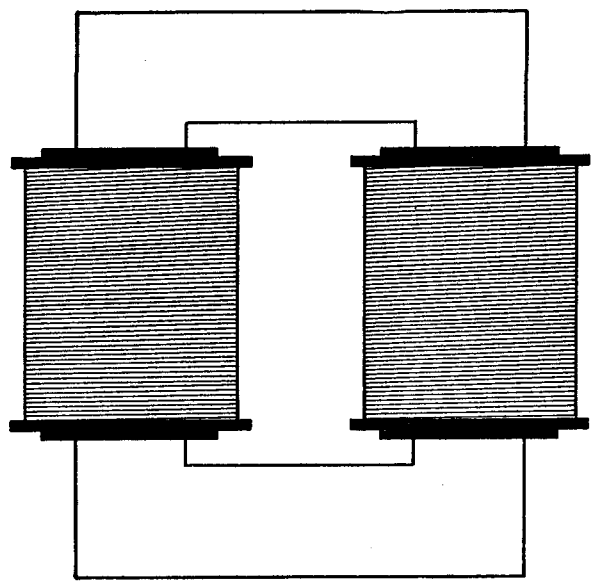

FIGURE 1. The electromagnet J, which has a laminated core of square cross-section of about 156 square centimeters area, and is built up of soft iron plates about one third of a millimeter thick.

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one has a hysteresis diagram for a given magnetic journey of the core, to predict accurately the form of a current curve corresponding to this journey under given electric conditions.

In the case of a commercial transformer, eddy currents in the laminated core affect slightly the form of a current curve at the very beginning, but nothing of the sort occurs in coils such as are used for loading telephone circuits. I shall assume, therefore, that the form of a current curve can be foretold if we have a permeability of the core for the given magnetic path, in the cases of such induction coils as we consider here.

The theory and method of investigation here proposed applies accurately to telephone loading coils with finely divided cores, and gives good approximation to correct results with commercial closed-core transformers.

The investigation comprises two parts: Part One, the establishment of the current in the primary coil of a transformer when the secondary is open; and Part Two, the growth of primary and secondary current, when the secondary is closed.

PART I. SECONDARY OPEN.

Theory.— Given a closed-core electromagnet of any form, perhaps one like J shown in Figure 1, suppose that the manner of growth of the flux of induction N in the core as the strength of the exciting current is made to grow larger by steps has been determined when the condition of the core at the outset is a certain definite one. Let the flux N be plotted against the ampere turns T of the exciting current, Curve $O\ B\ A$, Figure 2, and let one horizontal unit of the diagram represent a ampere turns, and one vertical unit represent m Maxwells. If the slope of the curve $O\ B\ A$ at any value T of ampere turns be $\Psi\ (T)$, then

$$\Psi\ (T) = \frac{m\ dN}{a\ dT} \tag{1}$$

If now an e. m. f. E be applied at the time $t = 0$ to the exciting circuit of resistance r, and if the exciting coil has n turns, and if the strength of the exciting current be i, and if we neglect the flux in the air through the turns of the coil, then

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$$E - n\frac{dN}{dt} = r\,i. \tag{2}$$

Let the slope $\Psi\,(T)$ be determined graphically from the curve $O\,B\,A$ described above, as a function of T. In practical units

$$E - \frac{n\,dN}{10^8 dt} = r\,i, \tag{3}$$

or

$$E - \frac{n\,a\,\Psi(T)}{10^8 m}\,\frac{dT}{dt} = \frac{r\,T}{n},$$

whence

$$\frac{n\,a\,\Psi(T)\,dT}{10^8 m\left(E - \frac{rT}{n}\right)} = dt \tag{4}$$

Call the left hand side of equation (4) $\Omega\,(T)\,dT$, then

$$\Omega\,(T)\,dT = dt.$$

If now $\Omega\,(T)$, computed from the known quantities of equation (4), be plotted as a function of T with one horizontal unit representing a ampere turns and one vertical unit v units of Ω, the area under the curve, thus plotted, from T_1 to T_2 when multiplied by av is equal to the time in seconds which elapses while the excitation is increasing from T_1 to T_2. It is evident that by taking successive values of T as T_1, T_2, T_3 etc., respectively equal to ni_1, ni_2, ni_3, etc., it is easy to obtain a graphical representation of T (and therefore of i) in terms of t. This gives in the absence of eddy currents the current curve.

We may illustrate the working of the foregoing theory by applying it to the electromagnet J, the form of which is shown in Figure 1.

Application to Magnet J.—With the core of J is magnetically neutral at the outset, the application of a series of currents, each a little stronger than the last, gave the numbers of Table I. With these results carefully plotted the slopes of the resulting curve furnished the numbers of Table II.

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TABLE I.

Ampere Turns (T)	Flux of Magnetic Induction N through the Core in Thousands of Maxwells
100	35
200	146
300	386
400	622
500	787
600	929
700	1013
800	1086
900	1137
1000	1176
1100	1208
1200	1238
1300	1262
1400	1285
1500	1309
1600	1331
1700	1352
1800	1371
1900	1390
2000	1409

Figure 2 represents Table I graphically in the full curve, $O\ B\ A$; one vertical unit corresponding to a hundred thousand maxwells, and one horizontal unit to a hundred ampere-turns. There is evidence of slight eddy-currents at the outset, but the effect quickly disappears and the magnetization curve follows closely the march of the current. The slopes of the curve $O\ B\ A$, were found in the following way: After the curve had been drawn on a large scale to represent the numbers of Table I, a zinc templet was made from it, as accurately as possible; this was fastened down on a drawing-board over a large sheet of coordinate paper and the value of the slope $\Psi\ (T)$ was determined by measuring on the paper the position of a straight-edge which was held tangent to the templet at any desired point. These slopes

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are recorded in Table II and plotted as the dotted curve of the same figure, Figure 2, with one division of ordinate equal to 200 $\frac{\text{Maxwell}}{\text{Ampere-turn}}$.

TABLE II.

Ampere Turns (T)	dN/dT
0	151
50	308
100	545
150	1255
200	2102
250	2639
300	2596
350	2374
400	2080
450	1786
500	1521
600	1119
700	818
800	617
900	437
1000	344
1100	301
1200	273
1300	251
1400	237
1500	222
1600	215
1800	208
2000	201

If after the core of the electromagnet J has been thoroughly demagnetized, a steady electromotive force E is applied to the exciting coil, which consists of n turns of wire of resistance r ohms, the march of the current can be predicted by the aid of equation (4). The values of the slope $\Psi(T)$ substituted in equation (4) permits the computation of $\Omega(T)$. In a particular case with $E = 10$, $r = 1$ and $n = 100$, the values obtained for $\Omega(T)$ are recorded in Table III, and plotted in the boundary of the shaded area of Figure 3. The actual curve which

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bounds the shaded area was plotted on a large scale, one vertical unit corresponding to $10^4\ \Omega\ (T)$ and one horizontal unit to a hundred T, and the area X from $T = 0$, to $T = T$, for a number of different values of T was measured with the aid of a good planimeter in terms of the unit square of the figure. A few values of the areas X are shown in Table III. These areas divided by a hundred express the time in seconds to which T corresponds. The resultant curve, $O\ P\ Q$, Figure 3, is the desired curve for growth of the current.

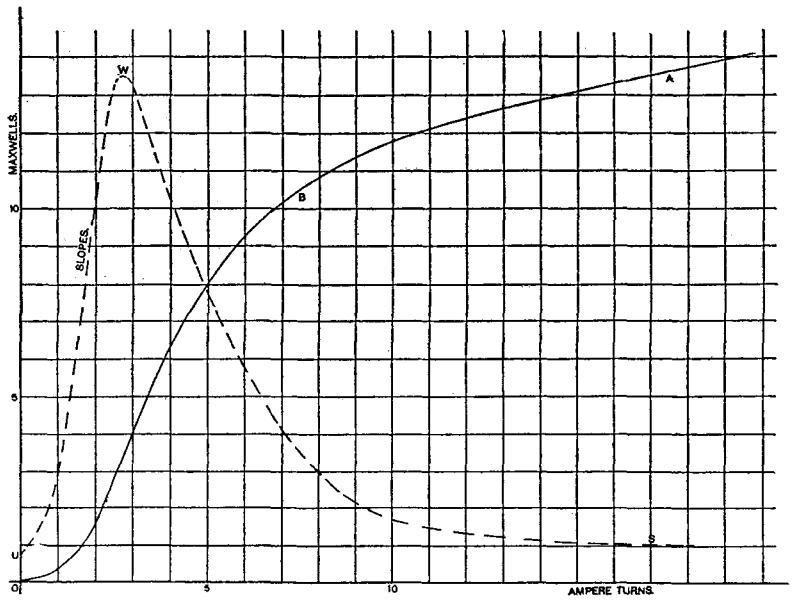

FIGURE 2. Magnetization curve, OBA, for the electromagnet J which at the outset is in a neutral state. Each horizontal division is 100 Ampere-turns and each vertical division is 100,000 Maxwells. The ordinates of the dotted curve represent the slopes of the magnetization curve, with each vertical division equal to 200 $\frac{\text{Maxwell}}{\text{Ampere-turn}}$.

Further Facts Regarding Electromagnet J.— We may add to the foregoing discussion, of the characteristics of the electromagnet J the results of a series of hysteresis cycles obtained some years ago. An outline of this magnet is shown in Figure 1. It weighs about 300 kilograms: the core has a square cross-section of 156 square centimeters area, and was built from varnished sheets of soft iron

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about one third of a millimeter thick. The exciting coil consists of 1394 turns of well insulated copper wire.

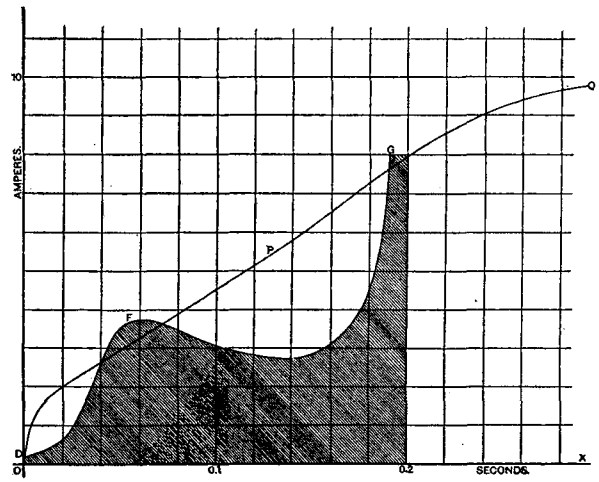

FIGURE 3. The ordinates of the boundary of the shaded area represent $10^4 \Omega(T)$ for $E = 10$, $r = 1$, and $n = 100$. The curve OPQ, shows the theoretical form of the corresponding current curve.

TABLE III.

T	$10^4 \Omega(T)$	X
0	0.151	0.000
50	0.324	0.123
100	0.605	0.336
150	1.477	0.790
200	2.629	1.887
250	3.519	3.477
300	3.710	4.612
400	3.467	8.157
500	3.042	11.404
600	2.800	14.318
700	2.727	17.058
800	3.085	19.884
900	4.370	23.489
950	7.800	26.279

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It is difficult to obtain an accurate hysteresis diagram for transformers containing massive iron cores, by the "step-by-step" ballistic method, with a short period galvanometer. Although, eddy currents may be nonexistent the time-lag of magnetization necessitates the use of a long period instrument, when ballistic methods are employed. The galvanometer used throughout these experiments was of the d'Arsonval type, and had a period of about four minutes. The accuracy of the ballistic method was tested by comparing the results, thus obtained, of a corresponding hysteresis cycle for an excitation of

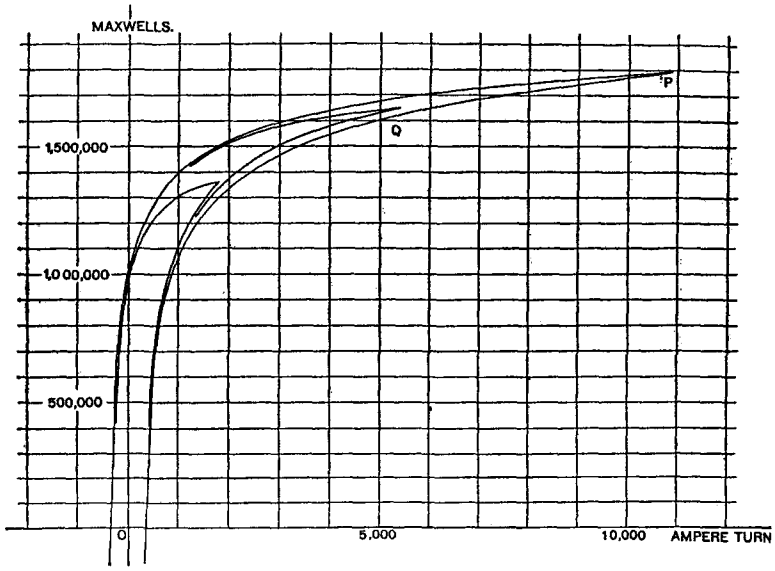

FIGURE 4. Hysteresis diagrams for the core of the electromagnet J.

1812 ampere turns, with a series of results reduced from oscillograph records. Throughout the comparison the agreement lay within a tenth of one per cent, and this may be considered close since it is not always possible to make a large mass of iron travel exactly the same magnetic journey twice.

Computation of Current Growth on Reversal of E. M. F.— Figure 4 shows a series of hysteresis diagrams for the electromagnet *J* obtained by decreasing and reversing the exciting current step by step after maximum excitations of 1812, 5370, and 10880 ampere-turns. The results of measurements of the flux changes in the core for the

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first of these cycles are given in Table IV. The next diagram, Figure 5, shows the slopes of the curve corresponding to Table IV, as a function of the strength of the exciting current.

From equation (4) it will be seen that the march of current on reversal of e. m. f. may be obtained. If the slope for any point of the flux curve is multiplied by $n^2/10^8$ $(E-ri)$, the result is the value of dt/di, for the reversed current curve, when the constant voltage E is applied to the exciting coil and reversed, where the coil consists of n turns of wire with resistance r ohms. Figure 5 exhibits dt/di for $E = 19.5$, $r = 15$, and $n = 1394$.

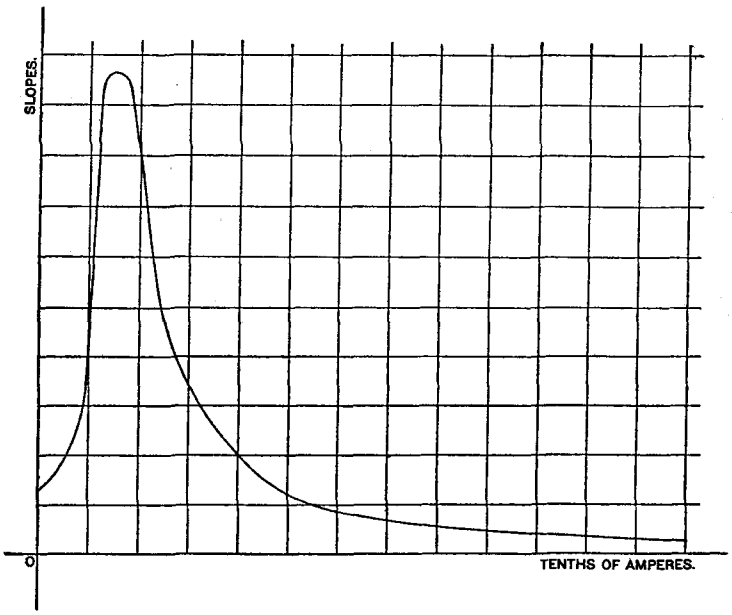

FIGURE 5.

If now the area X, underneath the curve, Figure 7, from $x = 0$ to $x = i$, for a number of different values of i, be measured in terms of the unit square of the figure; this area expresses the time required for the reverse current to attain the strength i. Table V contains a few values of X which were measured with a planimeter, and from which the desired reverse current curve, shown in Figure 7, was plotted.

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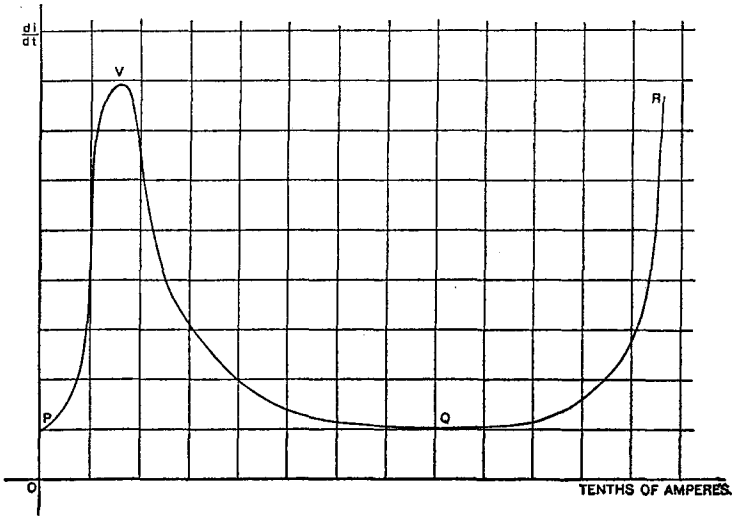

FIGURE 6. The value of di/dt for a reverse current in the coil of the magnet J when $E = 19.5$ and $r = 15$.

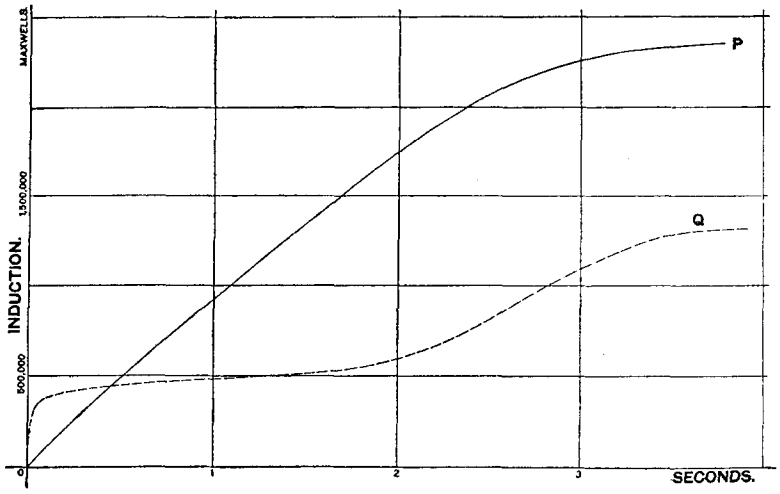

FIGURE 7. The full curve, OP, shows the rate of increase of the flux of magnetic induction through the core of the magnet J while a reverse current of 1.3 amperes is being established in the exciting coil of 1394 turns. The current curve is shown on an arbitrary scale by the dotted line.

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TABLE IV.

FLUX ON GRADUALLY DECREASING AND REVERSING EXCITING CURRENT.

Ampere Turns	Flux in Thousands of Maxwells	Ampere Turns	Flux in Thousands of Maxwells
1812	1371	−131	772
1394	1351	−148	734
1255	1340	−181	552
1031	1316	−234	332
809	1285	−294	22
474	1211	−392	−465
392	1186	−474	−661
294	1148	−809	−1010
234	1121	−1031	−1128
181	1099	−1255	−1214
148	1070	−1394	−1265
131	1060	−1821	−1371
000	953		

TABLE V.

i	$X/10$	i	$X/10$
0.05	0.057	0.50	1.750
0.10	0.115	0.60	1.875
0.15	0.494	0.70	1.985
0.20	0.878	0.80	2.088
0.25	1.141	0.90	2.188
0.30	1.325	1.00	2.294
0.35	1.471	1.10	2.412
0.40	1.579	1.20	2.682

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PART II. SECONDARY CLOSED.

Theory.— In an iron core built up of varnished sheets of metal there is usually a noticeable amount of magnetic leakage, but in a toroidal core made of one piece of varnished, very fine, soft iron wire, upon which two transformer coils are uniformly and closely wound, the leakage is generally negligible and we may assume with a close approximation to the truth, that if the coils consist of n_1 and n_2 turns respectively, and if the resistances of their circuits are r_1 and r_2, the excitation of the core in ampere turns is at any time,

$$T = n_1 i_1 + n_2 i_2$$

and if N is the flux of magnetic induction through the core, the corresponding fluxes through the two circuits are $n_1 N$ and $n_2 N$ respectively. Let N be determined experimentally at many points of a journey from one magnetic state of the core to another by a given path, and let the results be plotted in the form of a curve, the unknown equation of which may be written in the form $N = \Phi(T)$. Let the slope of this curve be obtained at a large number of points so as to give the values of dN/dT or $\Psi(T)$ for many values of T, then it is possible to predict the form of a building up curve for a current in the exciting coil which will carry the core over a part, or the whole, of the magnetic path for which we know the values of N as a function of T.

The current equations are

$$E_1 - \frac{n_1}{10^8} \cdot \frac{dN}{dt} = r_1 \cdot i_1, \tag{5}$$

$$-\frac{n_2}{10^8} \cdot \frac{dN}{dt} = r_2 \cdot i_2, \tag{6}$$

whence

$$E_1 n_2 = n_2 \cdot r_1 \cdot i_1 - n_1 \cdot r_2 \cdot i_2, \tag{7}$$

$$i_1 = \frac{n_1 \cdot r_2 \cdot T + n_2^2 \cdot E_1}{n_1^2 \cdot r_2 + n_2^2 \cdot r_1}, \tag{8}$$

$$i_2 = \frac{n_2 \cdot r_1 \cdot T - n_1 \cdot n_2 \cdot E_1}{n_1^2 \cdot r_2 + n_2^2 \cdot r_1}, \tag{9}$$

$$\frac{n_1}{10^8} \cdot \frac{dN}{dt} \cdot \frac{dT}{dt} = \frac{n_1^2 \cdot r_2 \cdot E_1 - n_1 \cdot r_1 \cdot r_2 \cdot T}{n_1^2 \cdot r_2 + n_2^2 \cdot r_1}, \tag{10}$$

$$\frac{n_1 (n_1^2 \cdot r_2 + n_2^2 \cdot r_1) \cdot \Psi(T) \cdot dT}{10^8 (n_1^2 \cdot r_2 \cdot E_1 - n_1 \cdot r_1 \cdot r_2 \cdot T)} \equiv \Omega(T) \cdot dT = dt. \tag{11}$$

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If now $\Omega(T)$ be accurately plotted against T, the area under this curve from T_1 to T_2 is equal to the time in seconds which elapses while the excitation is growing from T_1 to T_2. If a large number of these areas be measured by aid of a planimeter, it is easy to give a graphical representation of T (and therefore of i_1 and i_2) in terms of t, and we may expect the current curves thus obtained to correspond closely with the oscillograph records for the same case. If r_2, r_2, n_1, n_1, and E_1 are all increased in a given ratio λ, the quantity $\Omega(T)$ and therefore, dt, will be increased in the same ratio.

Application to Transformer with Fine Wire Core.— It will be instructive to apply the foregoing theory to a certain transformer (DN), in which magnetic leakage and eddy currents were negligible. This transformer was constructed in the form of a toroid, about 41 centimeters mean diameter, the core of which was made of about 25 kilograms of fine, soft, varnished iron wire.

After the core of the transformer had been thoroughly demagnetized the magnetic flux through the core, due to an ascending series of steady currents in the exciting coil, was determined. The results of the long series of measurements, which were taken with a slow period ballistic galvanometer, are given in Table VI. The full curve, OK, Figure 8, reproduces the table graphically: one vertical unit corresponds to fifty thousand maxwells, and one horizontal unit to a thousand ampere-turns. The ordinates of the dotted curve exhibit, on an arbitrary scale, the corresponding slopes of the other. A few values of the slope, dN/dT, are given in Table VII.

From the foregoing theoretical discussion, and the numbers given in Tables VI and VII, it is always possible to predict under fixed conditions the growth of the excitation in the core, the march of the current in the primary and secondary coils, and the manner of increase of the flux of magnetic induction in the core of the transformer in question. It will be seen from equation (11), when $E = 10$, $n_2 = 1000$, $n_1 = 100$, $r_2 = 10$, and $r_1 = 1$, that if the slope of any point of the curve, OK, is multiplied by $11/10^4\ (1000-T)$, the result is the value of $\Omega(T)$ for the given value of T. A few values of $\Omega(T)$ are shown in Table VIII. If now $\Omega(T)$ be plotted as a function of T, and the area from $T = 0$ to $T = T_1$, for a number of different values of T be measured in terms of the unit square of the figure, this area gives the time in seconds for the excitation in the core to attain the value T.

The curve, $VWSZ$, bounding the shaded area (Fig. 9), reproduces the first two columns of Table VIII graphically; that is, $\Omega(T)$ as a function of T. For convenience $10^4\,\Omega(T)/5$ was taken as the vertical

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unit, and 100 ampere-turns as the horizontal unit. In consideration of the units here chosen the growth of the excitation in the core (in ampere-turns) would be given by the equation

$$t = 20 \int_0^{T_1} \Omega\,(T)\, dT \qquad (12)$$

Therefore, it is clear that the area, between the limits of $T = 0$ and $T = T_1$, divided by 20 gives the time in seconds for the excitation to grow from 0 to T_1. The third column of the table contains a number

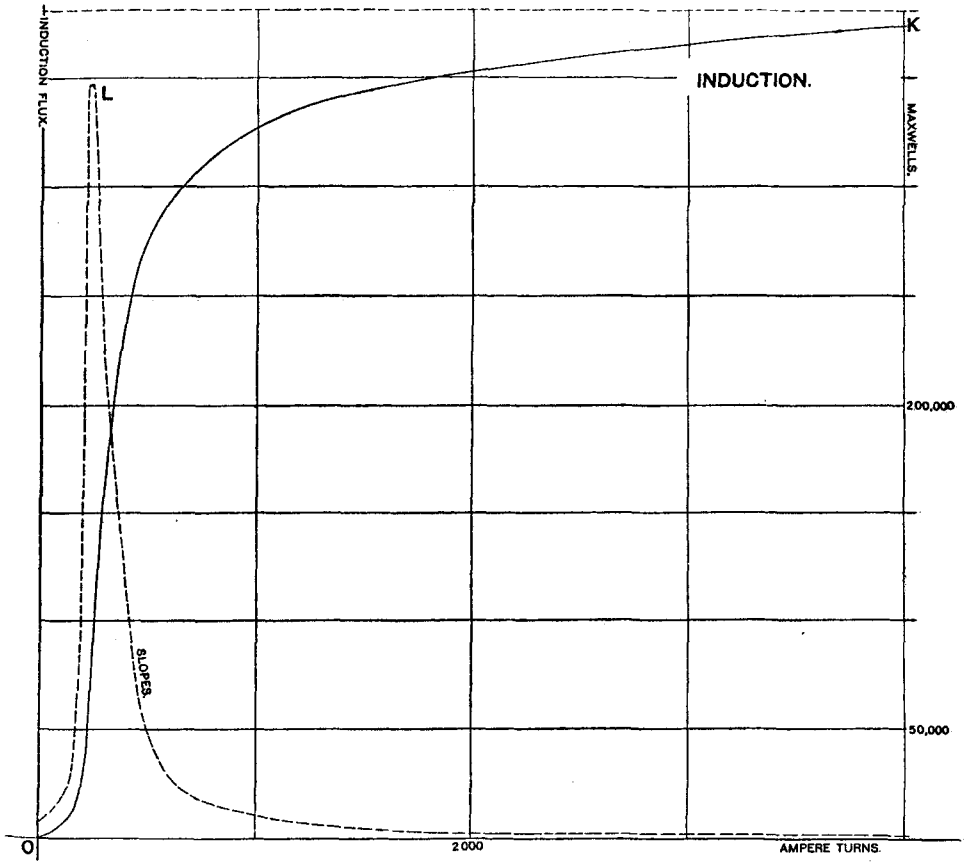

FIGURE 8. Magnetization curve for the finely divided core of the transformer (DN) which at the outset is in a neutral state. The dotted curve represents, on an arbitrary scale the slopes of the other.

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of areas, found by mechanical integration, for different values of T and the next column the corresponding time in seconds which the excitation took to rise from 0 to T. The curve $OBLP$ shows graphically the increase of the excitation as a function of the time.

TABLE VI.

Ampere Turns	Flux of Magnetic Induction (N) through the Core in Hundreds of Maxwells	Ampere Turns	Flux of Magnetic Induction (N) through the Core in Hundreds of Maxwells
23	10	600	2910
42	20	700	3032
87	50	800	3124
100	52	1000	3259
200	240	1200	3348
232	445	1400	3406
257	879	2000	3516
294	1520	2630	3602
300	1548	3000	3644
316	1733	3500	3690
400	2372	4160	3740
500	2736		

TABLE VII.

Ampere Turns (T)	dN/dT	Ampere Turns (T)	dN/dT
000	35	600	142
100	76	700	103
200	510	800	79.4
220	700	1000	53.9
240	1742	1200	36.2
260	1670	1400	30.5
280	1338	1600	20.5
320	950	1800	18.7
360	748	2000	13.6
400	552	3000	10.9
500	251	4000	4.8

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We may now determine the march of the current in the primary and the secondary coils of the transformer, for the given values of E_1, n_1, n_2, r_1, and r_2. Substituting in equations (21), and (22) we get

$$i_1 = \frac{T + 10^4}{1100}, \qquad i_2 = \frac{T - 10^3}{1100} \tag{13}$$

for any given T. Table VIII, columns five and six, contain values of i_1, and i_2 corresponding to the value of T which is given in the first column. Figure 10: the curve *PRH* shows graphically the march of

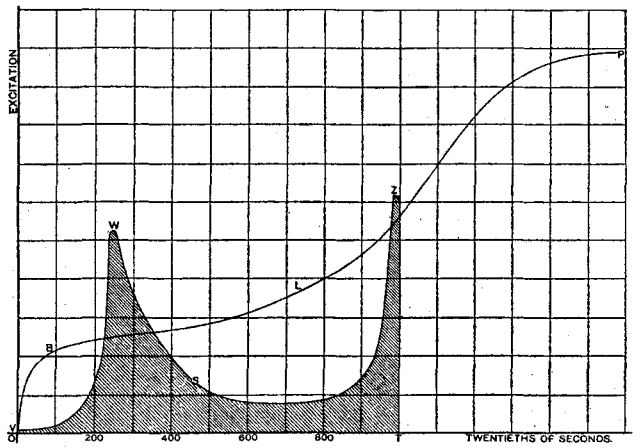

FIGURE 9. The ordinates of boundary of the shaded area represent $10^4\,\Omega\,(T)/5$, $E = 10$, $n_1 = 100$, $n_2 = 1000$, $r_1 = 1$, and $r_2 = 10$, the abscissas correspond to $100T$. The curve OLP shows the manner of growth of the excitation in the core of the transformer (DN), as a function of the time.

the current in the primary coil; and the curve *SEC*, bounding the shaded area at the bottom of the figure, the manner of decay of the current i_2 in the secondary. It will be seen from the figure that i_1, the current in the primary, builds up very rapidly at the start, but before reaching its maximum value it remains for a comparatively long time almost exactly parallel to the time axis. During this time the indication of an amperemeter in the circuit does not change perceptibly, and yet the flux of magnetic induction through the core is increasing at a very nearly constant rate. The shaded area, bounded by the curve *PRH* and its asymptote *KD*, which is a measure of the

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induction flux, is constantly growing. If the core of a transformer is very large the building-up time may be a minute or more, and the phenomenon may then become very striking.

The curve *ONM* (Fig. 10) represents graphically the flux of magnetic induction through the core as a function of the time. One vertical unit of the figure corresponds to thirty thousand maxwells, and one horizontal unit to a twentieth of a second.

A hysteresis diagram for the transformer under investigation is shown in Figure 11. The cycle corresponds to a maximum excitation

TABLE VIII.

Ampere Turns	$10^4\Omega(T)$	Area	t	i_1	i_2
00	0.385	0.000	000	9.092	−0.909
50	0.550	0.037	0.002	9.136	−0.864
100	0.929	0.123	0.006	9.182	−0.818
150	2.700	0.268	0.013	9.230	−0.773
200	7.013	0.692	0.035	9.273	−0.727
230	15.500	1.232	0.062	9.301	−0.700
250	26.300	2.169	0.108	9.320	−0.682
270	22.500	4.093	0.205	9.338	−0.664
300	17.950	5.313	0.266	9.365	−0.636
350	13.100	6.866	0.343	9.411	−0.591
400	10.120	7.988	0.399	9.458	−0.545
500	5.522	9.444	0.472	9.547	−0.454
600	3.905	10.297	0.515	9.636	−0.363
700	3.777	11.037	0.552	9.727	−0.273
800	4.367	11.814	0.596	9.800	−0.182
900	7.095	12.886	0.644	9.910	−0.091
950	12.980	13.786	0.689	9.957	−0.045
980	30.300	14.933	0.747		

of 4200 ampere-turns, and the results of measurements of the flux changes in the core for this cycle are given in Table IX. From what has been explained already, and the data given here one can predict with accuracy the characteristics of the core, and the current curves for this transformer, for any practical case within the limits of the above excitation.

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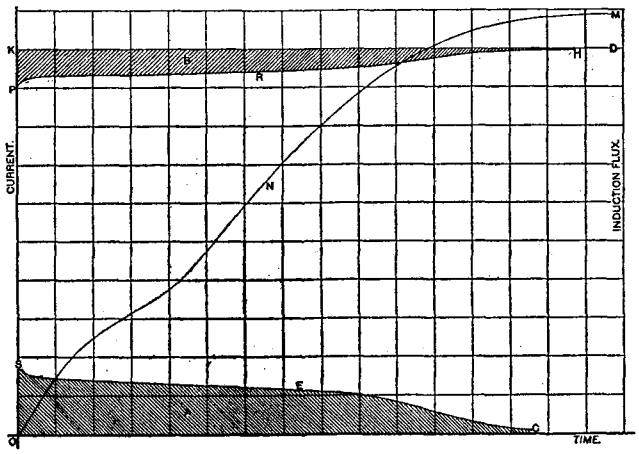

FIGURE 10. Transformer (DN): The curves, PRH and SEC, deduced from theoretical considerations, indicate the march of the current in the primary and secondary coils respectively. The increase of the flux of magnetic induction, through the core, with the time is shown by the curve ONM.

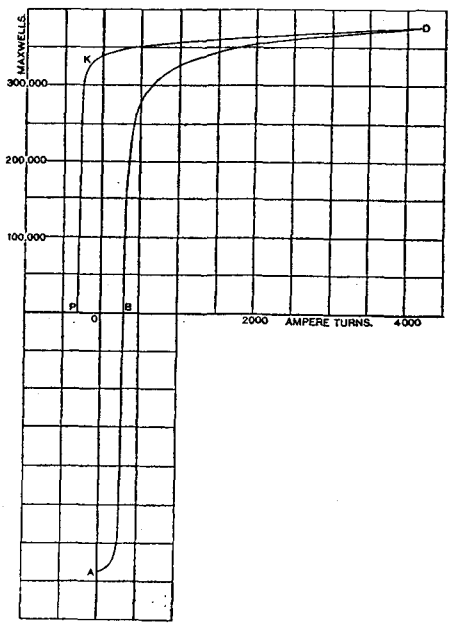

FIGURE 11. Hysteresis diagram for the core of the transformer (DN).

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TABLE IX.

Ampere Turns	Flux of Magnetic Induction through the Core in Hundreds of Maxwells.	Ampere Turns	Flux of Magnetic Induction through the Core in Hundreds of Maxwells
(Up)		1410	+3406
000	−3380	2190	+3580
203	−3241	3310	+3670
238	−2938	4200	+3750
262	−2321	(Down)	
303	+ 866	4000	+3730
317	+1276	3500	+3698
339	−1700	3000	+3667
360	+2030	2500	+3635
386	+2248	2000	+3603
422	+2482	1500	+3571
502	+2752	1000	+3531
685	+3048	500	+3482
1050	+3282	000	+3380

It is evident from the foregoing discussion that if eddy currents are nonexistent in the core of a transformer, a fair approximation to the form which the characteristic curves will have under any given circumstances can be made if one has an accurate statical hysteresis diagram of the core for the range required. That is, one can predict with accuracy the form of the current curves, the growth of excitation, and flux of magnetic induction in the core of a good transformer.

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A REVISION OF THE ATOMIC WEIGHT OF PRASEODYMIUM.

THE ANALYSIS OF PRASEODYMIUM CHLORIDE.

By Gregory Paul Baxter and Olus Jesse Stewart.

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Introduction.

Not many years ago the atomic weight of neodymium was investigated in this laboratory by the analysis of the anhydrous chloride.[1] Since considerable success was met both in preparing pure material and with the analytical method employed, the twin element praseodymium was investigated in a similar fashion; for, from the results of the earlier investigations upon praseodymium, it can readily be seen the value of its atomic weight is far from certain.

After Auer von Welsbach [2] first separated the old didymium, he determined the atomic weights of the constituents by Bunsen's method of converting oxide to sulphate. Apparently the results were

1 Proc. Amer. Acad., **46**, 213 (1911); Jour. Amer. Chem. Soc., **33**, 1; Zeit. anorg. Chem., **70**, 1.

2 Sitzungsb. Akad. Wiss. Wien, **92**, 317 (1885).

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interchanged in publication, as Brauner has suggested, for the lower value 140.8 was assigned to neodymium.

Next Brauner,[3] in 1898, starting with material purified by Shapleigh by crystallization of the double ammonium nitrate, continued the process of crystallizing this salt until neodymium was completely eliminated, then removed a trace of lanthanum by fusion with potassium nitrate, extraction of the praseodymium oxide with ammonium nitrate, and fractionation with ammonia and oxalic acid. By both the analysis of the oxalate and synthesis of the sulphate thirteen results were obtained between 140.84 and 141.19, with an average of 140.95.

Jones,[4] in the same year, further purified double ammonium nitrate furnished by the Welsbach Light Company by crystallization until the neodymium content was about 0.06 percent, as determined by comparison with neodymium solutions of known concentration. Cerium was removed by the basic nitrate process, and traces of lanthanum by further crystallization of the double ammonium nitrate. Spectroscopically only a trace of lanthanum and no cerium could be detected. The oxalate was converted to trioxide by ignition in air and then in hydrogen, and after being weighed the oxide was converted to sulphate. Twelve determinations between 140.38 and 140.54 give an average of 140.46.

Schéele[5] also purified material first by crystallization of the double ammonium nitrate, next by extracting the black oxide with ammonium nitrate, and then by precipitation of the oxalate. In a final series of determinations oxalate was converted to trioxide in a current of hydrogen, and the oxide in turn to sulphate. From the ratio of trioxide to sulphate five values between 140.48 and 140.61 resulted.

In 1901 Brauner[6] confirmed his earlier work by four different methods, using similarly purified material. By igniting weighed quantities of octahydrated sulphate to the black oxide and correcting for the oxidizing power of the oxide as determined iodimetrically, he obtained from the ratio $Pr_2O_3 : Pr_2(SO_4)_3\ 8\ H_2O$ two results, 141.13 and 141.04. Weighed amounts of carefully dehydrated sulphate were then converted to oxide in the same way, yielding the values 140.96 and 140.94. Air-dried oxalate was weighed and ignited, and

3 Proc. Chem. Soc., (1898) 70.
4 Am. Chem. Jour., **20**, 345 (1898).
5 Zeit. anorg. Chem., **17**, 310 (1898).
6 Proc. Chem. Soc. (1901) 65; Abegg, Handb. d. anorg. Chem., **3** (1) 263 (1906).

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the oxidizing power of the oxide was determined. Other weighed portions of oxalate were oxidized with permanganate. The ratio Pr_2O_3: $3C_2O_3$ gave the average result 140.98. Finally, weighed amounts of oxalate, the praseodymium content of which had been found as above, were ignited to black oxide and this in turn to trioxide in hydrogen. The trioxide was changed to sulphate by solution in nitric acid and evaporation with sulphuric acid. Excess of acid retained by the salt was found by titration. In eight experiments the ratio Pr_2O_3: $Pr_2(SO_4)_3$ yielded an average value 140.96. The mean of the four methods is 140.97, which is essentially identical with the result of Brauner's earlier work.

Auer von Welsbach [7] next published the results of three determinations by the Bunsen method, without details, 140.64, 140.50, 140.56, average 140.57.

Finally Feit and Przibylla [8] purified praseodymium material from neodymium by crystallization of the double magnesium nitrate, and from lanthanum by crystallization of the nitrate from nitric acid solution. The higher oxide, prepared by ignition of the oxalate, was dissolved in standard sulphuric acid and the oxygen evolved was measured, as well as the excess of sulphuric acid. The oxygen evolved was subtracted from the weight of the black oxide before computing the atomic weight from the relation of trioxide to sulphuric acid used. The average result of three experiments is 140.54.

Thus it can be seen that while the investigations of Jones, Schéele, von Welsbach, and Feit and Przibylla indicate a value for the atomic weight of praseodymium between 140.5 and 140.6, that of Brauner, which was carried out with equal or greater care, and with material of undoubted purity, points to a value at least as high as 140.9. The International Committee upon Atomic Weights has chosen the lower figure, and recommends the value 140.6.

The various difficulties likely to be met in carrying out the methods used in the earlier determinations have been many times discussed, and a résumé of the situation is given in the paper by Baxter and Chapin on the atomic weight of neodymium.[9] It is worth pointing out that in addition to the dangers there mentioned, methods involving the use of praseodymium oxide are subject to error from the tendency of this substance to form a higher oxide of somewhat uncertain com-

7 Sitsungsb. Akad. Wiss. Wien, **112**, 1037 (1903).
8 Zeit. anorg. Chem., **50**, 258 (1906).
9 Loc. cit.

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position. While ignition in hydrogen causes reduction to the trioxide, yet it is not easy to make certain that no higher oxide is retained in the lower one. On the other hand the analysis of the anhydrous chloride served so satisfactorily with neodymium that it seemed worth while to apply the same method to praseodymium. The results amply justified our expectations.

The Purification of the Praseodymium Material.

Through the great kindness of Dr. H. S. Miner of the Welsbach Light Co., Gloucester City, N. J., we were fortunate enough to secure as a starting point about 10 kilograms of praseodymium ammonium nitrate containing about 50 per cent. of the corresponding lanthanum and cerium salts as well as a small proportion of neodymium. Since one of the most rapid and effective methods of freeing praseodymium from the closely related elements, lanthanum, cerium, neodymium, and samarium, is the fractional crystallization of the above salt, this method of purification was chosen. According to Auer von Welsbach,[10] the bases separate in the order, lanthanum, cerium, praseodymium, neodymium, samarium, terbium and yttrium earths. The salt was crystallized in the usual way, that is, a concentrated, hot solution containing a small amount of nitric acid was allowed to cool and deposit the excess of salt, a period of 24 hours being allowed to secure equilibrium between the crystals and liquid. The separation of crystals and liquid was not completed by centrifugal drainage, because the labor and time involved in this operation are not repaid by any considerable increased speed of purification. The details of the crystallization are shown in the diagrams on pages 175 and 176. In any given series of crystallizations a lower number always indicates a less soluble fraction. A line not connecting an end fraction with any fraction in a subsequent series indicates rejection. This crystallization was begun by Mr. W. H. Whitcomb, continued by Mr. B. W. Grimes and Mr. C. C. Wallace and completed by Mr. Stewart.

In the early part of the crystallization it became obvious that the original material consisted largely of the lanthanum salt, the least soluble fraction quickly becoming essentially colorless. At the same time the neodymium absorption bands, which were readily visible in the original material, rapidly strengthened in the extreme mother liquor. When this mother liquor was reduced to a volume of about 20 cc., fraction 63, its absorption spectrum was examined visually in

10 Sitzungsb. Akad. Wiss. Wien, **112**, 1043 (1903).

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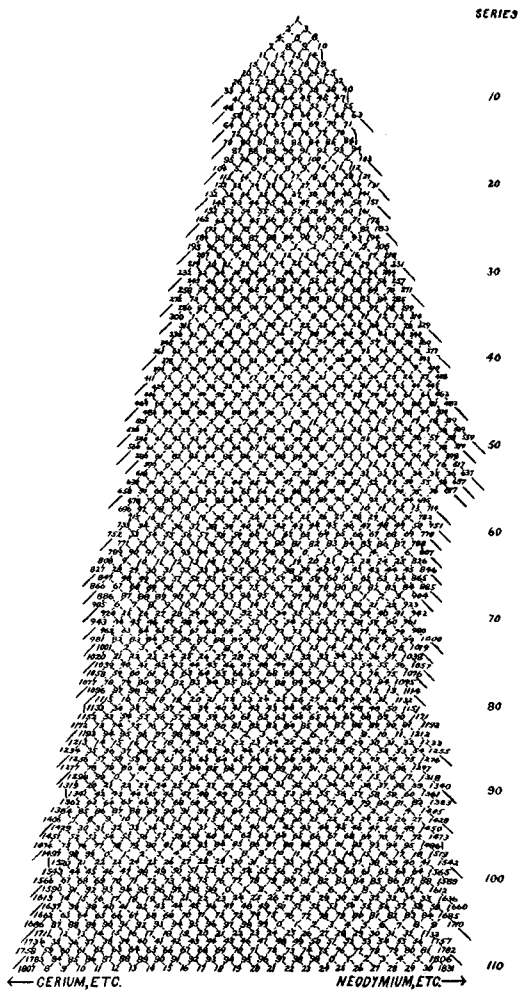
a Hilger wave length spectroscope. The only absorption bands which could be detected were those of praseodymium and neodymium. No sign of any of the samarium bands could be seen. Photography of the



ultraviolet absorption spectrum with a Féry quartz spectrograph also failed to indicate even a trace of samarium, for the band λ 401 was absolutely lacking. The crystallization proceeded with the occasional

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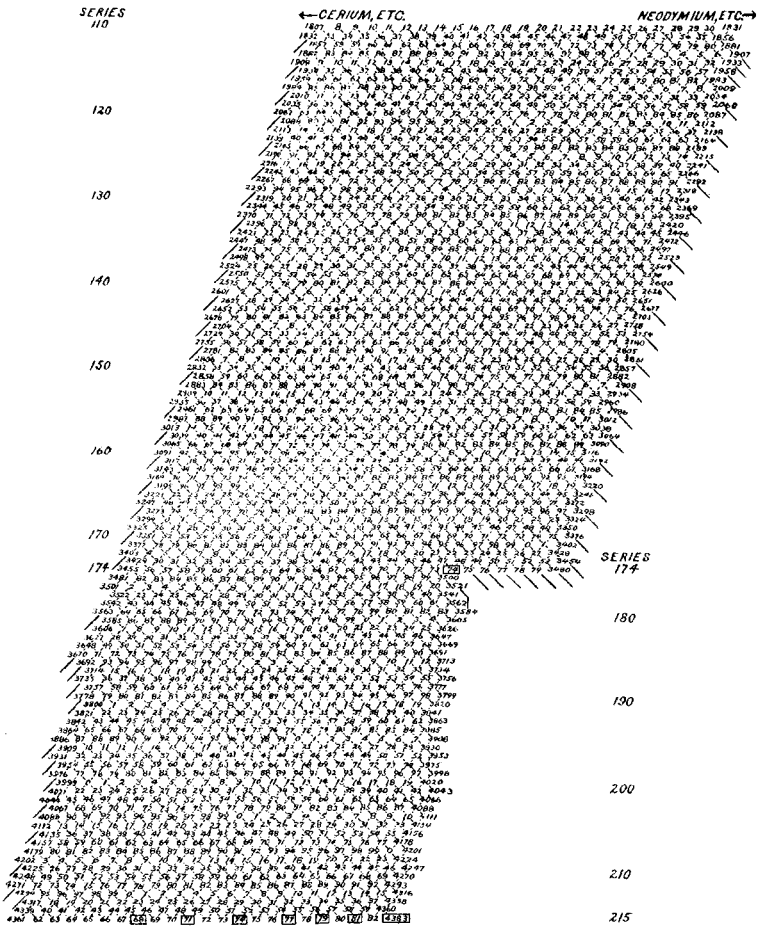
rejection of fairly large fractions, chiefly lanthanum and cerium, at the less soluble end of each series, and the rejection of much smaller fractions, partly neodymium, at the more soluble end, until 174 series of crystallizations had been made. From the diagrams it can be seen that the number of fractions rapidly increased from the outset to a fairly



constant number between 20 and 25. In series 174 a spectroscopic examination of the more soluble fractions showed that fraction 3478 contained a very small quantity of neodymium, the absorption band at

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520 being faintly visible. The proportion of neodymium in this fraction we estimate to be at least as small as 0.05 percent. In fraction 3476 the same neodymium band could scarcely be detected. At this point fraction 3474 was removed for analysis, since it seemed likely that this fraction was as pure as any in the series 174. The fractions 3475–3480 were rejected. Fractional crystallization was then continued in a similar way with the remaining fractions, except that while the extreme crystal fractions were occasionally rejected to remove cerium and lanthanum, no fractions were rejected from the more soluble end. When the extreme fraction at this end became very small, it was set aside to be added to a subsequent similar one, and the fractionation then continued. After 41 more series of crystallizations the process was discontinued because very careful spectroscopic examination of the extreme mother liquor, fraction 4383, in saturated solution with a 10 cm. layer showed no sign of the neodymium absorption band λ 520. The quantitative examination of selected fractions was then undertaken. Those chosen were 3474, 4383, 4381, 4379, 4377, 4374, 4371 and 4368. Since the less soluble fractions beyond 4368 were believed to contain cerium, they were rejected. In fact, an analysis of fraction 4368 showed it to contain about 0.4 of one percent of this element, a proportion, however, which is hardly perceptible in the atomic weight.

The Purity of the Fractions of Praseodymium Material.

The purity of the fractions in the final series was determined as follows: In neither fraction 3474 nor the more soluble fractions of the last series, 4383 and 4381, could any of the neodymium absorption lines be detected, either visually with a Hilger wave length spectroscope or by photography with a Hilger quartz spectrograph of the Féry type. With the latter instrument photographs were made with various depths of solution and with widely varying exposures, but the results were less satisfactory than the visual ones. In order to find out what proportion of neodymium could be detected in praseodymium material, measured portions of a standard solution of neodymium ammonium nitrate were added to weighed portions of the double ammonium nitrate of fraction 4367, which was as free from neodymium as any. By using concentrated solutions and a 10 cm. layer it was found that 0.05 percent of neodymium could be detected with ease through the absorption band λ 520, and it was evident that the sensi-

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tiveness of the method could have been increased still further if necessary. Since even 0.05 percent of neodymium would raise the atomic weight of praseodymium by only 0.002 unit, it is obvious that the purity of the material so far as neodymium is concerned is amply sufficient.

Attempts were made also to detect neodymium by means of the spark spectrum between copper electrodes with the quartz spectograph, but the absence of strong neodymium emission lines at points in the praseodymium spectrum which are comparatively free from lines prevents this method from being at all satisfactory.

At the other end of the series the detection of cerium was undertaken. Since lanthanum ammonium nitrate is even less soluble than the corresponding cerium salt, the absence of cerium in the praseodymium material is sufficient proof of the absence of lanthanum also. In the spark spectrum of cerium fortunately there is a strong line of wave length 306 located at a point in the praseodymium spectrum which is comparatively free from lines. By photographing the spectrum of the spark, between copper electrodes, of praseodymium material originally free from cerium (fraction 4380) but diluted with known percentages of cerium, it was found that the limit of detection of cerium in this way lay between 0.5 and 1.0 percent. Assuming the atomic weights of cerium and praseodymium to be 140.3 and 140.9 respectively, one percent of cerium, if in the trivalent condition, would lower the atomic weight of praseodymium by 0.006 unit. Such a difference is difficult to detect by the method which we are using for the determination of the atomic weight of praseodymium. Upon photographing the spark spectra of the extreme fractions 4361, 4364 and 4368 (the most impure analyzed), it was found by comparison that the first fraction of the three was rich in cerium, the second contained much less, and fraction 4368 contained no more at any rate than 1 percent.

Because of the uncertainty in estimating proportions of impurity from the intensity of the spectrum lines, fractions 4365, 4368, and 4371 were further tested for cerium as follows: The solution was precipitated with an excess of sodium hydroxide and the precipitated hydroxides were washed several times. Carefully scrubbed chlorine gas was next passed into the solution in order to dissolve the praseodymium hydroxide. The residual ceric hydroxide was dissolved and the process was repeated. The second residue was collected upon a filter paper, washed and ignited. Then the dissolved praseodymium was precipitated as oxalate and ignited to oxide which was

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weighed. In this way fraction 4365 was found to contain 2.5 percent, fraction 4368, 0.4 percent and fraction 4371, 0.1 percent of cerium. Besides the small and unimportant percentage of cerium in fractions 4368 and 4371, which were analyzed, the rapid falling off of the cerium percentage is worth noting.

The Absorption Spectrum of Praseodymium Chloride.

The absorption spectrum of praseodymium chloride prepared from fraction 4181 was measured in the region of the visible spectrum by means of a Hilger wavelength spectroscope, provided with an extra dense prism and achromatic lenses. The accuracy of measurement with this spectroscope was not far from 0.1 $\mu\mu$ even for the longer wavelengths. Of the four broad absorption bands shown by concentrated solutions, only the one in the yellow visibly resolves into two narrower ones as the dilution increases. The wavelengths of the middle of each band at the lowest concentration at which it could be plainly seen are given below, together with observations by some other observers.[11] Praseodymium nitrate was found to give an exactly similar absorption spectrum.

Baxter and Stewart (1914)	Aufrecht (1904)	Auer (1903)	Böhm (1902)
597	595.5	596	
587	590.1	592–587	589.6
481.5	482.0	482–481	481.2
469	468.9	470.6–466.2	469.0
443.5	444.0	447.8–440.3	444.0

Photographs of the absorption spectrum also were taken with a Féry quartz spectrograph. When the source of illumination was a Nernst filament, no absorption bands beyond the visible region could be found with any exposure or any concentration of solution. When the spark from "Nichrome" wire was employed, however, it was found that the light is completely cut off at from λ 280 to λ 270 according to the concentration of the solution, and that the solution is opaque up to the extreme limit of the spectrograms, about λ 210. In these spectro-

11 Kayser, Handb. d. Spectr., III, 440.

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grams there were noticeable at certain concentrations very faint minima of absorption in the middle of all three bands in the blue. These minima could be seen over a considerable range of concentration in all three cases but the particular concentration at which they were most marked was different for the three bands. Measurements were made of the position of these minima with a comparator and the cadmium spark spectrum as a standard, and the minima were found to coincide with the centres of the absorption bands observed with the most dilute solutions both visually and in the spectrograms.

The ultraviolet absorption bands at 354 and 353 reported by Forsling [12] and the one at 346 given by Exner [13] correspond to strong absorption bands of neodymium. As we could not find the least trace of these bands in the spectrograms, it seems probable that they were produced by neodymium impurity.

The Preparation of Praseodymium Chloride.

Each fraction of double nitrate investigated was converted to chloride as follows: The salt was dissolved, and the solution, after dilution to about two liters, was filtered. A considerable quantity of nitric acid was added, the solution was heated to boiling and praseodymium oxalate was precipitated by an excess of oxalic acid. After the precipitate had been thoroughly washed by decantation, it was collected upon a disk of filter paper in a large porcelain Gooch crucible, and dried in an electric oven at 105°. In order to change the oxalate to oxide it was heated to dull redness in a platinum boat in an electrically heated porcelain muffle. Care was taken that the temperature should not be high enough to vaporize platinum from the boat into the oxide.[14] The black oxide was next dissolved in a quartz dish in a large excess of nitric acid which had been distilled through a quartz condenser, and the oxalate was reprecipitated from dilute solution by adding a solution of twice recrystallized ammonium oxalate. After thorough washing, the oxalate was collected, dried and ignited as before. The chloride was now prepared by dissolving the oxide in a quartz dish in hydrochloric acid which had been distilled through quartz. Free chlorine was expelled by heating on an electric stove,

12 Kayser, loc. cit.
13 Ibid.
14 See Baxter and Chapin, Jour. Amer. Chem. Soc., **33**, 16 (1911).

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and the salt was crystallized three or four times from concentrated solution by "salting out" at 0° with hydrochloric acid gas made by boiling the fuming solution and conducting the gas to the solution through a quartz tube. The crystals were each time centrifugally drained and rinsed in platinum Gooch crucibles.[15] The product was preserved in quartz in a vacuum desiccator over fused potassium hydroxide.

The Purification of Silver and Reagents.

The greater part of the silver used in this work was prepared by Mr. W. H. Whitcomb for an investigation upon the atomic weight of neodymium, which will be published shortly. No innovations were made in the processes of purification which have been frequently described in papers from the Harvard Laboratory. These processes were in brief as follows: Crude silver was dissolved in nitric acid, and the chloride precipitated with a large excess of hydrochloric acid. The precipitate, after being washed, was dissolved in ammonia and reprecipitated with nitric acid. Then the silver chloride was reduced with sodium hydroxide and sugar, and the metal was fused on charcoal before a blast lamp. The metallic buttons were cleansed by scouring and etching, dissolved in distilled nitric acid, and the metal reprecipitated with ammonium formate made from distilled ammonia and formic acid. After thorough washing the product was again fused on the purest lime before a blast lamp. Electrolytic deposition, with silver nitrate as the electrolyte and with a dissolving anode of the pure silver buttons, followed and the electrolytic crystals were fused in a current of electrolytic hydrogen on a pure lime boat. Adhering lime was removed by etching with nitric acid, and the buttons were washed with water and ammonia, dried, and heated to about 500° in a vacuum. The silver was preserved over potassium hydroxide in a desiccator. In some of the later analyses the silver used had been purified exactly as described above by Mr. F. L. Grover for work upon the atomic weight of lead, or by Dr. H. C. Chapin for the investigation upon neodymium.

In Analyses 1 and 2 silver nitrate was employed which had been freed from chloride by repeated crystallization. This material was

15 Baxter, Jour. Amer. Chem. Soc., **30**, 286 (1908).

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prepared by Dr. Grinnell Jones for work on the atomic weight of phosphorus.[16]

In the preparation of reagents the precautions usual in exact work were taken. The ordinary distilled water of the laboratory was twice redistilled, once from alkaline permanganate and once alone, through block-tin condensers. Hydrochloric and nitric acids were distilled through quartz condensers, in the case of the hydrochloric acid the first and last running being rejected, in the case of the nitric acid two distillations being carried out, the first third being rejected in each distillation. Nitric acid distilled in this way does not contain more than the merest trace of chlorine, if the original acid is nearly free from the latter element.

Quartz or platinum utensils were employed wherever glass would have introduced objectionable impurities, and electrical heaters were used whenever the products of combustion of illuminating gas were to be avoided. In the crystallization of solids centrifugal drainage was always used to assist in the mechanical removal of mother liquor from crystals, except in the fractional crystallization of the praseodymium material where it would have been of little assistance.

The Drying of Praseodymium Chloride.

The drying of the chloride for analysis was effected according to the recommendations of Matignon [17] and in very much the same way that neodymium chloride was dried by Baxter and Chapin, except that while the neodymium salt was not fused, and hence retained a trace of water, which was subsequently determined, the praseodymium chloride was rendered anhydrous by fusion. Bearing in mind the earlier experience with neodymium chloride, that the dehydration of the salt must be made as complete as possible before the actual fusion occurs, in order to prevent the formation of basic salt, the salt was caused to lose its crystal water by a process of efflorescence in a current of dry nitrogen and hydrochloric acid gases at gradually increasing temperatures. Richards [18] has pointed out that a hydrated salt may be freed from moisture much more effectively in this way than when melting is allowed to take place. We found the transition tem-

16 Proc. Amer. Acad., **45**, 137 (1909); Jour. Amer. Chem. Soc., **31**, 298.
17 Compt. rend., **134**, 427 (1902).
18 Zeit. physik. Chem., **46**, 194 (1903).

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perature of the heptahydrate [19] to be 111° but when the salt is heated in a current of hydrochloric acid gas, the melting point is somewhat lowered. Therefore until a very considerable proportion of the water had been expelled, the temperature was kept below 100°. The temperature was then raised to about 165° where the last molecule of crystal water begins to evaporate, according to Matignon,[20] and when the salt was essentially anhydrous it was gradually heated to about 350°. During the latter part of the heating only hydrochloric acid was passed through the tube. The aluminum block oven [21] which had been used for heating the salt up to this point was now replaced by a sleeve which could be heated electrically and the salt was first heated to dull redness for a few minutes and then quickly to its fusing point, which Matignon [22] states to be 818°.

The platinum boat with the salt was placed in a quartz tube which formed part of the "bottling apparatus" [23] containing the weighing bottle with its stopper in which the boat originally had been weighed, and the bottling apparatus was connected with systems for the production of pure dry hydrochloric acid, nitrogen, and air. The hydrochloric acid gas was generated by adding C. P. concentrated sulphuric acid to C. P. fuming hydrochloric acid, and the gas was dried by passing through five towers filled with beads moistened with concentrated sulphuric acid which had previously been heated nearly to its boiling point. Nitrogen was prepared by Wanklyn's method of saturating air with ammonia and passing the mixture over hot copper gauze. The excess of ammonia was removed by dilute sulphuric acid, and the nitrogen was further purified and dried in towers containing glass beads moistened with silver nitrate solution, fused potassium hydroxide, concentrated sulphuric acid, and phosphorus pentoxide. Nitrogen made in this way invariably contains a small proportion of hydrogen,[24] owing to catalytic decomposition of the excess of ammonia in the copper tube, but this gas would do no harm in the present instance. Air was purified and dried by reagents similar to those used for purifying the nitrogen. The apparatus was constructed wholly of glass with either ground or fused joints, except at the beginning of the air and nitrogen apparatus. By means of stop-cocks any one of the

19 Matignon found 105°. Loc. cit.
20 Loc. cit.
21 Proc. Amer. Acad., **44**, 184 (1909); Jour. Amer. Chem. Soc., **31**, 206.
22 Compt. rend., **140**, 1340 (1905).
23 Richards and Parker, Proc. Amer. Acad., **32**, 59 (1896).
24 Dr. R. C. Wells first pointed out this fact.

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three gases could be delivered to the bottling apparatus at will. This apparatus is identical with that used by Baxter and Hartmann [25] for the drying of cadmium chloride, and by Baxter and Grover [26] for the drying of lead chloride. After the fusion of the neodymium chloride, the hydrochloric acid was displaced by nitrogen and the nitrogen in turn by air. Then the boat and contents were transferred to the weighing bottle, which was allowed to stand in a desiccator near the balance for some hours before being weighed.

A solution of praseodymium chloride which has been fused in this way, sometimes contains a small amount of insoluble glistening particles, practically invisible unless examined in a strong light in a vessel whose curvature magnifies their apparent size. A similar difficulty was met by Baxter and Chapin in the preparation of pure anhydrous neodymium chloride. Under unfavorable conditions, which will be described shortly, the amount of this insoluble material may be considerably augmented, but when prepared under the most favorable conditions the salt dissolves rapidly without leaving even a trace of insoluble matter. Even when a small amount of insoluble material is formed, a clear solution is usually obtained after a day or two. The conditions under which the extent of the difficulty is too small to have an appreciable effect upon the results, have been found to be very careful preliminary dehydration of the salt, and a very short period of fusion. These conditions were maintained in the preparation of all of the specimens of material which were analyzed, but only in Analyses 5, 6, 9, 10, 16, 24, 25, 30, 31, 38 and 39 was the solution of the salt absolutely clear at the start.

Many experiments were carried out, however, first, to find out the nature of the insoluble matter, and second, to discover the extent of the difficulty. At the outset there seemed to be three possibilities as to the nature of the substance. The most probable one was that the substance is a basic chloride, but it was possible that it might be either an insoluble allotropic form of the trichloride or a lower chloride, i. e. praseodymium dichloride. The fact that prolonged fusion of the salt invariably resulted in the formation of relatively large amounts of insoluble material seemed to point toward one of the last two explanations, although there seems to be no certain evidence of the existence of such compounds in the case either of praseodymium or of other rare earths.

25 Jour. Amer. Chem. Soc., **37**, 113 (1915).
26 Investigation not yet published.

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In order to find out the composition of the insoluble substance, amounts large enough to be analyzed were prepared by dehydrating the crystals with the usual care, and then fusing the salt for periods from one-half hour to one hour in a current of dry hydrochloric acid gas. The product was then treated with water, and as soon as the soluble portion of the salt had dissolved, the insoluble residue was collected upon a small weighed platinum-sponge Gooch crucible, dried and weighed. The residue, which was distinctly crystalline, did not change in appearance during the drying, so that it is improbable that its composition was appreciably altered during this treatment. The praseodymium content of the residue was then determined by dissolving the residue from the crucible in dilute hydrochloric acid, precipitating the base with ammonium oxalate, collecting the precipitate upon a filter, and igniting in a weighed platinum boat. The weight of the black oxide Pr_4O_7 was checked by igniting the residue in a stream of hydrogen and reweighing. The weights of oxide obtained in two experiments correspond very closely to the weight which should be obtained, assuming the residue to be praseodymium oxychloride, PrOCl. About 6 grams of anhydrous salt were used in each experiment.

	I	II
Period of Fusion	35 min.	60 min.
Wt. of Insoluble Residue	0.2002 gm.	0.0131 gm.
Wt. of Pr_4O_7 Observed	0.1760 "	0.0117 "
Wt. of Pr_4O_7 Calculated from PrOCl	0.1757 "	0.0115 "
Wt. of Pr_4O_7 Calculated from $PrCl_3$	0.1366 "	0.0090 "
Wt. of Pr_4O_7 Calculated from $PrCl_2$	0.1597 "	0.0104 "
Wt. of Pr_2O_3 Observed	0.1700 "	0.0114 "
Wt. of Pr_2O_3 Calculated from PrOCl	0.1715 "	0.0112 "

The composition of the residue was further checked by determining the chlorine content. Residues were collected and weighed as above, and then after solution in dilute nitric acid, the chlorine was precipitated as silver chloride, collected and weighed. During the solution of the residue the crucible and contents were placed in a closed tube through which a current of air was passed into a solution of ammonia, so that in case chlorine was liberated during the solution, it would be

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caught in the ammonia. The results of the chlorine determinations check closely those of the praseodymium determinations.

	I	II
Period of Fusion	60 min.	60 min.
Wt. of Insoluble Residue	0.0146 gm.	0.0111 gm.
Wt. of AgCl Observed	0.0110 "	0.0079 "
Wt. of AgCl Calculated from PrOCl	0.0109 "	0.0083 "
Wt. of AgCl Calculated from $PrCl_3$	0.0252 "	0.0193 "
Wt. of AgCl Calculated from $PrCl_2$	0.0198 "	0.0150 "

Since it is obvious that, if the oxychloride is formed from the trichloride, a loss in weight must occur, further experiments were carried out to discover whether during the formation of the larger amounts of insoluble residue, when the fusion is prolonged, the loss in weight of the salt corresponds to the weight of residue produced. In these experiments the hydrated salt was first very carefully dried, then quickly fused and weighed. During this treatment little if any insoluble material is formed. Then the boat with the salt was returned to the quartz tube, and after the apparatus had been thoroughly swept out with dry hydrochloric acid, the salt was brought to the fusing temperature, which was maintained for one hour in every case. After the salt had been reweighed, the insoluble residue was determined as previously described and a few tenths of a milligram of salt which sublimed from the boat to the quartz tube were dissolved in water, the solution was evaporated and the residue was heated and weighed. The weight of sublimed material was of course added to the weight of the fused salt before determining the loss in weight during fusion.

	I	II
Period of Fusion	60 min.	60 min.
Loss on Fusion	0.0063 gm.	0.0034 gm.
Wt. of Insoluble Residue Observed	0.0234 "	0.0111 "
Wt. of Insoluble Residue Calculated as PrOCl	0.0220 "	0.0119 "
Wt. of Insoluble Residue Calculated as $PrCl_2$	0.0376 "	0.0203 "

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These experiments agree very satisfactorily with those in which the praseodymium and chlorine were actually determined, in indicating beyond question that the insoluble residue is the oxychloride.

The surprising feature of these results is that the amount of insoluble matter increases with the period of fusion instead of decreasing or even remaining constant. The obvious conclusion is that some source of oxygen exists in the fusion atmosphere. To be sure, in the first of the experiments for the determination of the insoluble residue a tiny hole was discovered in one of the fused joints of the hydrochloric acid apparatus, and in this experiment the weight of insoluble residue was found to be about ten times as large as in the subsequent experiments. Fortunately the defective joint was a comparatively new one, and could have affected only four of the fusions of the salt for analyses. The Analyses involved are Nos. 12, 13, 14, 15, 34, 35, 36 and 37. The hole was excessively small, however, and it is not at all certain that any real difficulty was produced. It seemed possible that the hydrochloric acid gas might contain air, originating in the reagents used for generating the gas, or possibly from incomplete sweeping out of the purifying train. By passing the hydrochloric acid gas into water under an inverted tube after the apparatus had been thoroughly swept out, we found that it actually did contain a trace of air. We tried in some experiments to avoid the first difficulty by passing the hydrochloric acid gas through a hydrochloric acid solution of cuprous chloride before it entered the drying towers, and in order to avoid the second difficulty the apparatus was swept out even longer than before previous to the fusion of the salt, but neither of these remedies seemed to have any effect upon the formation of the basic salt. Another source of oxygen might be moisture. However, concentrated sulphuric acid has been found by Morley[27] to be a very effective drying agent. One liter of gas passed over concentrated sulphuric acid retains only 0.003 mg. of moisture. In order to make sure that the hydrochloric acid was really as dry as this, the gas was passed for several hours through a U- tube cooled with alcohol and solid carbon dioxide. A very small amount of white solid was condensed in the tube, presumably a hydrate of hydrochloric acid. But even on the assumption that all of the moisture is removed from the gas by the salt during the fusion, it seems impossible that the residue should have formed wholly from moisture contained by the acid gas. Still another possibility was that the quartz tube was attacked by the acid gas to yield moisture and a chloride of silicon. This point was tested by

[27] Am. J. Sci., **30**, 141 (1885).

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drying and fusing the salt in a platinum tube instead of in a quartz tube, but even under these conditions the insoluble residue was formed. We are still somewhat uncertain as to the cause of the formation of the basic salt during prolonged fusion, although incomplete preliminary drying is invariably the cause when the fusion period is short, but we are inclined to the opinion that the difficulty is due to a trace of air in the hydrochloric acid gas.

Since we were unable to avoid the formation of the insoluble material in every case, in two experiments in which the salt was dried as for analysis and then fused for a short period, the insoluble residue was collected and weighed. The results of these experiments follow. As before, about 6 grams of anhydrous salt were used in each experiment.

Period of Fusion	0.5 min.	4.0 min.
Wt. of Insoluble Residue	0.0001 gm.	0.0010 gm.

In order to discover whether the basic salt dissolves to an appreciable extent, in experiments with three separate portions of material which had been collected, dried and weighed upon a platinum-sponge crucible, about a liter of water was allowed to flow slowly through the crucible during the course of an hour. The crucible and contents were then dried and reweighed. The losses in weight per liter of water were 0.9, 0.5 and 0.3 mg. While these figures are undoubtedly somewhat less than the real solubility in water, they are probably far greater than the solubility in dilute praseodymium chloride solution. This explains the extreme slowness with which a mere trace of the basic salt dissolves in the solution of the neutral salt and supports the conclusion that the quantity of basic salt present in the material fused for a short time is very small.

In preparing the salt for the actual analyses the period of fusion was less than one minute, so that the weight of residue certainly must have been considerably less than 1 mg. But since even as large a proportion as 1 mg. of oxychloride in 5 gm. of salt, if it dissolves eventually, would raise the apparent atomic weight of praseodymium by only 0.028, we feel that little danger is introduced by not attempting to apply any correction for the residue.

In a research upon the atomic weight of neodymium carried out immediately at the conclusion of this research, similar evidence was obtained as to the nature of the insoluble neodymium compound which forms under essentially the same conditions.

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The Method of Analysis.

After the salt had been fused and weighed, the boat was transferred to a glass-stoppered Erlenmeyer flask and treated with about 500 cc. of pure water. In eleven of the analyses the salt dissolved immediately leaving an absolutely clear solution. In the remainder of the analyses, after the bulk of the salt was in solution, by close inspection a small quantity of the insoluble basic chloride could be seen. On allowing the flask to stand for a considerable period, 24 to 48 hours in general, this basic salt disappeared, and in a few instances gentle heating was used to accelerate its solution. When the solution was clear, it was transferred quantitatively to the 3 or 4 liter glass-stoppered Erlenmeyer precipitating flask. The rinsings of the weighing bottle were added and the solution diluted to a volume of from 1200 to 1500 cc. In the mean time a quantity of pure silver as nearly as possible equivalent to the praseodymium chloride was dissolved in nitric acid, in a dissolving flask provided with a column of bulbs to prevent loss by spattering while the metal was dissolving. After diluting the solution of silver nitrate, it was heated to eliminate nitrous acid, and then further diluted to about the same volume as the praseodymium chloride solution. The silver nitrate solution was poured into the chloride solution, both being cold, in small portions with continual gentle agitation. When the silver nitrate had been completely transferred to the precipitating flask, the flask was stoppered and gently shaken to insure thorough mixing, and allowed to stand for several days with occasional shaking. Then it was cooled with ice-water in order to lower the solubility of the silver chloride, and after standing for a day in the ice-bath, portions of the mother liquor were tested for excess of silver or chloride by adding to separate portions equal amounts of 0.01 normal chloride and silver solutions and comparing the opalescences in a nephelometer. If a deficiency of either chloride or silver was found, this deficiency was made up by adding 0.01 normal silver or chloride solution. The amount of either added, expressed in the silver equivalent, is given in the tables of results under the heading "Silver added or subtracted." When equilibrium had been reached, 0.05 gm. of dissolved silver nitrate for each liter of supernatant liquid was added to render the silver chloride less soluble. After the solution had been allowed to stand for at least a day, usually much longer, at room temperature, the precipitate was washed many times by decantation with a solution containing 0.05 gm. of silver nitrate per liter, and several times with ice-cold water, and transferred with cold water to a large weighed platinum-sponge Gooch crucible.

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The crucible with its contents was dried for at least 12 hours in an electric air-bath at 160°, cooled in a desiccator and weighed. The crucible had originally been dried in exactly the same way. In order to find out how much moisture had been retained by the dried silver chloride, the greater part of the salt was transferred to a porcelain crucible which was weighed. Then the crucible was heated to the fusing point of the silver chloride and reweighed. The loss in weight is assumed to represent residual moisture. On an average 0.004 percent of moisture was found, a proportion which is in accord with earlier experience in the Harvard laboratories.

The solubility of silver chloride in the filtrate and silver nitrate wash waters, which both contained 0.05 gm. of silver nitrate per liter, was computed from the solubility products as found by Kohlrausch [28] at 20° and 25°, 1×10^{-10} and 1.7×10^{-10} respectively. At 20° the solubility in 0.0003 normal silver nitrate solution is 0.05 mg., at 25° 0.08 mg. per liter. The former correction was used during the colder months when the laboratory was maintained at about 20°, the latter correction in four analyses which were completed in summer. The total correction in most cases fell between 0.2 and 0.3 mg. The silver chloride dissolved in the aqueous rinsings as well as that obtained by rinsing the precipitating flask with ammonia was determined by comparison with standard solutions of chloride in a nephelometer, the usual precautions being taken to secure uniformity of precipitation. As in earlier researches it was found desirable to dissolve in ammonia the cloud of silver chloride in both the standard solution and that being analyzed, and then to reprecipitate with nitric acid.

Corrections were of course applied for any chloride introduced in order to compensate for excess of silver, and also for the standard chloride solution added to the portions which were tested in the comparison, for these portions were always returned to the precipitating flask. The latter quantity amounts to 1.328 mg. for each test which was made in the comparison. Because of the comparatively large size of this correction, the standard silver and chloride solutions were made up, preserved, and measured with great care.

Weighings were carried out on a No. 10 Troemner balance, sensitive at least to 0.02 mg. In order to avoid difficulties from changes in atmospheric humidity and density, the receptacles containing the salts were always weighed by substitution for counterpoises as nearly as possible like the objects both in material, volume, and external surface. The balance case was provided with a small amount of radio-active

[28] Zeit. physik. Chem., **64**, 167 (1908).

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material to dissipate electrical charges produced during the transference of the objects to the balance pans. The weights were standardized to hundredths of a milligram by the method described by Richards.[29] Vacuum corrections were applied as follows:

	Specific Gravity	Vacuum Correction Per Gram
Weights	8.3	
$PrCl_3$	4.020[30]	+0.000154
AgCl	5.56	+0.000071
Ag	10.49	−0.000031

Results and Discussion.

In order to show that no considerable error occurred owing either to occlusion by silver chloride or from loss of silver chloride in solution, the ratio between the silver used and the silver chloride obtained has been calculated in the eighteen pairs of analyses for which the data are available.

Analyses	Ag:AgCl
3 and 21	0.752573
4 and 23	0.752711
5 and 24	0.752628
6 and 25	0.752597
7 and 26	0.752723
8 and 28	0.752660
9 and 30	0.752644
10 and 31	0.752671
11 and 33	0.752735
12 and 34	0.752784
13 and 35	0.752716
14 and 36	0.752610
15 and 37	0.752587
16 and 38	0.752647
17 and 40	0.752664
18 and 41	0.752637
19 and 42	0.752685
20 and 43	0.752630
Average	0.752661

29 Jour. Amer. Chem. Soc., **22**, 144 (1900).

30 Determined at 25° by Mr. C. F. Hawkins, not yet published. Matignon found 4.017 at 18°. Compt. rend., **140**, 1340 (1905).

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The Atomic Weight of Praseodymium.

Series I $PrCl_3$:3AgCl

Ag = 107.880 Cl = 35.457

Number of analysis	Fraction	Weight of $PrCl_3$ in vacuum	Weight of AgCl in vacuum	Loss on fusion	Dissolved AgCl	Corrected Weight of AgCl in vacuum	Ratio $PrCl_3$: 3AgCl	Atomic Weight of Praseodymium
		gm.	gm.	gm.	gm.	gm.		
1	3474	4.12848	7.17753	0.00004	0.00166	7.17915	0.575065	140.913
2	3474	6.91605	12.02366	0.00016	0.00183	12.02533	0.575123	140.939
3	3474	7.66554	13.32562	0.00023	0.00392	13.32931	0.575089	140.923
4	3474	5.01155	8.71232	0.00026	0.00173	8.71379	0.575128	140.941
5	4383	6.04235	10.50645	0.00027	0.00126	10.50744	0.575054	140.909
6	4383	6.14745	10.68933	0.00029	0.00146	10.69050	0.575039	140.902
7	4381	6.32550	10.99679	0.00030	0.00206	10.99855	0.575121	140.937
8	4381	4.59463	7.98763	0.00022	0.00199	7.98940	0.575090	140.925
9	4379	5.96661	10.37370	0.00040	0.00224	10.37554	0.575065	140.913
10	4379	6.87536	11.95356	0.00030	0.00182	11.95508	0.575100	140.928
11	4377	4.64585	8.07649	0.00052	0.00161	8.07758	0.575154	140.951
12	4374	6.85492	11.91575	0.00000	0.00199	11.91774	0.575186	140.965
13	4374	8.01711	13.93737	0.00008	0.00224	13.93953	0.575135	140.943
14	4374	6.17045	10.72789	0.00005	0.00049	10.72833	0.575155	140.952
15	4374	6.90040	11.99805	0.00022	0.00163	11.99946	0.575059	140.911
16	4374	7.08498	12.31948	0.00043	0.00114	12.32019	0.575071	140.916
17	4371	6.20845	10.79439	0.00003	0.00116	10.79552	0.575095	140.926
18	4371	7.07590	12.30310	0.00025	0.00116	12.30401	0.575089	140.924
19	4368	5.77646	10.04305	0.00021	0.00119	10.04403	0.575113	140.935
20	4368	5.72002	9.94547	0.00021	0.00125	9.94651	0.575078	140.919
						Average	0.575108	140.932

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The Atomic Weight of Praseodymium.

Series II $PrCl_3$:3Ag

Ag = 107.880 Cl = 35.457

Number of analysis	Fraction	Weight of $PrCl_3$ in vacuum	Weight of Ag in vacuum	Weight of Ag added	Corrected Weight of Ag in vacuum	Ratio $PrCl_3$: 3Ag	Atomic Weight of Praseodymium
		gm.	gm.	gm.	gm.		
21	3474	7.66554	10.02872	0.00257	10.03129	0.764163	140.943
22	3474	8.78959	11.49955	0.00356	11.50311	0.764106	140.924
23	3474	5.01155	6.55888	0.00010	6.55898	0.764075	140.914
24	4383	6.04235	7.90826	−0.00006	7.90820	0.764061	140.910
25	4383	6.14745	8.04493	0.00070	8.04563	0.764073	140.914
26	4381	6.32550	8.27440	0.00446	8.27886	0.764054	140.908
27	4381	5.12982	6.71246	0.00113	6.71359	0.764095	140.920
28	4381	4.59463	6.01206	0.00125	6.01331	0.764077	140.915
29	4381	4.77556	6.24933	0.00061	6.24994	0.764097	140.921
30	4379	5.96661	7.80878	0.00030	7.80908	0.764060	140.910
31	4379	6.87536	8.99794	0.00030	8.99824	0.764079	140.915
32	4377	5.73602	7.50710	−0.00003	7.50707	0.764082	140.917
33	4377	4.64585	6.08004	0.00025	6.08029	0.764084	140.917
34	4374	6.85492	8.97128	0.00020	8.97148	0.764079	140.916
35	4374	8.01711	10.49205	0.00045	10.49250	0.764080	140.916
36	4374	6.17045	8.07517	−0.00092	8.07425	0.764214	140.959
37	4374	6.90040	9.03078	−0.00014	9.03064	0.764110	140.926
38	4374	7.08498	9.27297	−0.00023	9.27274	0.764065	140.911
39	4374	5.81310	7.60811	0.00000	7.60811	0.764066	140.911
40	4371	6.20845	8.12468	0.00073	8.12541	0.764079	140.915
41	4371	7.07590	9.26026	0.00019	9.26045	0.764099	140.922
42	4368	5.77646	7.56013	−0.00013	7.56000	0.764082	140.916
43	4368	5.72002	7.48696	−0.00100	7.48596	0.764100	140.922
					Average	0.764090	140.919

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The average ratio of silver to silver chloride is within less than four thousandths of a percent of that obtained by Richards and Wells,[31] 0.752634:1.000000. Part of the discrepancy is due to the fact that in a few analyses, notably Nos. 12, 13 and 14, the platinum sponge crucible allowed traces of solid silver chloride to pass through. The difficulty was not detected in time to determine the extent of the error, but in one of these analyses silver chloride was found to have settled out of the mother liquor on long standing. This is undoubtedly the reason why the atomic weight of praseodymium determined in these analyses is slightly higher than in most other experiments. Needless to say, the crucible was repaired and in Analyses 15 and 16, made subsequently with the same fraction of material as Analyses 12, 13 and 14, the results are in accord with the general mean of all the determinations. These defective analyses as well as a very few other discrepant determinations, have not been omitted, in computing averages, since, because of the large number of experiments, their influence is not large.

RESULTS AVERAGED BY FRACTIONS.

Fraction	$PrCl_3$: 3Ag	$PrCl_3$: 3AgCl	Average of Individual Determinations	Average Corrected for Cerium Content
3474	140.927	140.929	140.928	140.928
4383	140.912	140.906	140.909	140.909
4381	140.916	140.931	140.921	140.921
4379	140.913	140.921	140.917	140.917
4377	140.917	140.951	140.928	140.928
4374	140.923	140.937	140.930	140.930
4371	140.919	140.925	140.922	140.923
4368	140.919	140.927	140.923	140.925
			Average .	140.923
	Average of all Individual Determinations			140.924

When the results are averaged by fractions, further evidence is

31 Pub. Car. Inst., No. 28 (1905); Jour. Amer. Chem. Soc., 28, 456.

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added to that obtained by the spectroscope, that all the fractions analyzed were of a high degree of purity.

Fraction 4383, which contained all the neodymium which may have accumulated in forty-one series of crystallizations, gives a slightly lower instead of higher figure than the average, while fraction 4368 which contained more cerium than any other fraction gives a result identical with the average. This close similarity is very striking when one considers that a range of sixteen fractions is included in those examined. Fraction 3474, selected at an earlier stage of the fractionation as being very pure, also shows no sign of being different from the other fractions.

The foregoing results leave little opportunity for choice. There seems to be no reason for preferring the value obtained with any one or more fractions to those resulting with the others. Possibly the average of Analyses 5, 6, 9, 10, 16, 24, 25, 30, 31, 38 and 39, 140.913, is more reliable than the average of all, for in these analyses the praseodymium chloride dissolved rapidly and completely at the start, and was probably entirely free from basic salt, but we believe that the final corrected average of all the experiments, 140.92, represents fairly the best material we succeeded in preparing. This result is not far from the value obtained by Brauner in both his researches, but is over three tenths of a unit higher than the choice of the International Committee Upon Atomic Weights.

We are indebted particularly to the Carnegie Institution of Washington for pecuniary assistance in carrying out this investigation, and also to the Cyrus M. Warren Fund for Research in Harvard University for indispensable platinum vessels, as well as to Dr. H. S. Miner of the Welsbach Light Company for the praseodymium material.

CAMBRIDGE, MASS.

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GEOMETRY WHOSE ELEMENT OF ARC IS A LINEAR DIFFERENTIAL FORM, WITH APPLICATION TO THE STUDY OF MINIMUM DEVELOPABLES.

By C. L. E. Moore.

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GEOMETRY WHOSE ELEMENT OF ARC IS A LINEAR DIFFERENTIAL FORM, WITH APPLICATION TO THE STUDY OF MINIMUM DEVELOPABLES.

By C. L. E. Moore.

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We are accustomed to think of the element of arc as being defined by the square root of a quadratic differential form. This defines length as a pure number which is always taken as positive. Under this definition the distance between two points cannot have a sign attached unless it is fixed arbitrarily for each direction of the surface. There are however two well known classes of surfaces for which the element of arc is a linear differential form, viz., minimum planes and minimum developables. For the minimum plane the element of arc is an exact differential, while for the minimum developable the element of arc is not an exact differential. These are the only two possibilities for a linear differential. By a proper choice of variable one exact (inexact) differential form can be transformed into any other exact (inexact) form but a form of one kind cannot be transformed into the other kind. The linear form of the arc length shows us first of all that on both these surfaces distance is a directed quantity since the sign depends on the direction of integration. On a minimum plane the length of any closed curve is zero while on the minimum developable this is not necessarily true. These surfaces then become all the more interesting because they differ in such a marked respect from ordinary surfaces.

The element of arc being linear, it is not possible on either of these surfaces to define geodesics as the curves which minimize the integral,

$$\int (X dx + Y dy),$$

for such an integral, by varying the path can take any value whatever. For the study of the geometry on such a surface then it is necessary to find some other means of defining what corresponds to straight lines in the plane. As for the minimum plane, straight lines are already defined by means of linear equations in the parameters ordinarily used or by the intersection of the minimum plane with an ordinary plane. However the ordinary definition of angle breaks down so

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that in the study of such a geometry angle has to be defined anew. This was done by Beck.[1] A geometry analogous to that in the minimum plane was discussed by Phillips and Moore [2] and many of the properties of the triangle ecc. derived analytically by Beck were there obtained synthetically. Such a geometry was also mentioned by Wilson and Lewis [3] and the analytic definition of angle given.

Phillips and Moore also discussed a second kind of geometry in which the element of arc is not an exact differential. In this paper I shall discuss these two geometries analytically and apply the results to the study of the geometry on the minimum plane and the minimum developable.

In these two geometries, in order to define distance, and angle, a fundamental line f and a fundamental point F were assumed. In the first kind the line passed through the point and in the second kind it did not. The equality of two distances on the same line was then defined as follows, let A, B, C, D, be four points on a line cutting f in P, then AB = CD if A, D and B, C have the same harmonic point with respect to P. The same definition for equality holds for two segments on parallel lines (lines intersecting on f). For any four points on a line,

$$AB = \lambda CD,$$

where

$$\lambda = (AC \mid DP) - (BC \mid DP).$$

This also gives a way of comparing distances on two parallel lines. From this definition it is easily seen that there is only one point on a line through A which is at a given distance from A. If we then assume that the locus of points equidistant from A is an analytic curve it is easily shown that it is a straight line. Knowing this locus we can now compare distances on any two lines for on any line through A we can mark off a distance AM which has a given ratio to a given distance. As soon then as we have assumed a unit of length we can measure all distances. Distance being defined in terms of double ratios the discussion which follows can be considered as a problem in pure projective geometry. It is to be noted that the above does not give any way of comparing distances on lines through F or on f. The same

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1 Zur Geometrie in der Minimalebene. Archiv der Mathematik und Physik, **20**, (1913).

2 An algebra of plane projective geometry. These Proceedings, **47** (1912). In what follows I shall refer to this paper as A. P. G.

3 The space-time manifold of relativity. The non-euclidean geometry of mechanics and electrodynamics. These Proceedings, **48** (1913).

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thing, however, happens in euclidean geometry as there is no way of comparing intervals on minimum lines or on the line at infinity.

Case I.

The fundamental facts of the first kind of geometry are:

Distance is a directed quantity, that is when a positive direction is fixed on one line of the plane it is fixed for each line of the plane.

The locus of points equidistant from a fixed point A is a line passing through F.

The distance between two points A and B is the same as the distance between any point on AF and any point on BF.

The sum of the sides of a triangle is zero.

The distance from a point, not on f, to a point (not F) on f is infinite. The distance from a point not on F is indeterminate. The distance from a point A to any point on AF is zero.

Angle is a directed quantity, that is when a positive rotation is fixed about one point of the plane it is fixed about each point of the plane.

Lines making a fixed angle with a given line a pass through the same point on f.

The angle between two lines a and b is the same as the angle between any line passing through (a, f) and any line passing through (b, f).

The sum of the angles of a triangle is zero.

The angle between a line, not through F, and a line (not f) through F is infinite. The angle between f and a line through F is indeterminate. The angle between two lines intersecting on f is zero.

If a, b, c, denote the sides of a triangle and A, B, C, the angles opposite, the relations connecting them were found to be,

$$\mathrm{A} + \mathrm{B} + \mathrm{C} = 0, \qquad a + b + c = 0,$$

$$\frac{a}{\mathrm{A}} = \frac{b}{\mathrm{B}} = \frac{c}{\mathrm{C}}.$$

Defining the line area of a triangle to be such a scalar function of the sides taken in a definite order around the triangle, that triangles having the same vertex and bases on the same line shall have areas proportional to their bases, the following formula for area was derived,

$$\text{Line area} = ab\mathrm{C} = ac\mathrm{B} = bc\mathrm{A}.$$

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A dual definition for angle area led to,

$$\text{Angle area} = \text{AB}c = \text{AC}b = \text{BC}a.$$

Coördinate systems.

The fact that the locus of points equidistant from a given point is a line suggests that one convenient coördinate system would be a point 0 and a line l passing through it. The coördinates of a point A are then the distance 0A and the angle which 0A makes with l. These numbers will fix a point uniquely. The distance between two points (ρ_1, θ_1) and (ρ_2, θ_2) is then,

$$d = \rho_2 - \rho_1.$$

The angle between the two lines is,

$$\theta = \theta_2 - \theta_1.$$

The element of arc is,

$$ds = d\rho.$$

This coördinate system is not suitable for a great many problems, for the coördinates of the points of the line 0F are indeterminate, each point having the coördinates $(0, \infty)$. For this system of coördinates the equation of a straight line is not linear which is another disadvantage.

A second and more convenient coördinate system is obtained by taking the three points 0 (not on f), F, P (on f), as the vertices of a triangle of reference. Number the points along 0P by beginning with 0 and laying off equal distances along 0P. Draw lines from F to these points and number them to correspond. Number the lines through 0 by beginning with 0P and constructing equal angles about 0. Number the points in which these lines cut f to correspond. Finally number the points on 0F by taking an assigned interval 0Q as unity and measuring off equal intervals by choosing Q_1, to satisfy the relation $(f\text{Q} \mid 0\text{Q}_1) = -1$, then choose Q_2 so that $(f\text{Q}_1 \mid 0\text{Q}_2) = -1$ *ecc.* If these relations are satisfied we will say that the intervals 0Q, QQ_1, Q_1Q_2, ecc. are equal. Draw lines from P to these points and number them to correspond. The coördinates of a point will then be the projection of the point on 0F and 0P from P and F. We will call the numbers along 0F, y, and those along 0P, x, then it is seen from the fundamental properties of distance that the distance between (x_1, y_1) and (x_2, y_2) is,

$$d = x_1 - x_2,$$

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that is the distance between two points is the same as the distance between the points into which they project from F.

With this system of coördinates the equation $y = kx$, represents a line through 0. For let 0Q be a line passing through 0, Figure 1,

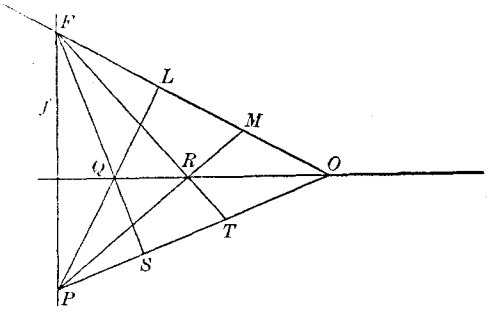

FIGURE 1.

and let (x_1, y_1) be the coördinates of R and (x, y), the coördinates of Q. Then from the properties of distance it is seen at once that, 0Q = 0S, 0R = 0T. From the definition of measure on 0F and the comparison of two distances on any line we see that,

$$0L : 0M = 0Q : 0R$$

Therefore

$$\frac{0L}{0M} = \frac{0S}{0T}, \text{ or } \frac{y}{y_1} = \frac{x}{x_1}, \text{ or } y = \frac{y_1}{x_1} x.$$

In the system of measure established on f we will take the coördinates of the point in which the line $y = kx$ cuts f as k. Then the coefficient k will represent the angle which 0Q makes with 0P.

To find the equation of a line joining two points, we will take as the defining property of the line the fact that the area of the triangle formed by any three of its points is zero. Let $Q_1(x_1, y_1)$, $Q_2(x_2, y_2)$ be two fixed points and Q (x, y) be any point on the line joining them, then

$$QQ_1Q_2 = 0QQ_1 + 0QQ_2 - 0Q_2Q_1.$$

Using the formula for area this becomes

$$\left(\frac{y}{x} - \frac{y_1}{x_1}\right) xx_1 - \left(\frac{y}{x} - \frac{y_2}{x_2}\right) - \left(\frac{y_2}{x_2} - \frac{y_1}{x_1}\right) x_1 x_2 = 0,$$

or
$$(x - x_1)(y_1 - y_2) = (y - y_1)(x_1 - x_2).$$

That is the equation of the straight line is of the first degree.

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The coördinates of a line will be the dual of the coördinates of a point and therefore will be the numbers corresponding to the points in which the line cuts f and 0P. The angle between two lines will be the difference between the numbers corresponding to the points in which it cuts f. The number corresponding to the point in which the line,

$$ux + vy = w,$$

cuts f is $-\frac{u}{v}$ Hence the angle between the lines

$$u_1x_1 + v_1y_1 = w_1,$$
$$u_2x_2 + v_2y_2 = w_2,$$

is

$$\theta = \frac{u_1v_2 - u_2v_1}{v_1v_2} \tag{2}$$

Curvature. Using the ordinary definition of curvature we can now derive the expression for the curvature, in this geometry, for a curve

$$y = f(x).$$

The tangent at the point (x_1, y_1) is

$$y - y_1 = f'(x)\,(x - x_1),$$

and the tangent at the near by point $(x_1 + dx, y_1 + dy)$ is,

$$y - y_1 - dy = f'(x_1 + dx)\,(x - x_1 - dx).$$

The angle between the tangents is then

$$f'(x_1 + dx) - f'(x_2).$$

Hence dividing this by the element of arc we have for the curvature,

$$K = \frac{f(x) + f''(x)\,dx + f'''(x)\,dx^2 + \cdots - f'(x_1)}{dx} = f''(x).$$

From this we see at once that the curves of zero curvature are straight lines. The curves of constant curvature are defined by the differential equation,

$$\frac{d^2y}{dx^2} = k$$

That is the curves of constant curvature are

$$y = \frac{k}{2}x^2 + c_1x + c_2. \tag{4}$$

This curve is then the analog of the circle in the euclidean plane. It

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is a conic tangent to the line f at the point F. In the minimum plane this is the curve which Study calls the parabolic circle.

Area. The line area of the triangle whose vertices are the points (x_1, y_1), (x_2, y_2), (x_3, y_3), is

$$\left[\frac{y_1 - y_2}{x_1 - x_2} - \frac{y_1 - y_3}{x_1 - x_3}\right](x_1 - x_2)\,(x_3 - x_1) =$$

$$(y_3x_1 - y_1x_3) + (y_1x_2 - y_2x_1) + (y_2x_3 - y_3x_2) =$$

$$\begin{vmatrix} 1 & y_1 & x_1 \\ 1 & y_2 & x_2 \\ 1 & y_3 & x_3 \end{vmatrix}$$

which agrees with the ordinary formula for area. The angle area is

$$\frac{1}{(x_1 - x_2)\,(x_2 - x_3)\,(x_3 - x_1)} \begin{vmatrix} 1 & y_1 & x_1 \\ 1 & y_2 & x_2 \\ 1 & y_3 & x_3 \end{vmatrix}$$

That is,

Line area = angle area multiplied by the product of the three sides.

The curvature of the parabolic circle circumscribing the above triangle is

$$\frac{2 \begin{vmatrix} 1 & y_1 & x_1 \\ 1 & y_2 & x_2 \\ 1 & y_3 & x_3 \end{vmatrix}}{(x_1 - x_2)\,(x_2 - x_3)\,(x_3 - x_1)} = 2 \times \text{Angle area} = 2 \times \text{line area} \div \text{abc.}$$

This agrees with the curvature of a circle in euclidean geometry except for a factor 2. This is due to the definition of area but there seems to be no need to complicate the formula just to make a closer agreement.

The condition for a point of inflection is the same here as in ordinary geometry,

$$\frac{d^2y}{dx^2} = 0.$$

Collineations of the plane. We saw from the definition of distance that the sides and angles of a triangle are in a measure independent. In fact if the vertices are moved along the lines which join them to the point F the sides are not changed in length while the angles can be made to vary from zero to infinity. We should then expect to find collineations which leave distance invariant and change angle and vice versa. If distance is to be preserved both the point F and the

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line f must be left invariant and $x_1 - x_2$ must be transformed into $x^1_1 - x^1_2$. Therefore the collineations which do this are,

$$(5) \qquad \begin{aligned} x^1 &= x + c_1, \\ y^1 &= a_2 x + b_2 y + c_2. \end{aligned}$$

These transformations, as we shall see multiply angle by a constant, therefore there is a four parameter group of transformations which leave distance invariant and multiply angle by a constant. The five parameter group of collineations,

$$(6) \qquad \begin{aligned} x^1 &= b_1 x + c_1, \\ y^1 &= a_2 x + b_2 y + c_2, \end{aligned}$$

is a magnification for both distance and angle. The ratio of magnification for distance is at once seen to be b_1. Since distance and angle are dual conceptions, there is a four parameter group of collineations which leave angle invariant and multiply distance by a constant. We could have taken the equations in angle coördinates and then the transformations leaving angle invariant would have had a form exactly similar to (5).

Similarity transformations do not exist in this plane in the same sense that they exist in the euclidean plane for if the sides of a triangle are left invariant the angles are not necessarily left invariant. If we apply the transformation (5) to the two lines,

$$\begin{aligned} u_1 x + v_1 y &= 0, \\ u_2 x + v_2 y &= 0, \end{aligned}$$

the angle between the transformed lines becomes,

$$\frac{1}{b_2}\left(\frac{u_1}{v_1} - \frac{u_2}{v_2}\right).$$

Hence by these transformations angle is multiplied by $\frac{1}{b_2}$. The collineations which preserve both distance and angle are the transformations of the three parameter group,

$$\begin{aligned} x^1 &= x + c_1, \\ y^1 &= a_2 x + y + c_2. \end{aligned}$$

If we apply the transformation (6) to the above two lines, the angle between the transformed lines becomes,

$$\frac{b_1}{b_2}\left(\frac{u_1}{v_1} - \frac{u_2}{v_2}\right).$$

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Then the subgroup of the similarity group which preserves angle are those for which $b_1 = b_2$.

The transformations which preserve distance and reverse angle are

$$x^1 = x + c_1,$$
$$y^1 = a_2 x - y + c_2.$$

Those which preserve angle and reverse distance are,

$$x^1 = -x + c_1,$$
$$y^1 = a_2 x + y + c_2.$$

Those which reverse both distance and angle are,

$$x^1 = -x + c_1,$$
$$y^1 = a_2 x - y + c_2.$$

In this geometry instead of having the ordinary similarity transformations we have two kinds, one which multiplies distance leaving angle invariant and one which multiplies angle leaving distance invariant. There is a three parameter group of motions leaving both invariant. We have also the "umlegung" which reverses distance and the dual which reverses angle.

Since the distance between two points is the same as the distance between any other two points, one on each line joining the two given points to the point F, the question arises, what are the most general analytic transformation of the plane into itself which will preserve distance. In the first place f and F must be invariant. Let the transformation be,

$$x = f(x^1, y^1),$$
$$y = g(x^1, y^1).$$

If it preserves distance,

$$ds = dx = \frac{\partial f}{\partial x^1} dx^1 + \frac{\partial f}{\partial y^1} dy^1 = dx^1.$$

Hence,

$$\frac{\partial f}{\partial y^1} = 0,$$
$$\frac{\partial f}{\partial x^1} = 1.$$

Therefore $f(x^1, y^1)$ is independent of y^1 and must have x^1 with coefficient unity, that is,

$$f = x^1 + a,$$

and $g(x^1, y^1)$ is any arbitrary function. We will take g so that in a

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given region the transformation will be (1, 1), that is in the angle between two lines through F a branch of a curve will go into a single branch with ends on the lines through F into which the boundaries of the angle transforms.

By such a transformation the curves,

$$u(x + a) + vg(x, y) = w,$$

can be transformed into straight lines. So far then as length is concerned these curves could be taken for straight lines and angle defined accordingly. The whole geometry would then be the same. In this character this geometry differs very much from ordinary geometry.

Case II.

The second kind of geometry discussed in A. P. G. had for fundamental system a line f and a point F not on it. In this geometry angle and distance are so related that any three parts of a triangle determine it. A transformation then which preserves distance must also preserve angle and there is no separation of these groups as in Case I.

The general definitions of distance are the same here as before. There is a metric example here, however, which will serve to make it more concrete. Let the line f be the line at infinity. The distance here defined then becomes equal to the area of the rectangles of which BF is one diagonal and A one vertex. The properties of distance can then be verified in this metrical case.

Distance has the following properties:

AB = AC if BC meets AF on f. That is the locus of points equidistant from a given point is a straight line. This line can be any line of the plane, however, instead of a line through F as in Case I.

The distance from a point A, not on f, to a point P of f is infinite if F does not lie on AP. If F does lie on AP the distance is indeterminate.

The distance from a point A to a point on AF, not on f, is zero.

Distance is a directed quantity, that is if a positive direction is assigned for one line of the plane a positive direction is assigned for each line of the plane. A construction was given for measuring on any line beginning at an assigned point, a distance equal to a given distance.

From these most all properties of distance can be derived.

Angle was defined as the dual of distance and with reference to the same fundamental system. Angle then has the following properties:

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$ab = ac$ (where xy denotes the angle between the lines x and y) if the line joining the point of intersection of b and c to F meets a on f, that is the envelope of lines making equal angles with a given line is a point.

The angle which a line a, not through F makes with a line through F is infinite if a does not meet it in f. If this is the case the angle is indeterminate.

Angle is a directed quantity, that is if a positive rotation is assigned around one point of the plane it is assigned around each point of the plane. A construction was given for measuring about any point, beginning at a fixed line, an angle equal to a given angle.

The relations connecting distance and angle are the expressions for the angles of a triangle in terms of the sides. These are,

$$A = \frac{a+b+c}{bc}, \quad B = \frac{a+b+c}{ac}, \quad C = \frac{a+b+c}{ab}.$$

Solved for a, b, c, these relations become,

$$a = \frac{A+B+C}{BC}, \quad b = \frac{A+B+C}{AC}, \quad c = \frac{A+B+C}{AB}.$$

From these can be derived,

$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C},$$

which corresponds to the sine law of ordinary trigonometry. It should be kept in mind that distance is a directed quantity and therefore a direction around the triangle must be assumed.

Coördinate systems.

Having these triangle relations it is easy to set up a coördinate system. A very simple one is an ordinary polar system consisting of a point 0 from which distances are measured and a line l, through it from which angles are measured. The coördinates are then the distance from 0 and the direction θ, measured from l. This system does not have the indeterminate character of polar coördinates in the euclidean plane since a point will have just one set of coördinates. It does have the disadvantage, however, of having an indeterminate character for the points of the line 0F. The coördinates of any point on this line are $(0, \infty)$, since the distance from any point to F is zero and the angle which any line makes with a line through F is infinite.

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The distance between two points (ρ_1, θ_1), (ρ_2, θ_2) is,

$$d = \rho_2 - \rho_1 + \rho_1 \rho_2 (\theta_2 - \theta_1).$$

For the angle between $0P_2$ and $0P_1$ is $\theta_2 - \theta_1$ and expressing the angle in terms of the sides of a triangle, we have,

$$\theta_2 - \theta_1 = \frac{\rho_1 + d - \rho_2}{-\rho_1\rho_2}$$

which is the relation above.

The equation of a straight line is,

$$A\rho\theta + B\rho + C = 10,$$

which is obtained by finding the locus of a point at a constant distance from a fixed point. The angle which this line makes with l is,

$$\beta = \frac{1}{A + B},$$

Hence for lines through F, $A = -C$. The element of arc is,

$$ds = d\rho + \rho^2 d\theta.$$

A second and for most purposes more convenient system has two points of reference. Algebraically this is very similar to cartesian coördinates in euclidean geometry but geometrically like bipolar coördinates. Let X and Y be the points of reference so chosen that

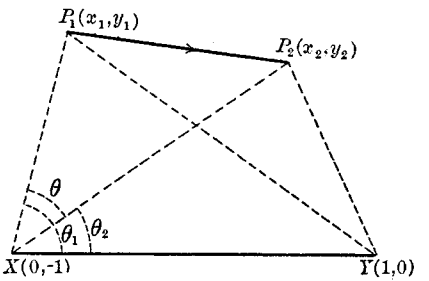

FIGURE 2.

the line XY does not pass through F. The coordinates of a point A is then the distances *from* X, Y *to* the point. The only indeterminate points for this system are those of the line f. For convenience we will choose the distance from X to Y to be unity. The coördinates of X and Y are $(0,-1)$, $(1, 0)$.

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The distance between two points (x_1, y_1), (x_2, y_2) is

$$d = x_1 y_2 - x_2 y_1.$$

For in the triangle XP_1Y, fig. 2, from the triangle relations,

$$\theta_1 = \frac{XP_1 + P_1X + YX}{XP_1 + YX} = \frac{x_1 - y_1 - 1}{-x_1}.$$

In the triangle XP_2Y,

$$\theta_2 = \frac{XP_2 + P_2X + YX}{XP_2 \cdot YX} = \frac{x_2 - y_2 - 1}{-x_2}.$$

Then,

$$\theta = \theta_1 - \theta_2 = \frac{y_1x_2 - y_2x_1 + x_2 - x_1}{x_1x_2}.$$

In the triangle XP_1P_2 we have,

$$\theta = \frac{XP_1 + P_1P_2 + P_2X}{XP_1 \cdot P_2X}$$

$$= \frac{x_1 + d - x_2}{-x_1x_2} = \frac{y_1x_2 - x_1y_2 + x_2 - x_1}{x_1x_2}$$

This solved for d gives the value above.

The equation of a straight line in this system of coördinates is linear as can easily be seen by finding the locus of points equidistant from a given point, this we saw was a straight line.

For measuring angles we shall take a third point of reference Z $(1, -1)$. We will then write the equation of the straight line in the form,

$$ux + vy = 1.$$

The equations of ZY, ZX and XY are respectively,

$$x = 1, \qquad y = -1, \qquad x - y = 1.$$

The equation of a line passing through F is

$$y = kx.$$

From the triangle PQZ, fig. 3,

$$\theta = \frac{PQ + QZ + ZP}{PQ \cdot ZP} = \frac{\frac{1 + v - u}{uv} + \frac{u - v - 1}{v} + \frac{v - u + 1}{u}}{\frac{v - u + 1}{u} \cdot \frac{v - u + 1}{uv}} = u.$$

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Similarly we find $\Psi = v$. (It is to be observed that for the angle between two lines, the one is always taken, which does not contain the point F. The other angle is the sum of two infinite angles). The angle between the lines,

$$u_1x + v_1y = 1,$$
$$u_2x + v_2y = 1,$$

is

$$\beta = u_1v_2 - u_2v_1.$$

These formulas for distance and angle show the directed character of both. For the angles of the triangle X Y Z we have,

$$X = 1, \qquad Y = 1, \qquad Z = -1,$$

corresponding to the lengths of the sides opposite,

$$ZY = 1, \qquad XZ = 1, \qquad YX = -1.$$

This triangle will do for triangle of reference for both distance and angle. The angle coördinates of a line are the angles which it makes

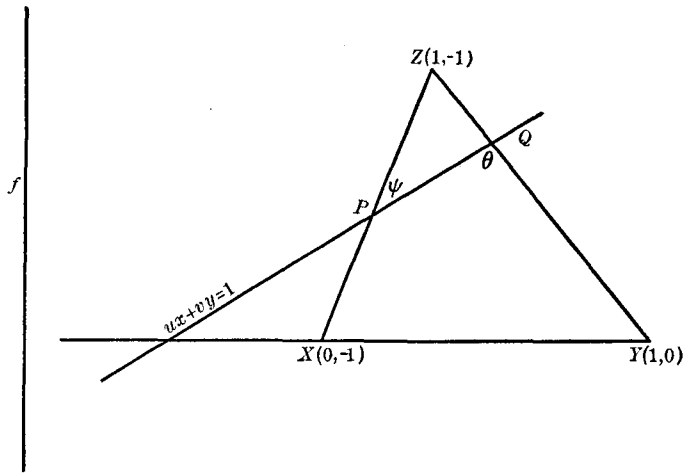

FIGURE 3.

with YZ and XZ. If the equation of a line be written in the form above it will be perfectly dual and the condition that the point (x, y) be on the line (u, v) is precisely,

$$ux + vy = 1.$$

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The element of arc in this system of coördinates is,

$$ds = xdy - ydx,$$

and likewise the angle between two nearby tangents of a curve is

$$d\sigma = udv - vdu,$$

where the curve is expressed in angle coördinates.

Using the definition for area as was used in Case I, it was shown in A. P. G. that here also we have two areas:

$$\text{Line area} = a + b + c = \text{A}bc = \text{C}ab = \text{B}ac.$$
$$\text{Angle area} = \text{A} + \text{B} + \text{C} = \text{AB}c = \text{AC}b = \text{BC}a.$$

From these formulas it is at once seen that the line area of any closed curve is equal to its length. That is for a closed curve,

$$\text{Line area} = \text{Length} = \int_c (xdy - ydx).$$

If the curve is given in angle coördinates,

$$\text{Angle area} = \text{Angle sum} = \int_c (udv - vdu).$$

If the equation of a curve is written in the form,

$$y = f(x),$$

the tangent at the point (x_1,y_1) is,

$$xdy - ydx = x_1dy - y_1dx \quad \text{or}$$
$$x\frac{dy}{ds} - y\frac{dx}{ds} = 1$$

If x and y are functions of s the tangent at a nearby point is

$$x\frac{d}{ds}y(s + ds) - y\frac{d}{ds}x(s + ds) = 1$$

or
$$x\left(\frac{dy}{ds} + \frac{d^2y}{ds^2}\cdots\right) - y\left(\frac{dx}{ds} + \frac{d^2x}{ds^2}\cdots\right) = 1.$$

The angle between these two tangents is,

$$d\sigma = \left(\frac{dy}{ds}\frac{d^2x}{ds^2} - \frac{dx}{ds}\frac{d^2y}{ds^2}\right)ds$$

The angle area then becomes,

$$\text{(7)} \qquad \text{Angle area} = \text{Angle sum} = \int\left(\frac{dy}{ds}\frac{d^2x}{ds^2} - \frac{dx}{ds}\frac{d^2y}{ds^2}\right)ds$$

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Curvature. Here as in case I we can define curvature as $\frac{d\sigma}{ds}$. We then have

$$\mathrm{K} = \frac{d\sigma}{ds} = \frac{dy}{ds}\frac{d^2x}{ds^2} - \frac{dx}{ds}\frac{d^2y}{ds^2}$$

$$= \left(\frac{dy}{ds}\right)^2 \frac{d}{ds}\left(\frac{\frac{dx}{ds}}{\frac{dy}{ds}}\right) \tag{8}$$

which is the same form as the formula for curvature in euclidean geometry. From (8) we see that curves of zero curvature are straight lines.

If x is taken as the independent variable the differential equation of the curves of constant curvature is,

$$\frac{d^2y}{dx^2} = \mathrm{K}\left(x\frac{dy}{dx} - y\right)^3,$$

the solution of which is,

$$c_1y - c_2x = \sqrt{(c_1 - \mathrm{K}x^2)}$$

or

$$(c_1y - c_2x)^2 + \mathrm{K}x^2 = c_1.$$

We will call these curves of constant curvature "pseudo circles." From the equations it is seen that these are conics with respect to which f is the polar of F. The points of a curve for which $\mathrm{K} = 0$, are points of inflection. Any collineation of the plane which leave f and F fixed have $\mathrm{K} = 0$, as an invariant.

Any conic of the form,

$$\mathrm{A}_1x^2 + 2\mathrm{A}_2xy + \mathrm{A}_3y^2 = 1,$$

is a pseudo circle whose curvature is,

$$\mathrm{K} = \frac{1}{4}(\mathrm{A}_1\mathrm{A}_3 - \mathrm{A}^2_2)$$

The line area of a triangle is the sum of the sides. As in Case I, this can be expressed as,

$$\text{Line Area} = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}.$$

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The area of any closed curve is its perimeter. The dual of this from formula (7), is

$$\text{Angle area} = \int \mathrm{K} ds.$$

For curves of constant curvature then,

$$\text{Angle Area} = \text{Line area} \times \mathrm{K}.$$

The area of a pseudo circle which does not cut f that is of the curve

$$Ax^2 + 2Bxy + Cy^2 = 1, \quad \text{where} \quad AC - B^2 > 0$$

is

$$\text{Line area} = \int (x dy - y dx) = \int x^2 d\left(\frac{y}{x}\right) = \int_{-\infty}^{\infty} \frac{d\left(\frac{y}{x}\right)}{A + 2B\left(\frac{y}{x}\right) + C\left(\frac{y}{x}\right)^2}$$

$$= \frac{2\pi,}{\sqrt{\Delta}} \qquad \text{where} \quad \Delta = AC - B^2.$$

but $\rho = \frac{1}{K} = \frac{4}{\Delta}$, hence $\rho = \frac{2}{\sqrt{\Delta}}$. The line area of the pseudo circle is $\pi\sqrt{\rho}$. If the pseudo circle cuts the line f the area is infinite.

Collineations of the plane. The general collineation of the plane, which leave F and f invariant is,

$$x^1 = ax + by,$$
$$y^1 = cx + dy.$$

If this transformation be applied to $x_1 y_2 - x_2 y_1$ we have

$$x^1_1 y^1_2 - x^1_2 y^1_1 = \mathrm{M}(x_1 y_2 - x_2 y_1), \qquad \text{where} \quad \mathrm{M} = ad - bc.$$

This is then a magnification for distance. The angle between the lines,

$$u_1 x + v_1 y = 1,$$
$$u_2 x + v_2 y = 1,$$

becomes

$$u^1_1 v^1_2 - u^1_2 v^1_1 = \frac{1}{\mathrm{M}}(u_1 v_2 - u_2 v_1).$$

The transformation then divides angle. If it is applied to a pseudo circle we have for curvature,

$$\mathrm{K}^1 = \frac{1}{\mathrm{M}^2}\mathrm{K}.$$

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If the collineation is a motion

$$ad - bc = 1.$$

These transformations are the area preserving collineations of the plane in euclidean geometry.

If

$$ad = bc = -1,$$

the transformation is an "umlegung," that is a transformation which reverses the sign of distance. For both motion and umlegung the curvature of a curve is an absolute invariant.

In this geometry rotation about a point A is equivalent to a translation along the series of lines passing through the point where the line AF meets f. If the rotation is to be about a finite point that point must remain invariant. But as f and F are left invariant no other finite point can be and hence there are no rotations except identity.

Parallel curves. Parallel curves are defined in ordinary geometry as the envelope of circles of constant radius having their centers on a fixed curve. In this geometry then a parallel to a given curve will be the envelope of the lines which are the loci of points at a constant distance from the points of the curve. The lines corresponding to the same point of the given curve all pass through the same point of f and hence are parallel. That is in a set of parallels to a curve the tangents at corresponding points are parallel. The parallels to the curve

$$f(x, y) = 0,$$

will have for equation the eliminant of

$$f(x_1, y_1) = 0, \qquad xy_1 - yx_1 = a.$$

This however is exactly the process for finding the tangential equation of the curve $f(x, y) = 0$ in euclidean geometry. Hence the parallels to a given curve have the order equal to the class of the original curve and vica versa.

The parallels to the pseudo circle is the eliminant of x_1, y_1 between,

$$Ax^2_1 + 2Bx_1 y_1 + Cy^2_1 = 1,$$

$$xy_1 - yx_1 = a,$$

which is

$$Ax^2 + 2Bxy + Cy^2 = a^2 (AC - B^2).$$

This is again a pseudo circle having the same pair of tangents from F.

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Conversely all pseudo circles having the same pair of tangents from F are parallel curves. The form of the equation shows that the parallel is the same whether a is positive or negative. Hence the lines at a distance $\pm a$ from the points of the given curve are parallel tangents to the same parallel curve.

All the lines which cut a given tangent to the curve $f(x, y) = 0$, at a constant angle will pass through the same point. The locus of this point will be the dual of the parallel to a given curve. If the curve is given in angle coördinates,

$$f(u, v) = 0$$

then the angle equation of this locus will have the same degree as the reciprocal of the given curve. The curve which corresponds to a pseudo circle is again a pseudo circle. We again have the same curve whether we take the fixed angle as $\pm \alpha$.

It was shown in A. P. G. that if the line l is the locus of points at a distance k from P then the lines through P all make with l angles equal to $\frac{1}{k}$. An easy calculation will show that the parallel to a pseudo circle corresponding to the distance k and the dual corresponding to the angle $\frac{1}{k}$ are one and the same curve. If we wish then to determine the point of contact of a given generator l of a parallel curve with its envelope all we need to do is to draw a line through the corresponding point of the original curve making an angle $-\frac{1}{k}$ with the tangent line. Where this line cuts l will be the point of contact.

Evolutes. In ordinary geometry the envelope of the normals of a curve is of considerable interest. Aside from the tangent the normal is the only unique line connected with a curve. Here however each line passing through a point of a curve is unique in the same sense, that is it makes a definite angle with the tangent line and there is no other line passing through this point making the same angle. Then connected with every curve there is an infinite number of involutes, that is, the envelope of lines making a fixed angle with the tangent lines. If the original curve is,

$$f(x, y) = 0,$$

the line making the angle k with the curve at the point (x_1, y_1) is

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$$(a - ky_1)x + (b + kx_1)y = ax_1 + by_1,$$

where

$$a = \frac{\dfrac{\partial f}{\partial x_1}}{x_1\dfrac{\partial f}{\partial x_1} + y_1\dfrac{\partial f}{\partial y_1}}, \qquad b = \frac{\dfrac{\partial f}{\partial y_1}}{x_1\dfrac{\partial f}{\partial x_1} + y_1\dfrac{\partial f}{\partial y_1}}.$$

The envelope of this line subject to the condition,

$$f(x_1, y_1) = 0,$$

will be the evolute. For the pseudo circle this is again a pseudo circle. In fact if the curve is of the form $f(x, y) = 1$, where $f(x, y)$ is a homogeneous function of x and y, the order of the evolute will be the same as the order of the original curve. The dual of this set of curves is the set traced by the points on the tangent line at a fixed distance from the point of tangency. In the case of a pseudo circle these curves are again pseudo circles.

The pedal curves of a given curve form another interesting configuration. Here for a given point there will be an infinite number of pedals depending on the size of the angle used. If we denote the given curve by C and the dual of the parallel by P and take A as the point with reference to which the pedal is taken a simple construction will show that the pedal curve is the locus of the intersections of the tangents to C with the lines drawn from the corresponding points of P through A. The point A will be a multiple point on the pedal of multiplicity equal to the class of C.

Length preserving transformations of the plane into itself. We found that there was a three papameter group of collineations which left distance invariant. It is evident that there are other transformations which leave distance invariant since the distance between two points measured along various curves may be the same as if measured along straight lines. The transformation,

$$(T) \qquad x = G(x^1, y^1), \qquad y = H(x^1, y^1),$$

will preserve length if,

$$xdy - ydx = G\left(\frac{\partial H}{\partial x^1}\,dx^1 + \frac{\partial H}{\partial y^1}\,dy^1\right) - H\left(\frac{\partial G}{\partial x^1}\,dx^1 + \frac{\partial G}{\partial y^1}\,dy^1\right)$$
$$= x^1\,dy^1 - y^1\,dx^1$$

from which we see that,

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$$(9) \qquad G\frac{\partial H}{\partial x^1}q - H\frac{\partial G}{\partial x^1} = -y^1,$$

$$G\frac{\partial H}{\partial y^1} - H\frac{\partial G}{\partial y^1} = x^1.$$

If we make the following simple transformation we can more readily draw conclusions from the above equations.

$$\frac{y}{x} = v, \quad x^2 = u, \quad \frac{y^1}{x^1} = v^1, \quad x^{1\,2} = u^1.$$

Suppose the transformation (T) then becomes,

$$u = f(u^1, v^1), \qquad v = g(u^1, v^1).$$

Then,

$$udv = f\left(\frac{\partial g}{\partial u^1}\,du^1 + \frac{\partial g}{\partial v^1}\,dv^1\right) = u^1\,dv^1,$$

and the relation (9) becomes,

$$f\frac{\partial g}{\partial u^1} = 0, \qquad f\frac{\partial g}{\partial v^1} = u^1.$$

The first equation says that g is a function of v^1 only. Since $\frac{1}{u}$, is an integrating factor and $g\,(v)$ is an arbitrary function of the solution of the differential equation we see from the second equation if written

$$f = u^1\frac{1}{\frac{\partial g}{\partial v^1}},$$

that $f(u^1, v^1)$ is the reciprocal of an integrating factor. We then have for the original transformation,

$$\frac{H}{G} = F\left(\frac{y^1}{x^1}\right),$$

and $G^2(x^1, y^1)$ is the reciprocal of an integrating factor. The transformation then is such that,

$$G = x^1 f\left(\frac{y^1}{x^1}\right),$$

$$H = g\left(\frac{y^1}{x^1}\right),$$

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where f and g are both arbitrary functions of $\frac{y^1}{x^1}$.

By this transformation a curve of the form,

$$u\,\mathrm{G} + v\,\mathrm{H} = 1,$$

will be transformed into a straight line and the theory of distance will be the same if these curves were used for straight lines. There is then an infinite group of transformations of the plane into itself which will preserve distance and consequently will preserve angle. The foregoing is then only one of an infinite number of geometries which can be built up and which are simply isomorphic with the one here discussed.

Application to Minimum Developables.

The equations of a minimum developable can be written in the form,

$$x = \mathrm{R}\left(\frac{u}{v}\right) + v^2\mathrm{R}'\left(\frac{u}{v}\right),$$

$$y = \mathrm{S}\left(\frac{u}{v}\right) + v^2\mathrm{S}'\left(\frac{u}{v}\right),$$

$$z = \mathrm{T}\left(\frac{u}{v}\right) + v^2\mathrm{T}'\left(\frac{u}{v}\right),$$

where $$\mathrm{R}'^2 + \mathrm{S}'^2 + \mathrm{T}'^2 = 0.$$

Primes denote differentiation with respect to $\frac{u}{v}$. Then

$$ds = \sqrt{\mathrm{R}''^2 + \mathrm{S}''^2 + \mathrm{T}''^2}\,(udv - vdu).$$

By a proper choice of variable the expression under the radical can be made equal to unity. In fact the change of variable is,

$$\frac{u^1}{v^1} = \int \frac{d\left(\frac{u}{v}\right)}{(\mathrm{R}''^2 + \mathrm{S}''^2 + \mathrm{T}''^2)^{\frac{1}{3}}}.$$

With this choice of variable we have,

$$ds = udv - vdu.$$

Since this form is the same for all minimum developables it follows that all such surfaces can be mapped on any one of them in such a

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way that length will be preserved and in fact this can be done in an infinite number of ways.[4] If we put,

$$u = x, \qquad v = y.$$

we will have the minimum developable mapped on the plane of A. P. G. in such a way that distance is preserved. In this depiction the generators of the developable are carried into the lines through F: the imaginary circle at infinity into the line f. The cuspidal edge will be transformed into the point F. This then differs from the development of the ordinary developable on the euclidean plane since a whole curve is transformed into a point. The transformation, however, does everywhere preserve length.

For lines analogous to geodesics on a minimum developable then we can take the lines which by this depiction go into straight lines. As we saw before this depiction can be made in an infinte number of ways and therefore on a minimum developable there are an infinite number of simply isomorphic geometries. The curves which we shall take as pseudo geodesics are,

$$au + bv = 1.$$

The distance between two points (u_1, v_1), (u_2, v_2) measured on one of these lines is

$$d = u_1 v_2 - u_2 v_1.$$

The angle between two lines

$$a_1 u + b_1 v = 1,$$
$$a_2 u + b_2 v = 1,$$

can then be defined as,

$$\beta = a_1 b_2 - a_2 b_1,$$

and will then be the exact dual of the distance between two points. The area of the triangle (u_1, v_1), (u_2, v_2), (u_3, v_3) will be

$$A = \begin{vmatrix} 1 & u_1 & v_1 \\ 1 & u_2 & v_2. \\ 1 & u_3 & v_3 \end{vmatrix}$$

4 From the ordinary formula the curvature of a minimum developable i indeterminate. However they can all be mapped on a minimum cone (poin t sphere). If the curvature of a sphere be taken as the reciprocal of the radius, the curvature of the point sphere is infinite. If then we say that applicable surfaces have the same curvature, the curvature of a minimum developable is infinite.

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A pseudo geodesic curvature of a curve can be defined as for the plane and will be found to be,

$$K = \frac{du}{ds}\frac{d^2v}{ds^2} - \frac{dv}{ds}\frac{d^2u}{ds^2},$$

where the curve is defined parametrically on terms of arc length. The pseudo geodesic circle of constant radius will be,

$$au + bv = 1,$$

And the pseudo geodesic circle of constant curvature will be

$$Au^2 + 2Buv + Cv^2 = 1,$$

and the curvature is,

$$K = \frac{AC - B^2}{4}.$$

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By Alfred Rehder.

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At the Arnold Arboretum there have been growing under the name of *Pyrus sinensis* several quite distinct trees which always have been a puzzle as regards their taxonomic standing. In working up the Pyrus of the Wilson collection I took the opportunity to essay a determination of all the Chinese Pyrus represented in the herbarium and grounds of the Arnold Arboretum. The first task, of course, was to decide which of the different forms represents the true *Pyrus sinensis* of Lindley. Professor Sargent, who had always taken much interest in the Chinese pear question, looked up Lindley's type in the Botanical Museum at Cambridge during his visit to England last summer and came to the conclusion that it does not agree with any of the forms now cultivated as *Pyrus sinensis*. He brought back an excellent photograph of the type specimen, which, together with Lindley's description and figure in the Botanical Register, convinced me, too, that *P. sinensis* of Lindley is quite different from the *P. sinensis* of subsequent authors, which is in most cases an aggregate of several species. To one of them belong the forms introduced some forty years ago from Japan into this country as Japanese or Chinese sand pears and which have given rise by crossing with the common pear to the Kieffer and similar forms. These Japanese pears are probably garden forms derived from a Chinese type or partly hybrids and, though differing in the fruit, are remarkably alike in foliage, as shown by an extensive collection from the Garden Herbarium of the Cornell University Experiment Station, kindly loaned by Professor L. H. Bailey. Two apparently wild forms, introduced from northern China, have been cultivated at the Arnold Arboretum since 1882, together with a third form of unknown native habitat received from Kew as *P. Simonii* which I have seen in France cultivated as *P. sinensis*. Two other forms have been introduced by Wilson from western China. Recently the Department of Agriculture has introduced a number of Chinese cultivated pears which exhibit a great variety in the size, shape, color and quality of the fruits, and, to a lesser degree, also in foliage. We are obliged to the Department for a series of photographs of fruits taken in China and of specimens of leaves from grafts

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growing at Chico, Cal. Little, however, can be said at present about the affinity of these pears, as the material is too insufficient; the leaves are apparently from the tips of young shoots and do not show the characteristic form of normal leaves, and most of the photographs fail to exhibit the important character of the persistent or deciduous calyx. We shall have to wait until these plants flower and fruit before we can attempt their classification.

The following Conspectus includes all the species hitherto known from China proper. Several of these species occur also outside of the limits of China; *Pyrus ussuriensis* ranges into Amurland, *P. Calleryana* into Korea and Central Japan, *P. Kochnei* into Formosa and *P. pashia* into the Himalayas. Besides these there occur in central and eastern Asia three species which are not found in China, though closely related to Chinese species; these are *P. Fauriei* Schneider in Korea, *P. Uyematsuana* Makino in central Japan, and *P. Jaquemontiana* Decaisne in the western Himalayas. All these are mentioned and briefly characterized in the following Enumeration under the species to which they are most nearly related. Another group of about nine species extends from Turkestan through Persia and Asia Minor into southern and western Europe and into northern Africa, but as none of them seems to be very closely related to any of the Chinese species, they do not concern us here. There is no true Pyrus in the southern hemisphere nor on the American continent.

CONSPECTUS SPECIERUM SINENSIUM.

Calyx persistens.
  Folia argute setoso-serrata.
    Pomum breviter pedicellatum, subglobosum: folia orbiculari-ovata vel ovata, ut inflorescentia ab initio glabra. 1. *P. ussuriensis.*
    Pomum longe pedicellatum, ovatum; folia oblongo-ovata v. ovata ut inflorescentia ab initio plus minusve floccoso-tomentosa . . . . . 2. *P. ovoidea.*
  Folia denticulata v. dentata dentibus non setoso-acuminatis: pomum longe pedicellatum, ovale . . . . . 3. *P. Lindleyi.*
Calyx deciduus.
  Folia setoso-serrata.
    Basis foliorum late cuneata: pomum flavum. 4. *P. Bretschneideri.*

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Basis foliorum subcordata vel rotundata: pomum fuscum. 5. *P. serotina.*

Folia argute serrulata v. dentato-serrata dentibus non setoso-acuminatis.

Styli 3–4: pomum 1.5–2 cm. diam.: folia basi plerumque rotundata.

Folia argute serrulata dentibus acutis vel acuminatis subaccumbentibus.....6. *P. serrulata.*

Folia dentato-serrata dentibus erecto-patentibus. 7. *P. phaeocarpa.*

Styli 2: pomum circiter 1 cm. diam.: folia grosse dentato-serrata, basi plerumque late cuneata...8. *P. betulaefolia.*

Folia crenata vel crenato-serrata.

Stamina circiter 20: folia et inflorescenta ab initio glabra.

Styli 2: folia ovata, basi plerumque rotundata. 9. *P. Calleryana.*

Styli 3–5.

Folia late ovata, crenato-dentata, basi subcordata. 10. *P. kolupana.*

Folia ovata, crenato-serrata, basi late cuneata vel rotundata. 11. *P. Koehnei.*

Stamina 25–30: folia ovata v. oblonga, crenato-serrata, basi plerumque rotundata, ut inflorescentia tomentosa vel fere glabra: styli 3–5.....12. *P. pashia.*

ENUMERATIO SPECIERUM.

1. **Pyrus ussuriensis** Maximowicz in *Bull. Acad. Sci. St. Pétersb.* XV. 132 (1857); in *Mém. Sav. Étr. Acad. Sci. St. Pétersb.* IX. 102 (*Prim. Fl. Amur.*) (1859).— Regel in *Gartenfl.* X. 374, t. 345 (1861).— Lauche in *Monatsschr. Ver. Beförd. Gartenb. Preuss.* XXII. 318, t. 4 (1879), pro parte, quoad flores.

Pyrus communis Bunge in *Mém. Sav. Étr. Acad. Sci. St. Pétersb.* II. 101 (*Enum. Pl. Chin. Bor.* 27) (1833), pro parte, non Linnaeus.

Pyrus sinensis Decaisne, *Jard. Fruit.* I. t. 5 (1872), non Poiret, nec Lindley.— Maximowicz in *Bull. Acad. Sci. St. Pétersb.* XIX 172 (1873); in *Mél. Biol.* IX. 168 (1873), pro parte.— Komarov in *Act. Hort. Petrop.* XXII. 476 (*Fl. Manshur.*) (1904), pro parte.— Schneider, *Ill. Handb. Laubholzk.* I. 663, fig. 361 q (1906), pro parte, quoad folia.

Pyrus Simonii Carrière in *Rev. Hort.* 1872, 28, fig. 3.

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Pirus sinensis a. *ussuriensis* Makino in *Tokyo Bot. Mag.* XXII. 69 (1908).
Pirus sinensis a. *silvestris* Makino msc. ex Makino, l. c., quasi synon.

CHILI: "China bor.," ex herb. Bunge (Gray Herb.); Hsiao Wu-tai-shan, August 20, 1913, *F. N. Meyer* (No. 1232). AMURLAND: "montes Burejae," 1859, *Maximowicz* (Gray Herb.); "Amur medius," May 18, 1891, *S. Korshinsky*. USSURI: without precise locality, *Maximowicz* (Gray Herb.). MANCHURIA: Kabarovsk, cultivated, August 23, 1903, *C. S. Sargent*.

This species differs from the allied species chiefly in the short stalk of the globose fruit with persistent calyx, in the broad, often nearly orbicular, strongly setosely serrate leaves and in the lighter yellowish brown color of the branches; the flower clusters are, owing to the short stalks rather dense and hemispherical; the petals are obovate and rather gradually narrowed toward the base; the styles are distinctly pilose near the base.

Pyrus ussuriensis was first introduced into cultivation by Richard Maack who sent seeds to the Botanic Garden at St. Petersburg, but the tree is still rare in gardens.

2. **Pyrus ovoidea** Rehder, sp. n.

?*Pyrus chinensis* Roxburgh, *Hort. Bengal.* 38 (1814), nomen nudum, non *Pyrus sinensis* Poiret; *Fl. Ind.* ed. 2, II. 511 (1832).
Pyrus sinensis Hemsley in *Jour. Linn. Soc.* XXIII. 257 (1887), pro parte, non Poiret, nec Lindley.— Diels in *Bot. Jahrb.* XXIX. 38 (1900), pro parte.— Schneider, *Ill. Handb. Laubholzk.* I. 663, fig. 364 c–d (1906), pro parte.
Pyrus Simonii Hort., non Carrière.

Arbor pyramidalis, 10–15-metralis, inermis; ramuli hornotini maturi purpureo-fusci v. flavo-fusci, nitiduli, vetustiores castanei, lenticellis parvis paucis conspersi; gemmae conico-ovoideae, castaneae, glabrae. Folia ovato-oblonga, rarius ovata, acuminata, basi rotundata v. subcordata, argute setoso-serrata dentibus erecti-patentibus v. interdum accumbentibus, 7–12 cm. longa et 4–6.5 cm. lata, initio ad marginem tantum et subtus ad costam tomento floccoso fulvo cito evanescente praedita, mox glaberrima, supra luteo-viridia, lucida, subtus paullo pallidiora, maturitate chartacea et autumno colore pulchro purpureo-scarlatino et aurantiaco gaudentia, utrinsecus nervis 9–10 supra et subtus leviter elevatis; petioli graciles, 2.5–5 cm. longi, initio sparse floccoso-lanati, mox glaberrimi. Flores circiter 3 cm. diam., in racemis umbelliformibus 5–7-floris glabris v. interdum tomento floccoso cito evanescente vestitis; pedicelli 2.5–4 cm. longi;

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calyx ut receptaculum extus glaber; sepala e basi late triangulari-lanceolata, denticulata, intus ad basin dense lanata; petala late obovata v. late ovalia, circiter 12 mm. longa, basi subito brevissime unguiculata, glabra, alba; stamina circiter 20, dimidiam partem petalorum vix aequantia, antheris purpureis; styli 5, distincti, basi pilosi, staminibus longioribus paullo breviores. Pomum ovoideum, basi rotundatum impressum, apicem versus attenuatum calyce persistente erecto vel incurvo coronatum, pedunculo gracili 2–4 cm. longo insidens, flavum, punctatum, circiter 4–4.5 cm. longum et 3.5–4 cm. diam., sapore grato leviter adstringente; semina ovoideo-oblonga, compressa, 9–10 mm. longa et 6 mm. lata, castanea, nitida.

Cultivated in the Arnold Arboretum under No. 4033 (received from Kew as *P. Simoni*), May 7 and 13, 1909, April 30, 1910, May 14, 1914, October 19, 1908, October 16, 1912 (type); Hort. Simon-Louis, Plantières near Metz, August 24, 1911, *A. Rehder* (as *P. sinensis*). Probably also the following specimens belong to this species: FOKIEN: *Dunn's* Exped. to Central Fokien, April to June, 1905 (Hongkong Herb. No. 2595). YUNNAN: Mengtze, cultivated, alt. 1500 m., *A. Henry* (No. 11058).

This species seems to be most closely related to *P. ussuriensis* Maximowicz which differs chiefly in the broader orbicular-ovate or ovate leaves, in the lighter colored branches, and in the short-stalked subglobose fruit with the persistent sepals spreading. The shape of the fruit of *P. ovoidea* is very unusual and quite distinct from any pear I know; the fruit is exactly ovate, broad and rounded at the base and tapering from the middle toward the truncate apex, as figured by Schneider (l. c. fig. 364 d). This may, however, not be a specific character and the shape of the fruit may vary in other specimens referable to this species. The Chinese material which I have seen and which might belong here is very meagre. The Fokien specimen is in young fruit which suggests a more pyriform shape, though tapering toward the apex and showing the same kind of persistent calyx; the serration of the leaves is more minute and more accumbent. The Yunnan specimen is in flower and differs somewhat in the more copious tomentum of the leaves and of the inflorescence and in the shorter nearly entire calyx-lobes.

It is not known when and whence this species was introduced. Possibly it was sent in the early sixties from northern China by G. E. Simon, or by A. David a little later from the same region or from Mongolia to the Museum in Paris and was afterwards distributed by Decaisne.

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3. **Pyrus Lindleyi** Rehder, nom. n.

Pyrus sinensis Lindley in *Trans. Hort. Soc. London*, VI. 396 (1826), non Poiret; in *Bot. Reg.* XV. t. 1248 (1829).

Lindley's *Pyrus sinensis* seems to have been much misunderstood, and the name has been applied to all Chinese pears characterized by setosely serrate leaves. Lindley's description, however, as well as his type specimen, of which I have a good photograph before me, show that the leaves of his species have short, rather small and not at all acuminate teeth. He certainly would have mentioned such a striking character as the setose teeth in his description, but in the description he simply says "foliis. . . . serratis" and in his comparison with *Pyrus communis* he does not mention the serration at all, which tends to show that he did not perceive much difference between the serration of *Pyrus communis* and that of his new species. The true *P. sinensis* of Lindley seems to have been lost to cultivation in Europe and in this country, for all the plants and specimens I have seen belong to species with setosely serrate leaves, and *Pyrus Lindleyi* rests at present only on Lindley's description and his type specimen.

In the *P. sinensis* of most authors three species seem to have been included, two of them with persistent calyx and one with deciduous calyx. In *P. Lindleyi* the calyx is persistent according to Lindley's description in the Botanical Register and according to the description, probably by the secretary of the society, in the Transactions of the Horticultural Society where only the fruit is fully described and nothing said about the leaves and the tree itself, with a note that "it has been named by Mr. Lindley *Pyrus sinensis*."

In Lindley's type specimen the leaves of the shoots are ovate, abruptly acuminate and rounded at the base, and those of the short branchlets mostly subcordate; all are closely serrate, with small appressed, acute teeth, or those of the short branchlets nearly crenately serrate. As much as can be judged from the photograph they appear to be quite glabrous and about 8 or 10 cm. long.

Pyrus Lindleyi is possibly not a wild species, but a cultivated form. At present, however, with our incomplete knowledge of the Chinese pears, it seems best to treat it as a species. *Pyrus communis* Loureiro (*Fl. Cochin.* 321. 1790) may belong here as a synonym, as the author describes the leaves as "subintegerrima."

The name *Pyrus sinensis* of Lindley cannot be maintained for this species on account of the older *P. sinensis* (Thouin) Poiret (*Encycl. Méth. Suppl.* IV. 452. 1816). Even though the latter species is now

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generally referred to Chaenomeles, it must be considered nomenclatorially a valid name and the combination cannot be used again for another species.

4. **Pyrus Bretschneideri** Rehder, sp. n.

Arbor mediocris; ramuli hornotini sparsissime lanuginosi, cito glabri, annotini fusco-purpurei, sparse lenticellati; gemmae ovatae, 4–5 mm. longae, perulis late ovatis manifeste mucronulatis castaneis extus margine villoso excepto glabris. Folia subchartacea, ovata vel elliptico-ovata, acuminata, basi late cuneata, rarissime fere rotundata, argute serrata dentibus initio setoso-acuminatis, demum manifeste acuminatis plerumque leviter accumbentibus, 5–11 cm. longa et 3.5–6 cm. lata, initio utrinque laxe araneoso-lanuginosa, cito glabra, supra saturate luteo-viridia, subtus paullo pallidiora, leviter reticulata; petioli graciles, 2.5–7 cm. longi, initio sparsissime lanuginosi, cito glabri. Inflorescentia umbellato-racemosa, 7–10-flora, rhachi, pedicellis, receptaculis sparse lanuginosis, mox glabris; pedicelli 1.5–3 cm. longi, bracteolis 2 subulatis circiter 5 mm. longis instructi; sepala e basi late triangulari acuminata, circiter 4 mm. longa, glanduloso-serrata, extus glabra, intus fulvo-lanuginosa; petala ovalia, apice plerumque irregulariter erosa, basi breviter unguiculata, 12–14 mm. longa et 10–12 mm. lata, alba; stamina circiter 20, petala dimidia aequantia; styli 5 vel interdum 4, stamina subaequantes, glabri. Pomum subglobosum vel globoso-ovoideum, 2.5–3 cm. longum et circiter 2.5 cm. diam., apice cicatrice impressa calycis decidui notatum, basi subito in pedicellum 3–4 cm. longum contractum, pendens, flavum, pallide et minute punctulatum, 5- vel rarius 4-loculare; semina obovoidea, leviter compressa, 6–7 mm. longa, castanea.

Arnold Arboretum, cultivated, April 22, 1910 and October, 1908 (type, raised from seed sent by Dr. E. Bretschneider from Peking in 1882).

This species seems nearest to *P. ovoidea* Rehder which is easily distinguished by the persistent calyx and by the more oblong-ovate leaves rounded or even subcordate at the base. In its deciduous calyx it agrees with *P. phaeocarpa* Rehder, but this species differs in its smaller 3–4-celled brown fruit and the coarser serration of the more oblong-ovate leaves. *Pyrus Bretschneideri* may be the pear alluded to by Bretschneider (*Hist. Eur. Bot. Discov.* 830) as pai-li (white pear), though this name may also apply to *P. ussuriensis* or to *P. ovoidea*.

5. **Pyrus serotina** Rehder, sp. n.

Arbor 7–15-metralis; ramuli hornotini glabri vel initio laxe villosuli, rarius tomento floccoso densius obtecti, mox glabri, annotini et vetu-

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stiores purpureo- vel fusco-brunnei, sparse lenticellati; gemmae acutiusculae, ad 1 cm. longae, perulis ovatis acutis margine excepto fere glabris fuscis. Folia subchartacea, ovato-oblonga vel rarius ovata, longe acuminata, basi rotundata, rarius subcordata, interdum late cuneata, arcte et argute serrata dentibus setoso-acuminatis subaccumbentibus, 7–12 cm. longa et 4–6.5 cm. lata, initio margine laxe villosa et subtus secus costam tenuiter araneoso-lanata vel fere glabra, rarius utrinque fere in tota facie tomento araneoso evanescente laxe obtecta, mox glaberrima, supra laete viridia, subtus paullo pallidiora, utrinque in sicco leviter reticulata, utrinsecus nervis 6–11 arcuatis; petioli graciles, 3–4.5 cm. longi, initio fere glabri vel plus minusve floccoso-tomentosi, mox glabri. Inflorescentia umbellato-racemosa, 6–9-flora, fere glabra vel plus minusve tomento floccoso flavescente vel canescente obtecta; pedicelli graciles, 3.5–5 cm. longi; sepala e basi triangulari-ovata, longe acuminata, 6–10 mm. longa, glanduloso-denticulata, patentia, receptaculum saepe fere duplo superantia, extus fere glabra vel plus minusve tomentosa, intus ad basin saltem fulvo-tomentosa; petala ovalia, 15–17 mm. longa, apice plerumque irregulariter erosa, breviter unguiculata; stamina circiter 20, dimidia petala aequantia; styli 5, rarius 4, glabri, staminibus fere aequilongi. Pomum subglobosum, apice leviter impressum, cicatrice calycis decidui notatum, basi subito in pedicellum gracilem 3.3–5 cm. longum contractum, circiter 3 cm. diam., fuscum, pallide punctulatum; semina cuneato-ovoidea, leviter compressa, basi in stipitem contracta, 8–10 mm. longa, atro-brunnea.

WESTERN HUPEH: Hsing-shan Hsien, woodlands, alt. 1300–2000 m., May and December 1907, *E. H. Wilson* (No. 479a, type); same locality, May and October 1907, *E. H. Wilson* (No. 2977); Patung Hsien, thickets, alt. 1000–1600 m., May 1907, *E. H. Wilson* (No. 556b); north and south of Ichang, thickets, alt. 600–1300 m., April 1907, *E. H. Wilson* (No. 479b); same locality, October 1907, *E. H. Wilson* (No. 415); Changlo Hsien, thickets, May 1907, *E. H. Wilson* (No. 556c); without precise locality, *A. Henry* (No. 5299). EASTERN SZECHUAN: without precise locality, *A. Henry* (No. 5875). WESTERN SZECHUAN: Tachienlu, alt. 2000–2600 m., October 1908, *E. H. Wilson* (No. 1293).

This species seems to be most closely related to *P. Bretschneideri* Rehder which is easily distinguished by the leaves being broadly cuneate at the base, by the smaller flowers and by the yellow color of the fruit. Its leaves resemble closely those of *P. ovoidea* Rehder so that it seems impossible to distinguish these two species with cer-

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tainty without flowers or fruits; in fruit, however, the persistent calyx of the ovate yellow fruit of *P. ovoidea* presents a good character, and the flowers of *P. ovoidea* may be distinguished by the styles being pubescent at the base. Wilson's No. 2977, of which the fruit is not known, differs from the type in its broader ovate or broadly ovate leaves with more appressed teeth and may represent a distinct variety or even another species. Henry's No. 5875 has the fruit pyriform and may belong to the following variety. Henry's No. 5299 is in flower only and agrees with this species in its glabrous styles; both specimens of Henry may be from cultivated plants.

This species was introduced by E. H. Wilson in 1909 and seeds were distributed by the Arnold Arboretum. According to a note received two years ago it is growing in the Botanic garden at Glasnevin.

This pear and probably other brown-fruited species are called by the Chinese tang-li.

Pyrus serotina var. **Stapfiana** Rehder, n. var.

Pyrus sinensis Stapf in *Bot. Mag.* CXXXIV. t. 8226 (1908), quoad plantam depictam, non Poiret, nec Lindley.

A typo recedit fructu pyriformi, foliorum dentibus minus adpressis, petalis satis sensim in unguiculum attenuatis.

Pyrus serotina var. **culta**, Rehder, comb. n.

?*Pyrus communis* Thunberg, *Fl. Jap.* 207 (1784), non Linnaeus.

?*Pyrus communis* c. *hiemalis* Siebold in *Verh. Bat. Genoot.* XII. pt. 1, 66 (*Syn. Pl. Oecon. Jap.* No. 349) (1830), nomen nudum.

?*Pyrus communis* β. sinensis K. Koch in *Ann. Mus. Lugd.-Bat.* III. 40 (1866).

Pyrus Sieboldi Carrière in *Rev. Hort.* 1880, 110, t., non Regel.

Pyrus sinensis Bailey, *Cycl. Am. Hort.* III. 1470 (1901), pro parte, non Lindley, nec Poiret.

Pyrus japonica Hort. ex Bailey, l. c. quasi synon., non Thunberg.

Pyrus sinensis β. *culta* Makino in *Tokyo Bot. Mag.* XXII. 69 (1908).— Koidzumi in *Jour. Coll. Sci. Tokyo*, XXXIV. art. 2, 54 (1913).

A typo recedit fructu majore pyriformi vel maliformi, foliis majoribus latioribusque ad 15 cm. longis et ad 8–10 cm. latis.

JAPAN: Arakawa, north of Tokyo, roadside, April 2, 1914, *E. H. Wilson* (No. 6541, bush 2 m.); Hatogaya near Tokyo, cultivated, April 29, 1914, *E. H. Wilson* (No. 6595); Tokyo, Sakurai's garden, April 24, 1912, *K. Sakurai;* slopes of Mt. Fuji, alt. 2600 m., cultivated, May 8, 1914, *E. H. Wilson* (No. 6668; small tree, 12–20

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feet); Tsuba-kura-dake, prov. Shinano, alt. 900 m., cultivated, September 15, 1914, *E. H. Wilson* (No. 7497; tree 10 m. tall, fruit russet, flesh white, sweet). Besides these specimens I have seen from the Herbarium of the Cornell University Experiment Station numerous specimens from plants cultivated in this country under the names: Oriental Pear, Japanese Pear, Chinese Sand Pear, Madame von Siebold, Daymio, Mikado, Rikiya and Gold Rust or Golden Russet.

The Japanese pear cultivated under the name "Madame von Siebold" may be considered as representing the type of this variety. It has large subglobose, somewhat depressed fruit with deciduous calyx and is well described and figured by Carrière (in *Rev. Hort.* 1879, 170, t.); by Ottolander in *Flor. & Pomol.* 1877, 100, fig. 2 and in *Nederl. Flora en Pomona* 69, t. 21; by S. Morris in *Am. Gard.* n. ser. XIII. 87 (1892); also the figure in *American Agriculturist*, XXX. 462 (1871) and in *Gard. Chron.* ser. 2, III. 106, fig. 17, 18 (1875) belong probably here. *Pyrus Sieboldi* Carrière (in *Rev. Hort.*, 1880, 110, t.) has large pear-shaped fruit with deciduous calyx, but according to Ottolander (in *Flor. & Pomol.*, 1877, 100, fig. 3) it has a persistent calyx. "Ottolander" has an oblong fruit with deciduous calyx (*Flor. & Pomol.*, 1877, 100, fig. 4; also in *Gard. Chron.* ser. 2, IV. 456, fig. 95. 1875, and ser. 3, XXVIII. 300, fig. 89. 1900). "Gold Rust" has, according to a photograph in the Cornell Garden Herbarium, a subglobose fruit without calyx. "Daymio" has a globose-ovoid yellowish fruit with persistent calyx (*Nederl. Flora en Pomona*, 69, t. 21, fruit at the left). "Mikado" has a broadly pear-shaped yellow fruit without calyx (*Rev. Hort.*, 1878, 310, t.). The last two forms are possibly hybrids. Some of the above named forms have hybridized with the Common Pear; the best known of these hybrids is the "Kieffer Pear" which has finely or nearly crenately serrulate leaves and a pear-shaped fruit with persistent calyx.

6. **Pyrus serrulata,** Rehder, sp. n.

Arbor 7–8-metralis; ramuli hornotini leviter lanati, mox glabri, annotini purpureo-fusci, sparse lenticellati; gemmae ovatae, fuscae, perulis ovatis acutis exterioribus margine ciliato excepto glabris. Folia chartacea, ovata vel ovato-oblonga, subito vel sensim acuminata, basi rotundata vel late cuneata, margine serrulata dentibus adpressis et plerumque leviter incurvis acutis vel breviter acuminatis, 5.5–11 cm. longa et 3.5–6.5 lata, initio subtus tomento araneoso-lanato fugace leviter obtecta, cito glabrata, supra ab initio glabra vel fere glabra, nervis utrinsecus 7–13 arcuatis, utrinque in sicco leviter reticulata; petioli graciles, 3.5–7.5 longi, initio leviter lanati, mox

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glabri. Inflorescentia racemoso-umbellata, 6–10-flora, rhachi circiter 1.5 cm. longa satis dense flavo-lanata; pedicelli ut receptaculum laxe lanati, 1.5–2 cm. longi; sepala triangulari-ovata, acuta vel acuminata, sparse glanduloso-denticulata, receptaculum subaequantia, circiter 3 mm. longa, extus sparse, intus dense lanata; petala late ovalia, alba, 10–12 mm. longa, subito breviter unguiculata, leviter et irregulariter erosa; stamina circiter 20, petalis circiter triente breviora; styli 3, rarius 4, stamina subaequantes, basi sparsissime pilosi. Pomum subglobosum vel globoso-obovoideum, 1.5–1.8 cm. longum, apice vix impresso cicatrice calycis decidui notatum, basi subito in petiolum 3–4 cm. longum contractum, fuscum, pallide lenticellatum, 3–4-loculare; semina obovoidea, 7 mm. longa et 4 mm. lata, castanea.

WESTERN HUPEH: Hsing-shan Hsien, thickets, alt. 1300–1600 m., May and December 1907, *E. H. Wilson* (No. 779, type); north and south of Ichang, alt. 600–1300 m., October 1907, *E. H. Wilson* (No. 479).

This species seems to be most closely related to *P. serotina* Rehder which is easily distinguished by the setosely serrate, generally longer leaves, by the larger flowers with usually 5 styles and long-acuminate sepals and by the larger fruit. The Japanese *P. Uyematsuana* Makino[1] seems to be most nearly related to *P. serrulata;* according to the description it differs from it in the often subcordate leaves and in the disk which is described as villose by Koidzumi; styles 3–5.

Pyrus serrulata was introduced by E. H. Wilson and seeds have been distributed through the Arnold Arboretum.

7. **Pyrus phaeocarpa** Rehder, sp. n.

Pyrus ussuriensis Lauche in *Monatschr. Ver. Beförd. Gartenb. Preuss.* XXII. 318, t. 4 (1879), quoad fructus, non Maximowicz.

Arbor mediocris; ramuli hornotini tomentosi, tarde glabrescentes, annotini glabri, purpureo-fusci, sparse lenticellati; gemmae oblongo-conicae, acutiusculae, 6–7 mm. longae, perulis castaneis vel partim griseis glabris vel fere glabris. Folia chartacea, elliptico-ovata vel oblongo-ovata, in acumen longum sensim attenuata, basi plerumque late cuneata, serrata dentibus acuminatis apice initio plus minusve incurvis demum patentibus sinubus apertis saepe fere rectangulis, 6–10 cm. longa et 3.5–5.5 cm. lata, initio laxe araneoso-lanata, mox

[1] *Pyrus Uyematsuana* Makino in *Tokyo Bot. Mag.*, **22**, 68 (1908).— Koidzumi in *Jour. Coll. Sci. Tokyo*, XXXIV., art. 2, 56 (1913).
JAPAN: Prov. Ise (ex Makino and Koidzumi).

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glaberrima, supra saturate luteo-viridia, subtus pallidiora, nervis utrinsecus 7–10 arcuatis, in sicco utrinque leviter reticulata; petioli initio albido-lanati, rarius fere glabri, mox omnino glabri, graciles, 2–6 cm. longi. Inflorescentia umbellato-racemosa, 5–7-flora, albido-lanata, raro fere glabra; pedicelli 2–2.5 cm. longi, ut receptaculum initio plus minusve lanati, rarius fere glabri, mox omnino glabri; sepala triangulari-lanceolata, acuminata, glandulosa-serrata, receptaculo paullo longiora, 4–5 mm. longa, extus sparse, intus densius lanata; petala ovalia, breviter unguiculata, 1–1.5 cm. longa et 0.8–1.2 cm. lata, glabra, alba; stamina circiter 20, dimidia petala subaequantia; styli glabri, 3–4, rarissime 2. Pomum pyriforme, 2–2.5 cm. longum et 1.5–2 cm. diam., graciliter pedicellatum pedicello 2–3 cm. longo, fuscum, pallide lenticellatum, 3–4-loculare, rarissime 2-loculare; semina obovoidea, compressa, circiter 7 mm. longa, fusco-castanea.

Arnold Arboretum, cultivated, May 12, 1909, and October 1908 (type, raised from seed sent by Dr. E. Bretschneider from Peking in 1882).

This species is most closely related to *P. betulaefolia* Bunge which is easily distinguished by its much smaller 2-celled fruit, the smaller flowers, the smaller and more coarsely serrate leaves and the denser grayish tomentum persisting on the branchlets, on the inflorescence and often on the under side of the leaves particularly on the midrib until autumn. In the shape of its leaves it has some resemblance to *P. Bretschneideri* Rehder, but that species has setosely serrate, generally larger and broader leaves and larger yellow subglobose fruit.

Pyrus phaeocarpa was apparently first introduced in its pear-shaped form to the Horticultural School at Potsdam, Germany, about 1870. At the Arnold Arboretum both the pear-shaped and the apple-shaped forms were raised from seed sent by Dr. Bretschneider from Peking in 1882.

Pyrus phaeocarpa f. globosa Rehder, forma n.

A typo recedit fructu globoso, 1.5–2. cm. diam. et foliis paullo latioribus, saepius ovatis et basi rotundatis.

8. **Pyrus betulaefolia** Bunge in *Mém. Sav. Étr. Acad. Sci. St. Pétersb.* II. 101 (*Enum. Pl. Chin. Bor.* 27) (1833).— Walpers, *Rep.* II. 53 (1843).— Decaisne, *Jard. Fruit.* I. t. 20 (1872).— Maximowicz in *Bull. Acad. Sci. St. Pétersb.* XIX. 172 (1873); in *Mél. Biol.* IX. 169 (1873).— Debeaux in *Act. Soc. Linn. Cherbourg*, XXXI. 156 (*Fl. Tchéfou*, 61) (1876).— Carrière in *Rev. Hort.* 1879, 318, fig. 68, 69.— Hemsley in *Jour. Linn. Soc.* XXIII. 256 (1887).— Sargent in *Gard. &*

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For. VII. 224, fig. 39 (1894).— Diels in *Bot. Jahrb.* XXIX. 387 (1900). — Schneider, *Ill. Handb. Laubholzk.* I. 665, fig. 363 o, 364 k–p (1906).— Pampanini in *Nuov. Giorn. Bot. Ital.* n. ser. XVII. 291 (1910).

CHILI: near Peking, *E. Bretschneider*, 1881 (seeds). SHANTUNG: Lau-shan, August 1907, *F. N. Meyer* (No. 308); without precise locality, September 1907, *F. N. Meyer* (No. 398). SHENSI: Yenan Fu, 1910, *W. Purdom* (No. 328); Poa ting Fu plain, 1909, *W. Purdom.* HUPEH: without precise locality, *A. Henry* (No. 1654).

Henry's specimen from Hupeh differs from the type in its broader and somewhat larger leaves.

This species was first introduced in the sixties by G. E. Simon to the Museum at Paris unintentionally as stock of a grafted Chinese pear. In 1882 it was introduced again to the Arnold Arboretum by Dr. Bretschneider from the mountains near Peking.

This pear is called by the Chinese t'ao-li (pea-pear).

9. **Pyrus Calleryana** Decaisne, *Jard. Fruit.* I. in textu ad t. 8 (1872).— Maximowicz in *Bull. Acad. Sci. St. Pétersb.* XIX. 172 (1873); in *Mél. Biol.* IX. 169 (1873); in *Bull. Soc. Nat. Mosc.* LIV, pt. 1, 18 (1879).— Hance in *Jour. Bot.* XXI. 298 (1883).— Franchet in *Nouv. Arch. Mus. Paris*, ser. 2, V. 272 (1883).— Schneider, *Ill. Handb. Laubholzk.* I. 666, fig. 363 p (1906).— Koidzumi in *Jour. Coll. Sci. Tokyo*, XXXIV. art. 2, 55 (1913).

WESTERN HUPEH: Hsing-shan Hsien, thickets, not common, alt. 1000–13000 m. May 14, 1907, *E. H. Wilson* (No. 2775); Changlo Hsien, thickets, alt. 1000–1500 m., December 1907, *E. H. Wilson* (No. 556); Patung Hsien, alt. 1000–1700 m., December 1907, *E. H. Wilson* (No. 556a); around Ichang, common, alt. 1000–1300, March and July 1907, *E. H. Wilson* (No. 2976); mountains north and south of Ichang, alt. 600–1500 m., April 1907, *E. H. Wilson* (No. 415a). KIANGSI: Kuling, side of streams, common, alt. 1300 m., July 29, 1907, *E. H. Wilson* (No. 1662). CHEKIANG: Ningpo, 1908, *D. Macgregor.* KWANGTUNG: without precise locality, *C. Ford* (No. 68); Botanic Garden, Hongkong, Nov. 4, 1903, *C. S. Sargent.*

Pyrus Calleryana is a widely distributed species and seems not uncommon on mountains at an altitude of from 1000 to 1500 m. It is easily recognizable by its comparatively small crenate leaves, like the inflorescence glabrous or nearly glabrous and by its small flowers with 2, rarely 3 styles. When unfolding most specimens show a loose and thin tomentum on the under side of the leaves which usually soon disappears, but in No. 1662 from Kuling even the fully grown leaves are loosely rusty tomentose on the midrib

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beneath. In No. 415a the leaves are longer, generally ovate-oblong, the pedicels very long and slender, about 3–4 cm. long and the sepals are mostly long-acuminate. The fruit of No. 556a is rather large, about 1–1.4 cm. in diameter, but a fruit examined proved to be 2-celled.[2]

This species was introduced by E. H. Wilson to the Arnold Arboretum in 1908 and the young plants seem to be hardy here.

The following Japanese pear is referred by Koidzumi as a variety to *P. Calleryana.*

Pyrus Calleryana var. **dimorphophylla** Koidzumi in *Jour. Coll. Sci. Tokyo,* XXXIV. art. 2, 56 (1913).

Pyrus Calleryana Maximowicz in *Bull. Acad. Sci. St. Pétersb.* XIX. 172 (1873); in *Mél. Biol.* IX. 169 (1873), quoad plantam japonicam.
Pyrus dimorphophylla Makino in *Tokyo Bot. Mag.* XXII. 65 (1908).

JAPAN: Prov. Ise and prov. Shinano (ex Makino and Koidzumi).[2]

10. **Pyrus kolupana** Schneider, *Ill. Handb. Laubholzk.* I. 665 (1906); in Fedde, *Rep. Nov. Spec.* III, 120 (1807).

SHENSI: Ko-lu-pa, *G. Giraldi* (Nos. 1050, 5105, ex Schneider).

This species is little known and not yet in cultivation.

11. **Pyrus Koehnei,** Schneider, *Ill. Handb. Laubholzk.* I. 665, fig. 363 m, 364 t–u (1906); in Fedde, *Rep. Nov. Spec.* III. 119 (1907).— Koidzumi in *Jour. Coll. Sci. Tokyo,* XXXIV. art. 2, 57 (1913).

Pyrus Kawakamii Hayata in *Jour. Coll. Sci. Tokyo,* XXX. art. 1, 99 (1911).

CHEKIANG: Tien-tai mountains, alt. 1000 m., *E. Faber* (ex (Schneider). FORMOSA: Nanto, *T. Kawakami* (ex Hayata).

This species like the preceding is as yet little known and is not in cultivation. As I have seen neither a specimen from the type locality nor from Formosa, I do not know whether Koidzumi is right in referring *P. Kawakamii* as a synonym to *P. Koehnei.*

12. **Pyrus pashia** Hamilton apud Don, *Prodr. Fl. Nepal.* 236 (1825).— G. Don, *Gen. Syst.* II. 622 (1832).— Decaisne, *Jard. Fruit.*

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2 A closely related species, *P. Fauriei* Schneider (*Ill. Handb. Laubholzk.* I. 666, fig. 363 d.' 1906) occurs in Korea; it differs chiefly in its much smaller leaves and fruits.

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I. 328, t. 7 (1872).— Wenzig in *Linnaea*, XXXVIII. 48 (1874).— Brandis, *Forest Fl. Brit. Ind.* 204 (1874); *Ind. Trees*, 291 (1906).— Kurz, *Forest Fl. Brit. Burma*, I. 441 (1877).— Hooker f., *Fl. Brit. Ind.* II. 374 (1879).— Collett, *Fl. Siml.* 169, fig. 47 (1902).— Schneider, *Ill. Handb. Laubholzk.* I. 664, fig. 363 h, 364 e–g (1906).

Pyrus variolosa Wallich, Cat. No. 680 (1828), nomen nudum.— G. Don, *Gen. Syst.* II. 622 (1832).

Pyrus verruculosa Bertoloni in *Mem. Accad. Sci. Bologna*, ser. 2, IV. 312 (*Piante As.* II.) (1864).[3]

Pyrus heterophylla Hort. ex Decaisne, *Jard. Fruit* I. 328, sub t. 7 (1872), quasi synon.

Pyrus nepalensis Herb. Hamilt. et Hort. ex Hooker, *Fl. Brit. Ind.* II, 374 (1879), quasi synon.

WESTERN SZECHUAN: Ching-chi Hsien, alt. 1500 m., October 1908, *E. H. Wilson* (No. 1335); same locality, open country, alt. 2600 m., October 1910, *E. H. Wilson* (No. 4132). YUNNAN: Mengtze, alt. 1400–1500 m., *A. Henry* (Nos. 10035, 10035 c). HIMALAYAS: Kashmir to Bhutan, Kashia Mountains, Ava (ex Hooker f.).

This species is not mentioned by Hemsley in his Index florae sinensis, though Hooker in 1879 includes Yunnan in the distribution of the species. Wilson's No. 1335 which is in ripe fruit agrees well with typical *P. pashia* and the young plants raised at the Arnold Arboretum from seeds of that number show exactly the kind of finely and sharply serrate mostly deeply lobed leaves, figured by Decaisne as the form occurring on suckers. No. 4132 differs in its much shorter and tomentose pedicels, only about 1.5 cm. long, and in the generally broader leaves mostly subcordate at the base; part of the fruits show a persistent calyx. Whether this is a variety of this species or a distinct species may be decided when the plants in cultivation flower and fruit.

Pyrus pashia was first introduced in 1825 into England from Nepal or Kumaon; in 1908 it was reintroduced by E. H. Wilson from western China and distributed through the Arnold Arboretum. Possibly the plant introduced in 1825 represents the following variety, as it is this variety which is now found occasionally in European collections.

Pyrus pashia var. **kumaoni** Stapf in *Bot. Mag.* CXXXV. t. 8256 (1909).

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[3] In the text the references to the type specimens of this species and of P. *granulosa* are interchanged; "Pyrus (c)" belongs to *P. granulosa* and "*P. variolosa* Trell. [sic] var." belongs to *P. verruculosa*.

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Pyrus Kumaoni Decaisne, *Jard. Fruit.* I. 328, sub t. 7 (1872).— Hooker f., *Fl. Brit. Ind.* II. 374 (1879).— Schneider, *Ill. Handb. Laubholzk.* I. 665 (1906).

Pyrus Wilhelmii Schneider, *Ill. Handb. Laubholzk.* I. 665, fig. 363 n (1906); in Fedde, *Rep. Nov. Spec.* III. 120 (1907).

YUNNAN: Mengtze, mountain woods, alt. 1600 m., *A. Henry* (No. 10035a, type); same locality, alt. 1400 m., *A. Henry* (No. 10035b). HIMALAYAS: Kashmir to Kumaon (ex Hooker).

This variety differs from the type in its glabrous or nearly glabrous inflorescence and leaves and in the ovate, broader and often obtuse calyx-lobes. I am unable to separate *P. Wilhelmii* specifically from this variety; broadly ovate leaves occur also in *P. pashia* and the statement that *P. Wilhelmii* has only 3 styles is not borne out by the specimen which represents Henry's No. 10035a, on which *P. Wilhelmii* is based, in the herbarium of the Arboretum, as the number of styles varies in that specimen from 3 to 5.

Besides *P. pashia* there occurs in the western Himalayas another closely related species, *P. Jacquemontii* Decaisne (*Jard. Fruit.* I. t. 8. 1872). This according to Decaisne differs from *P. pashia* in its smooth, not verruculose, young fruits. It is so far very imperfectly known, as neither the flowers nor the mature fruits have been described.

ARNOLD ARBORETUM,
Harvard University.

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CERTAIN OLD CHINESE NOTES.

By Andrew McFarland Davis.

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CERTAIN OLD CHINESE NOTES.

By Andrew McFarland Davis.

Presented, February 10, 1915. Received, February 19, 1915.

Chinese Paper Money.

The use in China, as a medium of trade, of a representative paper currency, based upon government credit dates back with reasonable certainty to the beginning of the ninth century of the Christian Era. The notes than in circulation are referred to by Chinese historians in such a way as to leave no doubt in the minds of investigators competent to analyze the literature of that country, as to the authenticity of the statement that at that date a government paper money was in circulation. There are fabulous assertions by chinese writers, as to the use of paper money many centuries before the birth of Christ. And there are specific assertions of the value assigned to white deer skins, for purposes of transfer under certain circumstances, by one of the emperors about a century and a half before Christ, which have led translators to speak of them as "Deer Skin Money." I take no consideration of this so-called money, for it was neither paper money, nor would the accounts that we have of it permit it to be defined as money at all. But the beginning of the ninth century of the Christian Era has been generally accepted by students of the subject as the period when it can be satisfactorily demonstrated that government notes were actually circulated in lieu of metallic money. There is indeed a chinese numismatical work [1] which furnishes pictorial representations of notes emitted as early as the middle of the seventh century. The compiler of that work must have had some authority for the designs of these earlier notes which he published. If we hesitate to accept this date, without further knowledge as to the authorities upon which are based the details given about these notes, it must nevertheless be recognized as possible that the date of 650 A.D. may ultimately be accepted as that of the earliest emission of notes by the chinese government of which we have at present any trace. The fact remains however that there is corroborative evidence from numer-

[1] Ch'ien Pu Tung Chih, or by others Chuan Pu Tung Chih.

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ous sources as to the notes emitted after 806 A.D. and that a few scattered specimens of some of the various emissions remain in existence, while it is said, that no other mention of those earlier notes has been met with in chinese literature than that accredited to this particular writer.

The acquisition by myself in 1910 of a Ming note emitted probably about 1375, led to correspondence and investigation on my part. Interest in the subject was revived in the fall of 1914 by my securing possession of fourteen of these old notes, two of which dated back to the Tang Dynasty, somewhere about 850 A.D., and of course investigation on my part was thereby stimulated. The circumstances connected with the purchase of the first of these notes were as follows:

In the latter part of the year 1910, I received a catalogue from a London book-seller which contained the following item:

"Chinese Bank Note. A genuine specimen of the earliest known Bank Note, being one issued during the reign of the Emperor, Hung Wu (1368–1398). 12½ by 8½ ins. The inscription enclosed by elaborate ornamental border, the whole being printed from a wood block; mounted on limp board, with embroidered work at back; worn in places."

The earliest European note was issued by the Bank of Stockholm about three centuries after the above. There is a similar example in the British Museum, which is the only one known to me.

Further correspondence revealed the fact that the description of the note, except as to the mounting was taken from labels at the British Museum. A friend in London, at my instance, took a look at the specimen which was offered for sale, and although not an expert in such matters, expressed himself as satisfied that it was genuine and I purchased it.

At that time I knew nothing about chinese paper-money. The statement that the specimen was a bank-note I rejected, as improbable, but the error of describing these government emissions as bank-notes is one that is frequently committed by writers on the subject, and in this particular case may perhaps be charged to the labels of the British Museum.

In the spring of 1911 I wrote to one of the curators in that institution asking about specimens of chinese currency in their possession, and found that the Museum at that time had two notes precisely alike, both of the Ming Dynasty (1368, 1398) and each for one kwan. One was procured in 1890. The other in 1902.

Marco Polo was in China about one hundred years before the date

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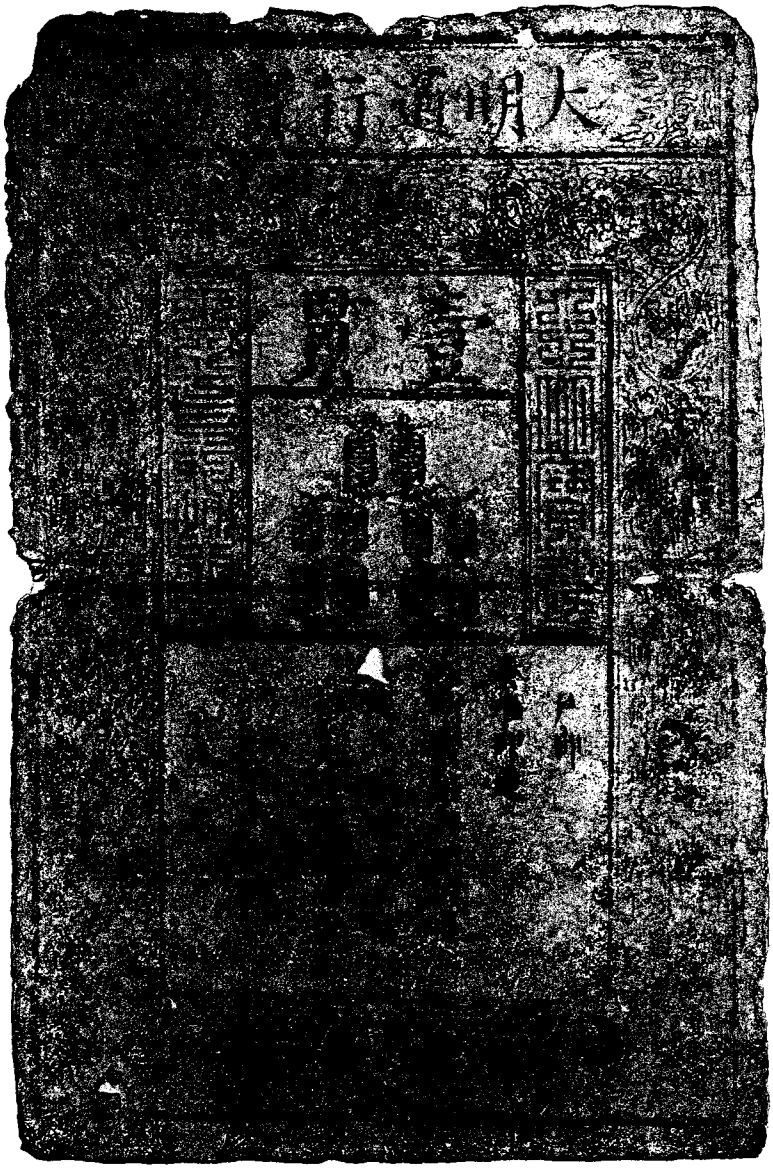

One kwan note, Ming Dynasty. Emperor, T'ai Tsu; Period, Hung Wu; A.D. 1368–1398.

Reproduced from the original, which is darker than this half-tone.

The paper measures 13½ by 8¾ inches; the impression, 12½ by 8¼ inches. The note had the value given on the back and originally bore two vermilion seals on the face and one on the back.

From the collection of Andrew McFarland Davis.

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of these notes, and he left behind him a full description of the paper money that he then found in circulation. I therefore turned to that source of information and examined Polo's description of the Mongol notes. At the same time I procured a copy of H. B. Morse's "Trade and Administration of the Chinese Empire," which was published in 1908 and contained a lithographic fac-simile of one of the one kwan Ming notes. Morse gives in this book a translation of the various inscriptions on the face of this note, whether written in ordinary chinese or in seal characters. The note pictured by Morse was emitted after an experience of upward of five, possibly of seven, centuries' use of paper money. It embodied therefore whatever there was that this long experience had taught the chinese to be essential in the form of the note for the circulation of paper money in China. It was of the same emission as the one which I had purchased. Translations of the inscriptions on the one kwan Ming notes are to be found in the works of several writers who treat on the subject of chinese paper money, but the most complete is that given by Morse. It is desirable to know at the outset precisely what is meant by an old chinese note. I therefore incorporate Morse's translation of the inscriptions on a Ming note and also include his story of the manner in which the note was acquired, which runs as follows:

"This 500-year old instrument of credit has a curious history furnishing an absolute guarantee of its authenticity. During the foreign occupation of Peking in 1900, some European soldiers had overthrown a sacred image of Buddha, in the grounds of the Summer Palace, and, deposited in the pedestal (as in the corner-stones of our public buildings), found gems and jewelry and ingots of gold and silver and a bundle of these notes. Contented with the loot having intrinsic value, the soldiers readily surrendered the bundle of notes to a by-stander, who was present 'unofficially,' Surgeon Major Louis Livingston Seaman, U. S. A., of New York and he gave to the Museum of St. John's College at Shanghai, the specimen which is here reproduced.

"The note is printed on mulberry-bark paper, which now is of a dark slate colour, the 'something resembling sheets of paper, but black' of Marco Polo's description. The sheet of paper is 13.5 by 8.75 inches and the design on the face is 12.6 by 8.3 inches. The border, 1.4 inches wide, is made of extended dragons filled around with an arabesque design, and is surmounted by a panel with the inscription (from right to left) 'circulating government note of the Ming Empire.' The space within the border is divided into two panels. The upper has on two sides in conventionalized square seal characters, on the right 'government note of the Ming Empire,' on the left 'circulating for ever and ever,' between these two inscriptions, above in large ordinary characters 'One *Kwan*,' (or *tiao* or string), and below a pictorial illustration representing

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ten hundreds of cash. The lower panel contains the following: 'The Imperial Board of Revenue having memorialized the Throne has received the Imperial sanction for the issue of government notes of the Ming Empire, to circulate on the same footing as standard cash. To counterfeit is death. The informant will receive 250 taels of silver and in addition the entire property of the criminal. Hung Wu....year....month...day.' A seal 3.25 inches square is impressed, once on the upper panel, once on the lower panel, bearing in square seal characters the legend 'The Seal of Government Note Administrators.' On the back of the note, above is impressed in vermilion a seal bearing in square seal characters the legend 'Seal for Circulating Government Notes'; below, within a border 6.2 by 4.1 inches is repeated the middle of the upper panel of the face — One *Kwan*, with a pictorial illustration representing ten hundreds of cash."

What was known about the Subject in the Fourteenth Century.

Such was the form of note in use in China after so many years of practical experience with paper money, a form it may be said which did not differ in any essential particular from that of the earliest notes which have been preserved, and which closely resembles that of our own greenbacks in its substantial features, even down to the recital of the law against counterfeiting. In the different chinese notes the names of the emperors and of the dynasties; of the denominational values; and of the rewards for information as to counterfeiters; are subject to change, but the general form was as we find it in the Ming note for one kwan.

Nonintercourse with China will not altogether explain why our ancestors were not able to profit by the experience of these orientals in the use of paper money, for during the 13th and 14th cen uries information concerning chinese paper money began to filter through. Incredulity and incapacity to comprehend what information was placed before the readers and students in Christendom, prevented them from taking advantage of what little was actually submitted to them. It is a matter of some interest to us in the examination of this subject to see what knowledge has been at different times during the past six hundred years at the command of one who would master the subject, and of what he can now avail himself.

The earliest mention of the chinese paper of early days to be found in European literature was made by a Franciscan friar named Rubruk [2]

2 Quoted by Vissering, "On Chinese Currency," p. 26. He gives as authority "Recueil de divers voyages curieux par P. Bergeron. Voyage de Rubruquis en Tartarie," p. 91. Leide, 1729.

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or Rubrouck who was in China a few years before Marco Polo and who simply said "The common money of Cathay consists of pieces of cotton-paper, about a palm in length and breadth, upon which certain lines are printed, resembling the seal of Mangu Khan." This mention anticipated by a few years the account given by Marco Polo who was in China for a period of about twenty-five years, covering practically the last quarter of the thirteenth century. Polo's description of the paper money which he then found in actual use in China as a medium of trade, was minute enough in detail and sufficiently suggestive in character to have aroused the financiers of Christendom to an appreciative sense of the possible uses of credit, had it reached the eyes of a people far enough advanced in knowledge of currency and banking to have comprehended what was thus laid before them. It must be remembered, however, that this description was made public in the days of Edward III, when banking as a profession was unknown, and at a time when students had still to wait about one hundred years before they should see a printed book. Authors then had to rely on manuscripts as the medium through which they could communicate with the public. Eighty-five of the Polo manuscripts which served the readers in those days, written in five languages, have been preserved. It is doubtless true that some have been destroyed, still it must be remembered that such manuscripts were in their day precious things, to be preserved with care, read and passed on. It is obvious that the number of manuscripts in circulation, which could have furnished the public information as to the conditions of life in China, and the number of persons who could have profited by their perusal were very limited. Moreover, it must be remembered that the stories of travellers like Marco Polo were received with incredulity, so that, even after it became possible through the invention of moveable types, for writers to reach a larger public by means of printed books, their influence was very slight. These facts adequately explain why Europe failed to benefit by the information furnished by Marco Polo. What might have been learned at that time concerning this subject, had the circumstances been favorable, is to be found in the following words, taken from Polo's account of his travels: [3]

"In this city of Kanbalu is the Mint of the grand Khan, who may truly be said to possess the secret of the alchemists, as he has the art of producing money by the following process. He causes the bark to be stripped from those mulberry-trees the leaves of which are used for feeding silk-worms, and takes

[3] The Travels of Marco Polo the Venetian, with an Introduction by John Masefield, London and New York (1907).

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from it that thin inner rind which lies between the coarser bark and the wood of the tree. This being steeped, and afterwards pounded in a mortar, until reduced to a pulp, is made into paper, resembling (in substance) that which is manufactured from cotton, but quite black. When ready for use, he has it cut into pieces of money of different sizes, nearly square, but somewhat longer than they are wide. Of these, the smallest pass for a denier tournois; the next size for a Venetian silver groat; others for two, five, and ten groats; others for one, two, three, and as far as ten besants of gold. The coinage of this paper money is authenticated with as much form and ceremony as if it were actually of pure gold or silver; for to each note a number of officers, specially appointed, not only subscribe their names, but affix their signets also; and when this has been regularly done by the whole of them, the principal officer, deputed by his majesty, having dipped into vermilion the royal seal committed to his custody, stamps with it the piece of paper, so that the form of the seal tinged with the vermilion remains impressed upon it, by which it receives full authenticity as current money, and the act of counterfeiting it is punished as a capital offence. When thus coined in large quantities, this paper currency is circulated in every part of the grand Khan's dominions; nor dares any person, at the peril of his life, refuse to accept it in payment. All his subjects receive it without hesitation, because, wherever their business may call them, they can dispose of it again in the purchase of merchandise they may have occasion for; such as pearls, jewels, gold, or silver. With it, in short, every article may be procured."

Farther on Polo says:

"When any persons happen to be possessed of paper money which from long use has become damaged, they carry it to the mint, where, upon the payment of only three per cent, they may receive fresh notes in exchange. Should any be desirous of procuring gold or silver for the purposes of manufacture, such as of drinking-cups, girdles, or other articles wrought of these metals, they in like manner apply at the mint, and for their paper obtain the bullion they require. All his majesty's armies are paid with this currency, which is to them of the same value as if it were gold or silver. Upon these grounds, it may certainly be affirmed that the grand Khan has a more extensive command of treasure than any other sovereign in the universe." [4]

The corroboration by Sir John de Mandeville, of the fact that paper money was in use in China at that time, will not perhaps be accepted as testimony of much value, but inasmuch as his account of travel is supposed to contain much that was appropriated from the stories of

[4] As will appear later, the whereabouts of existing examples of these Mongol notes are unknown. I am indebted to Mr. Worthington C. Ford for photostats of some of these latter from illustrations in Chuan Pu Tung Chih, a Chinese numismatical work.

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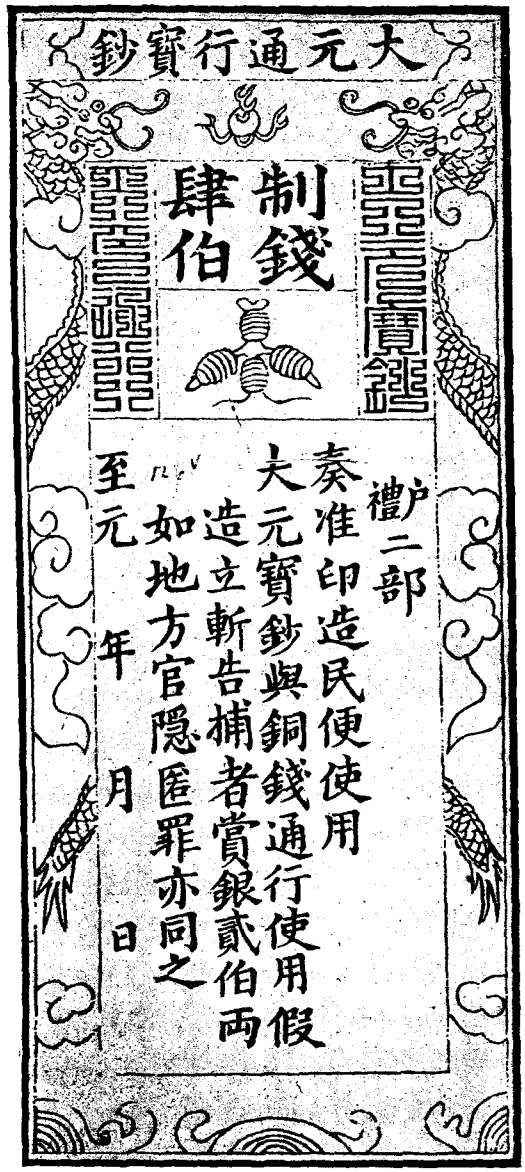

Four hundred wen note of the Yüan Dynasty. Emperor, She Tsu; Period, Che Yüan; A.D. 1264–1295. This form of note was for current use and represents the currency described by Marco Polo. Reproduced from a photostat, taken by the Massachusetts Historical Society, from the original illustration in *Ch'üan Pu T'ung Chih*. The size of the original is 8 by 3½ inches. The government seals are given separately.

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other writers, it may be that it will have some weight. He simply says that the emperor did not depend upon money but on "Lether emprented" or on "Papyre."

An Arabian traveller named Ibn Batuta covered a part of the ground of Polo's travels about fifty years after Polo, and left behind him an account of the chinese paper currency. This was written in Arabic and was not so full as that of Polo, but it contained much that might have stimulated financiers in the use of credit, if they could have read it. Nothing was known in Europe of this manuscript, unless the Moors in Spain knew about it, until the early part of the nineteenth century, and so far as European financiers are concerned, it cannot be said, that they were so situated as to have been subject to its influence. Nevertheless his story is well worthy of examination.

Like Marco Polo, Batuta was astonished to see pieces of paper used in place of coins and quaintly expressed himself in the following language: [5]

"Their transactions are carried on with paper:.... As to the paper every piece of it is in extent about the measure of the palm of the hand, and is stamped with the King's stamp. Five and twenty of such notes are termed a "shat," which means the same thing as the dinar with us. But when the papers happen to be torn, or worn out by use, they are carried to their house, which is just like a mint with us, and new ones are given by the King. This is done without interest, the profit arising from the circulation accruing to the King. When any one goes to the market with a dinar or a dirhem in his hand, no one will take it until it has been changed for their notes."

It will be remembered that Marco Polo in speaking of these notes had described the manner in which the fabric upon which they were printed was manufactured; had stated that the color of the paper was black, an error probably of the translator, since the actual color was dark gray; had compared the office from which they were issued to a mint; had spoken of the official seal which was placed upon them; had described the method of securing new notes in place of those which were worn and unfit for use, stating in this connection that a charge of three per cent was made by the government; had given the values of the different denominations of the notes; had alleged that they were of different sizes, the smaller the denominational value of the note the smaller the size; had stated that there was a law against

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5 The Travels of Ibn Batuta by The Rev. Samuel Lee. B. D. London, p. 209 (1829).

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counterfeiting these notes and finally had asserted that their circulation was compulsory.

It is somewhat singular that Ibn Batuta should have touched upon six of these nine points in Polo's description, omitting only the method of manufacture of the fabric of the notes, which he merely mentions as paper without stating the color, and omitting also any reference to the edict against counterfeiting, as well as any account of the compulsory nature of the circulation; although he does say that a traveller was obliged to convert his foreign coin into notes in order to make use of it in the market. He speaks of only one size of notes and those the small notes not larger than the palm of the hand. This omission may be explained by the fact that when he was there, Kublai Khan was no longer on the throne. He had been succeeded by Ch'ĭng Tsung, and the emissions during the reign of the latter were, we have reason to suppose, uniform in size.

Both Marco Polo and Ibn Batuta were in China at a time when the notes were greatly depreciated. Some writers have expressed wonder at their not being impressed with the fact of this depreciation. There is no occasion for wonder at this. The travellers were unfamiliar with paper money and were surrounded by conditions of life never before seen or heard of by them, but even had they been familiar with the currency and had they possessed such knowledge of prices as would have enabled them to measure the reduction of the purchasing power of the currency, they would probably have been impressed only with the high prices and would not have been likely to attribute them to the degradation of the currency, precisely as we see our government to-day investigating the question of high prices and simultaneously emitting emergency currency — thus adding fuel to one of the causes of high prices.

What was known in the Eighteenth Century.

The next allusion in European literature to the old chinese notes did not come from one who had seen them in circulation, nor did it contain within itself any description of the notes or of their current use, which would make it of value to us, unless an allusion to the abandonment of paper currency should prove of assistance in fixing the date of that event. What was of value, however, in this connection, was an illustration giving the inscriptions on the one kwan Ming notes, 1368–1398, with a translation. The work containing this

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allusion was put forth in Paris in 1735 by Jean Baptiste Du Halde, a Jesuit father. It consisted of a collation, made up from the *relations* of the Jesuit missionaries who from China sent to the central office of the order, observations on the geography, the zoölogy, the botany of the country, and also on the manners, the customs and the life of the people. At what date the *relation* accompanying the drawing of the note was sent does not appear, but it seems probable that it must have been in the latter part of the seventeenth century, or possibly early in the eighteenth. When Du Halde's description of China appeared, the financiers of Europe were no longer ignorant of the power of credit and of the dangers of its use in the form of paper money. France had just suffered from the miseries entailed by Law's Mississippi scheme and England had but just emerged from the troubles caused by the South Sea bubble. In Massachusetts Bay, paper money, after nearly fifty years of use, was still the only medium of trade in the Province. The value of Du Halde's book was recognized in England and a translation with some abridgments was published there as early as 1738. The brief account of the currency given by Du Halde was preserved and appears in the translation in the following words: [6]

"In the beginning of the reign of Hung-vû, Founder of the twenty-second dynasty, called Ming .... Money was become so very scarce that they paid the *Mandarins* and soldiers partly in Paper; giving them a sheet of paper sealed with an Imperial Seal, which passed for a thousand little copper pieces, or a tael of silver. Those sheets are much sought after by such as build, who hang them up as a rarity to the chief beam of the house, the people and even of the quality, being so as to imagine [sic] that it preserves them from all misfortunes."

Du Halde then goes on to state that this paper currency was abandoned by the same emperor who inaugurated or, perhaps I ought to say, revived it.

The works of a chinese author which have been made use of by writers on the topic of chinese paper money, the characters in the title of which have been anglicized as "*Tung Keen Kang Muh*" and translated, "Condensation of the Mirror of History," were rendered into French in 1779–1783 by Father Joseph de Mailla.[7] There was no

[6] A Description of the Empire of China, etc., from the French of P. J. Du Halde. **1**, p. 332 (1738).

[7] Histoire Generale de la Chine ou annales de cet Empire, traduites du Tong-Kieng-Kang-Mou. Paris (1779–1783).

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special reason why economists should turn their attention to the pages of Du Halde or to those of de Mailla, but it cannot be denied that there was information at their command in both these publications, and probably from time to time readers were stimulated by it to further research.

For a period of about forty years after this the subject does not appear to have excited either orientalists or economists. At any rate there are no contributions on the subject in the literature of the day during this period which have attracted attention enough to cause inclusion in this review of the authorities who have contributed to knowledge or stirred up interest in the subject. When next the matter is taken up, China is no longer a mysterious land shut off from the rest of the world. Students have mastered to some extent the difficulties which the language interposes to prevent free interchange of thought between the chinese and foreigners, and knowledge has been acquired that there is a vast amount of historical information treasured in the pages of chinese books on the shelves of their great libraries. The time has come when sinologues can make topical investigations of matters strictly chinese.

The Subject Fairly Opened Up.

The first approach to the subject of chinese paper money in a truly scientific spirit was made by Heinrich Julius Klaproth, a German oriental scholar and traveller, who published an article in the Paris "Journal Asiatique," in November, 1822, in which he furnished the European world with a brief account of the chinese paper money experience.

This article was translated and published in the Journal of the American Oriental Society,[8] by John Pickering, one of the founders of that Society and at one time President of the American Academy of Arts and Sciences, the matter by this publication being brought to the attention of American students.

In 1824, Klaproth published a little volume entitled "Mémoires relatifs a l'Asie," in which he incorporated his article on the paper currency. The subject was there treated in a general way and necessarily with great brevity, covering the dates from 807 A.D. down to

[8] Vol. I., p. 136, *et seq.*

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1455 A.D. This research has proved of great assistance to subsequent students and has been much cited and quoted. It is based upon three chinese works, historical in character and covering so much ground that a consecutive story of the experience in the use of paper money was not reasonably to be expected to be found in their pages. What they did disclose was, however, of enormous importance and of great value.

Two of the chinese books referred to by Klaproth are in the New York Public Library. Their titles indicate the broad field which they seek to cover. One of them has been translated by some "Antiquarian Researches," by others "General examination of Records and Scholars." The work was originally published in 1322 and afterwards continued in a supplement, in 1586.[9] The original and supplement are bound in twenty-three thick volumes and in the opinion of Mr. Wilberforce Eames, to whose courtesy I am indebted for the information herein given concerning these chinese books, they would if translated into some European language make about fifty or sixty volumes of the size of the Dictionary of National Biography. They constitute according to Mr. Eames, an encyclopaedia of every subject connected with the government, history, statistics, literature, religion etc., of the Chinese Empire.

The title of the other chinese work referred to as being in the New York Public Library is generally given as "Condensation of the Mirror of History."[10] It was originally written in the twelfth century and was afterwards continued to 1476 A.D. It was afterwards brought down to 1644, A.D., and was reprinted in the early part of the 19th century. It is bound in 24 volumes. The third work referred to by Klaproth is a literary and scientific encyclopaedia.[11] An edition in the British Museum bears date 1642.

Klaproth was the pioneer among topical investigators of chinese paper money. The conditions under which he made his research were such that had he not been attracted by the subject the opportunity afforded for work of this sort must soon have welcomed some other student. China no longer kept her doors closed against the entrance of foreigners. Sinologues were at work endeavoring to master the obscurities of the chinese language, and knowledge was being spread of the historical treasures hidden in the pages of the voluminous chinese

9 Wan Heen Tung Kaou, by Ma-twan-lin. The supplement was compiled by Wang Ke.
10 Tung Keen Kang Muh.
11 Keun Shoo Pe Kao.

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works which filled the shelves of their great libraries. If it had not been Klaproth it must soon have been somebody else.

In 1837, Edouard Biot, published a series of papers, or rather a single paper which was scattered through several numbers of the "Journal Asiatique." [12] It was based mainly on information derived from the chinese work by Ma-twan-lin, one of the authorities made use of by Klaproth, and although an interesting contribution towards knowledge of the subject, Biot's paper did not contain much that was new.

In 1842, Baron S. de Chaudoir published at St. Petersburg, a numismatical work on the coins of the Orient.[13] This volume which is folio in size, is illustrated by sixty plates on which are depicted upwards of one thousand coins. Fourteen pages are given to a history of the paper-money of China, and engravings of the face and back of a one kwan Ming note are to be found in the plates. Statistics as to the amount of paper-money which had been emitted at certain specified dates are given by Klaproth and Biot. Chaudoir adds to these a table of the annual emissions of Mongol notes from 1260 to 1330. Chaudoir relied upon an interpreter for the translation of the chinese works which he cited or quoted, among which that of Ma-twan-lin was conspicuous. His work is a distinct addition to the knowledge on the subject acquired by students up to that time, one might almost say up to the present time.

In 1847, Robert Montgomery Martin, published a work in two volumes, entitled "China, political, commercial and social," etc. In the first volume on pages 174 and 175, he gives a perfunctory account of chinese paper money, based apparently on Klaproth.

In 1848, Frederick Edwyn Forbes published "Five Years in China," in which he incorporated a brief, unsatisfactory description of the chinese paper money episode.

The chronological position of the publications of Martin and Forbes, entitles them to notice, even though their contributions to knowledge upon the subject were not of much value, for at the time when they wrote, English writers had not taken up the subject and whatever was contributed by those gentlemen, no matter how small the amount, was a challenge to the attention of students.

Henry Dunning MacLeod in his Dictionary of Political Economy,

12 Memoire sur le Systême Monetaire des Chinoise."

13 Recueil de monnaies de la Chine, du Japan, de la Corée, d'Anam et de Jave au nombre de plus de mille, Précédé d'une introduction historique sur les monnaies. St. Petersbourg (1842).

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London, 1863, under the titles of Banking and of Currency, deals with chinese paper money. He relies upon Klaproth and Biot, and under "Currency" gives a chronological narrative of the events connected with the episode. He says: "We have given this account of chinese paper money because we are not aware that any account of it has ever been published in English." Had the dictionary been completed this account might have attracted more attention, but as it was made public in an incomplete work and was hidden under the title "currency," it made little or no impression.

MacLeod also refers to chinese paper money in his Theory and Practice of Banking.

In 1873, in the Dictionnaire de l'Économic Politique, Courcelle Seneuil describes the chinese paper money experience at some length, under the title "Papier-Monnaie. He relies on Biot, and while he gives an account of the "Deer Skin Money," he adds in a note "ces morceaux de peau ne constituaient par, à proprement parler, un papier monnaie."

In 1874, A. M. Benadakis, communicated a paper on this subject to the Journal des Economistes etc. He quotes freely from the chinese work of Ma-twan-lin which has already been referred to several times, and speaks of the writer as "The learned Ma-twan-lin, author of an encyclopedia of 100 volumes" [14]— and adds an estimate of the value of his work, "He knows the true theory of money and of paper, and what he has written is striking in its precision and good sense."

In 1875, William Stanley Jevons in "Money and the Mechanism of Exchange gives a brief account of chinese paper money. He relies upon Benadakis and Courcelle Seneuil.

In 1877, W. Vissering, published at Leyden, a book on chinese coins and paper money.[15] The writer has incorporated in his publication, the more important sections of Ma-twan-lin's celebrated work bearing on this special subject. The chinese characters of the original publication are given by Vissering in connection with a translation, and this again is accompanied by foot-notes explanatory of the special meanings which he has given to some of these characters. The first paper money that Vissering finds was in use in 809 A.D. He stated what knowledge the chinese author furnishes on this and other emis-

14 Journal des Economistes, etc. March, pp. 366, 367 (1874).

15 On Chinese Currency, Coins and Paper money. W. Vissering. Leide, (1877).

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sions and also gives the alleged results arising from the emissions, down to about 1210 A.D. He does not deal with the continuation of the chinese work which was published in 1586.

In 1885, Alexander Del Mar briefly described chinese paper money in his History of Money in Ancient Countries. Klaproth was his main authority.

Topical Publications in the Orient.

In 1889, Shioda Saburo published a paper on chinese paper money in the Journal of the Peking Oriental Society, a contribution presumably made at a meeting of that society.[16] Saburo was from Japan and evidently attacked the subject with indefatigable industry. His obvious lack of knowledge as to the economic situation produced by the fluctuation of the quantity of currency in circulation impairs the value of his paper. It is, however, a learned dissertation notwithstanding much obvious confusion of thought on the part of the writer. The paper is founded upon translations from chinese authors but it is difficult to separate the expressions of opinion of the writer from those which he attributes to the authors whom he quotes.

In 1905, Joseph Edkins, a missionary to China, published a book which was devoted to the development from chinese sources of methods of revenue and taxation in the past in China; and incidentally to the story of the use of paper money, so far as that story was told by the chinese authors made use of by the writer.[17]

During a long life spent in China, Edkins had acquired a prodigious amount of knowledge of the chinese language and also of its literature, and had published numerous books on different subjects connected with chinese life and history mainly, however, devoted to philological topics. His "Banking and Prices" has, scattered through its pages in a desultory way, substantially the same knowledge concerning paper money as had already been furnished by Chaudoir, with a little additional information, but all of it buried under a mass of statements about other subjects, which make the matter relating especially to paper money difficult of discovery. A writer who made use of "Bank-

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16 The origin of the Paper Currency of China. Journal of the Peking Oriental Society, **3**, No. 4. I am indebted to Mr. Edward B. Drew for calling my attention to this paper.

17 Banking and Prices in China, by J. Edkins, D.D. Shanghai, printed at the Presbyterian Mission Press (1905).

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ing and Prices in China" characterized the book in the following words: "A rich mass of conglomerate for the patient student." The book has a distinct value, for it is the work of a learned sinologue, but the confusion in the arrangement of its contents and the difficulty experienced in determining whether one is meeting the opinions of some chinese historian or those of Joseph Edkins, tend greatly to obscure its merits. It is not to be found in the Harvard Library, or the Boston Public Library.

In 1907, Mr. H. B. Morse, a member of the staff of Sir Robert Hart, published in the Journal of the North China Branch of the Royal Asiatic Society,[18] a communication dealing with the currency question including paper money. This was illustrated with a lithographic reproduction of a one kwan Ming note, 1368–1398, in colors, and two engravings of government notes emitted during the Tae-Ping rebellion which are also printed in colors. This article was incorporated in Morse's book on the trade and administration of China, which appeared in 1908.[19] Morse relied largely on Klaproth and Edkins. It is from his book that the translation already given of the inscriptions on a Ming note was taken. If it may be said of Klaproth that he brought the subject of chinese paper money within easy reach of students, the claim may be advanced for Morse that his book was the first that actually brought the matter to public notice.

In 1911, H. A. Ramsden published in Yokohama, a pamphlet thirty-seven pages in length entitled, "Chinese paper money." It is in the nature of a numismatical manual and contains descriptions of a great number of these old chinese notes, sufficiently definite to enable a collector to identify a specimen which has come into his hands, provided it belongs to one of the emissions thus described. No information is furnished as to the language used in the inscriptions on the face of the notes. Ramsden acknowledges that he is indebted to a chinese work for much valuable information. A copy of this work,[20] which is an elaborate numismatical treatise profusely illustrated, with representations of coins and occasionally of a note, is to be found in the Essex Institute, at Salem. Examination shows that of these old notes, there are given in this publication eighty-one representations

18 Journal, &c., **38**, pp. 17–31.

19 The Trade and Administration of the Chinese Empire, by H. B. Morse. London (1908).

20 Chuan Pu Tung Chih or General account of money and coins — given by Ramsden as C'hien Pu Tung C'hih.

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or pictures, furnishing all the details of the inscriptions and drawings on the face of the notes as fully as an actual impression from the wood-cut or stereotype of the note itself would have done. What is perhaps more remarkable is that the inscriptions on the notes and their general arrangement, while not absolutely uniform from dynasty to dynasty, nevertheless correspond so closely with each other and with that of the Ming note, the inscriptions on which were translated by Morse, that one might almost conceive that many of the notes would have circulated in the days of the Ming dynasty without attracting much attention. The legends on the face of the notes are said to be legible with ease by any person who can read modern chinese characters and those inscriptions which are written in seal characters yield readily to the concentration of a little effort on the part of the student who is at all familiar with the various seal characters. The oldest among the notes delineated in this work was emitted in 650 A.D., about one hundred and seventy-five years before the quarrelsome bands of Anglo Saxons who constituted the Heptarchy were brought under one control and organized into the government which has become known as England. It seems incredible that at a time when English History as such had not yet begun, the chinese should have developed the manufacture of paper and cultivated the art of block printing, making use of characters which have persisted to this day. Yet that is what is revealed by the representation of the currency of this period to be found in this chinese numismatical work. My friend, Mr. Edward B. Drew, formerly a Commissioner of Customs on the staff of Sir Robert Hart, to whom I am deeply indebted for his patient examination of the inscriptions on these notes, at my instance scrutinized the date of the notes of this early emission carefully and he says there can be no doubt that the characters which fix the date of the issue actually represent a period covering the years 650–656 A. D. Ramsden hesitates to endorse the authenticity of this emission. He gives it place because the author or compiler of the chinese work above referred to claims that it was the first in use, but says in that connection, "I would like to add, however, that no other authority seems to share this contention, which is unsupported by historical or other data." As to this emission being the first — an examination of the inscriptions on the face of the notes will disclose the penalty for counterfeiting and the offer of a reward to whomsoever shall furnish information which will lead to the arrest of the counterfeiter. It is not probable that these clauses could have been incorporated in the inscriptions on the notes of the first emission. It would seem as

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Ten kwan note of the Tang Dynasty. Period, Yung Hui; A.D. 650–656. Reproduced from a photostat, taken by the Massachusetts Historical Society, from the original illustration in *Ch'üan Pu T'ung Chih*. The size of the original is 9 by 5⅞ inches. The government seal is given separately in *Ch'üan Pu T'ung Chih*.

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though the desirability of holding the threat of this penalty over the heads of evil-doers must have resulted from experience. If then we accept these illustrations in the chinese work, as representations of notes actually emitted about 650 A.D. it would follow that it is quite probable that there were previous issues.

On the point of there being no other authority to share the contention, the question may be asked, Where did the author procure his picture? It is not conceivable that the compiler of the numismatical work deliberately manufactured the design. If he did not, then his picture must represent an actual note that he has seen, or it must have been appropriated from some other numismatical work, the latter being the more probable supposition.

A sketch of the history of chinese paper money appeared in the North China Herald (Weekly Edition) December 21, 1912. It was dated at Ching Kiang and was signed T. M. Bowern. It covered the period from the middle of the twelfth century down to the first half of the fifteenth and was obviously based on Klaproth and Morse. Mr. Bowern traces the ownership of a number of these one kwan Ming notes into the hands of different individuals. There is also some contributory information on this subject in the issue of December 14, 1912, of the same paper.

At this point the question of what sources of information have been at given times in the past and what now are at command of a student, may be abandoned. The matter has been followed down to 1912 and has brought to light Ramsden's manual, through which we learn that he has recorded the details of two hundred and sixty-nine varieties of notes with sufficient definiteness to permit of identification and classification. Scattered references to emissions are to be found on the pages of some of the works to which reference has been made. It may be said that we know something about at least three hundred varieties of these notes, and that there are still chronological gaps to be filled.

What Oriental Scholars know about existing Specimens of the Notes.

Up to this point no special effort has been made to indicate where a person interested can find specimens of these notes and what examples he can find at any given point. We know that chinese collectors have long treasured them and European residents in China agree that speci-

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mens are to be found in the hands of these collectors. This knowledge does not advance us much in our pursuit of information which can be made use of by an investigator who is not in the Orient and who does not speak or read chinese. We can, however, turn to the pages of those who have written on the subject and especially to those works which have been illustrated by engravings or photographic pictures of notes. In some instances we shall find that the author has furnished a translation of the inscriptions or legends on the notes depicted in his works. This examination may not reveal much that is of value, but it must be made if we would cover the ground thoroughly.

The oldest representation of a chinese note, excluding of course from this statement all illustrated chinese numismatical works, is to be found in Du Halde. It is a line engraving, giving the outline of the panelling of the face and back of a one kwan Ming note of the Hung Wu period, without the pictorial representation of the ten strings and without the ornamental decorations of the border. The inscriptions are however reproduced whether horizontal or vertical, whether in ordinary or square seal characters, precisely as in the original. The notes of this emission apparently had three government seals impressed on them, two on the face and one on the back. The engraving, in du Halde, gives no indication that there was any seal either on the face or on the back of the notes, but in Du Halde's text it will be remembered that he spoke of payment being made to mandarins and to soldiers by giving them a piece of paper sealed with the Imperial seal. He therefore furnishes a detached picture of the government seal without stating that it was stamped upon the note. The back of the note is also represented showing the panelling, but as in the picture of the face of the same, the panel, which should contain the ten strings of cash, is vacant. A translation of the inscriptions is furnished showing phonetically in our alphabet what the chinese words are and also giving the French equivalents. What Du Halde says about the rarity of these notes and about their being treasured and hung up in the houses in China has already been stated. It depends somewhat upon the date of the *relations* which treated of this note, what the value of this observation amounts to. The Manchus succeeded the Mings about the middle of the seventeenth century. If at that date these notes were thus treasured, it would tend to confirm those who assert that the abandonment of paper money took place under the Ming dynasty and not at the accession of the Manchus. Any evidence that tends to determine the exact time when paper money was abandoned and specie resumed is welcome

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Chaudoir is authority for the statement that in 1842 there were three one kwan notes of the Ming dynasty in The Imperial Academy of Science at St. Petersburg, two in the museum and one in the cabinet. He gave on his plates an engraved representation of one of these with a single impression of the government seal in vermilion on the lower panel. The picture of the ten strings of cash was given in place on the face of the note, but the figure of the panel on the back of the note which ought to contain these strings is left vacant. The decoration of the border of the note is floral in character there being nothing in the pattern that would at first sight suggest the dragon patterns. The peculiarities of these particular representations are the single seal on the note and the lack of the picture of the ten strings on the back. The floral pattern for the decoration of the border is also unusual.

It is undoubtedly the case that the artists who have undertaken to furnish complete and perfect copies of these notes have labored under great disadvantages and have been obliged to make use of the imagination to fill in existing vacancies. The paper of which the notes are made has little or no sizing, has never been calendered and is more like a felt than a paper. On such a surface as this the impression from the wood cut or stereotype plate of the bold chinese characters would be more distinct than the delicate ornamentation of the borders and would prove to be more lasting when the note was subjected to use. Such also would be the relative wearing capacity of the vermilion government seals, especially when superimposed. The red ink did not seem to have the same permanency of form as the black. It preserved its color indefinitely, but apparently did not adhere closely to the paper. On some of the badly worn specimens which have been preserved there is scarce a trace of the red seals left. It may be that the note which is delineated in Chaudoir originally had more than one seal and it is probable that the engraver must have called largely upon his imagination to restore the single seal that he gives, and to depict the perfect ornamentation of the floral border.

Chaudoir makes the following statement relative to the notes which have been preserved: "It seems that these notes, namely: those of 1000 cash, are the only ones that have come down to us, for according to the statement of persons attached to the Russian Mission at Peking, they do not know of any other even in China."

Col. Henry Yule in his edition of Polo's travels,[21] published in

21 The Book of Marco Polo, etc. London, Note to Ch. XXIV, 1, p. 414 (1875).

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1875, gives a reduced photographic picture of the face of one of these notes, bearing two seals and adds that there was difficulty in making a legible copy of the inscriptions on the seals. He asserts that he has "never heard of the preservation of any note of the Mongols," but adds that "some of the Ming survive and are highly valued as curiosities in China." He procured his picture at St. Petersburg.

Vissering in his work on chinese currency gives a photographic process picture of the face of a one kwan Ming note and also furnishes, in black ink, drawings of the two red seals which originally were to be found there. He confesses that the work of the engraver in constructing a representation of the impression of the seals has not been without its difficulties but he believes that the pictures of the impressions are substantially correct. Vissering also went to St. Petersburg for his note.

Edkins says that collectors obtain notes of the Mongol dynasty from Japan and adds:[22] "Notes are found in the possession of chinese men of wealth. It is a rare thing to see them in shops. There is no fixed price for those curious relics of the 13th and 14th centuries." The statement that Mongol notes are obtained from Japan must pass for what it is worth. There is no evidence of its truth at hand to-day. It is curious, however, to note the time limitation which he puts when he speaks of the prices of the notes. He absolutely excludes from consideration all the earlier emissions.

T. Dyer Ball, a member of the civil service at Hong Kong, a resident in China for upwards of forty years and the author of several books on China, conceived the idea some years since of publishing a chinese common-place book, in which under alphabetical arrangement, information was furnished concerning selected topics touching on life in China.[23] Under the title "Banks and Bank Notes," he tells what he knows concerning old and new chinese notes. He says: "The earliest specimen known to exist in any country was purchased in 1890 by the British Museum, where it may be seen in the King's Library placed under a glass case."

H. B. Morse, writing in 1908, said: [24] "I have been unable to obtain a copy of a Mongol government note. . . . . I give, however, a reduced representation of a note for 1000 cash, issued by the first Ming Emperor, (Hung Wu, A.D., 1368–1398), who may be assumed to have followed closely the procedure and copied the forms of his predecessor."

22 Banking and Prices in China, p. 232.
23 Things Chinese, fourth Edition, p. 79 (1906).
24 The Trade and Administration of the Chinese Empire, p. 141.

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In the "Imprint" (a publication of the American Bank-Note Co.) for November, 1908, there is a photographic process picture of a chinese government note for one kwan. The note is there described as a "Chinese Bank Note of the Ming Dynasty about 1400."

The numismatic manual published in 1911, by H. A. Ramsden, President of the Yokohama Numismatical Society, was illustrated with a flawless picture of a one kwan Ming note, every detail of the ornamental border being represented in absolutely perfect condition. Mr. Ramsden says with reference to this, "The reduced reproduction figuring in Plate I of this Manual, was taken from a clear and genuine specimen in my own collection and is consequently a true and faithful copy of the original Notes themselves." This is probably true, so far as the impression from the plate goes, but it must be remarked that there is not a sign in the picture of the government seals.

The Numismatist, a New York illustrated monthly devoted to the science of Numismatics, gives a process picture of the American Bank Note Company's note in Vol. XXV, No. 5, May, 1912.

This review of the information to be culled from the works of the most conspicuous writers on the subject of chinese paper money as to what they know about existing specimens has resulted in showing that the one kwan note of the Ming dynasty, and of the period of Hung Wu, is the only note made use of by these authors to illustrate their works, and further has disclosed the fact that several of these writers have frankly declared that so far as they knew, no other of these old chinese notes has been preserved. Ramsden intimates that it would not be difficult to make a collection of chinese notes, but when it comes to illustrating his Manual he makes use of a one kwan Ming note of the Hung Wu period.

The only writer other than chinese who has furnished any pictorial illustration of any other form of note than the one above mentioned is Dr. S. W. Bushell,[25] who has given us through the publications of the Peking Oriental Society, first, a representation of a fragment of a wood-cut for a note of the second half of the twelfth century, second, the full text of the inscriptions on a note issued in 1214, and finally the picture of what appears to be the stereotype plate of a two hundred cash note of the Ming dynasty, Hung Wu period. Dr. Bushell asserts that this plate is an actual coin, and that the inscription on its face shows that it belongs to the period of T'sung Chên and to the

25 In a communication published in the Journal of the Peking Oriental Society. **3**, No. 4, pp. 308–312.

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year 1639. If the Doctor is correct, the value attached to the dates on the face of these notes, to which implicit faith has been given as indicating the period of their emission, will be greatly diminished, for here we have what seems in the picture to be a Hung Wu note, which is said to bear an inscription of a later date. Whatever it is, whether note or coin, the illustration given by Dr. Bushell, in no way contravenes or limits what was said about the one kwan notes of that particular period being the only existing notes at command of illustrators, since Dr. Bushell says that he extracted what he furnishes concerning the Ming note from a chinese work, the *Chi Chin So Chien Lu*, and further the text of the note of 1214 given by the Doctor is also taken from a chinese book.

Twelve Old Chinese Notes and Four of the Tae Ping Rebellion Period.

I have already stated the circumstances connected with my acquisition of the first of these chinese notes. This particular note, it will be remembered, was described as "mounted on limp board with embroidered work at back." My examination of the authorities treating of chinese notes disclosed to me the meaning of this mounting, which was all the more plain when, on receiving the note, I found that the "embroidered work at back" proved to be chinese brocade. It was evident that I had received one of those notes of which the Jesuit fathers wrote, according to Du Halde, that they are hung up "as a rarity to the chief beam of the house, the people and even of the quality being so as to imagine that it preserves them from all mischief." Notwithstanding the loss of opportunity to inspect the back of the note through the mounting on limp board, the added interest given to the specimen from the fact that it had actually hung upon the walls of a chinese house, possibly for two or three centuries, was more than a compensation.

My examination of the authorities who had written upon the subject of chinese paper money had shown me that government notes had already been in use certainly between five and six hundred years, and perhaps longer, when the specimen that was in my possession was emitted. Concerning existing specimens I had ascertained that outside the collections of chinese men of means, there were none other known than the one kwan Ming note emitted somewhere about 1375.

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Moreover, it was plain from the pains taken by the North China Herald to trace the ownership of specimens of those notes, that even in China, it was a matter of interest, among the English residents, to follow up the destination of such specimens, in the evident belief that they were the only existing representatives of the great paper money experience of China in early days, which were likely to be found outside chinese collections.

In the year 1912, my interest was greatly stimulated by receiving by mail a cutting from the Sunday New York Sun of an article on chinese paper money, which was illustrated by a picture of a one kwan Ming note, having the legend on the vertical panels enclosing the characters which state the denominational value of the note and also enclosing the pictorial representation of the ten strings of cash, written in ordinary chinese instead of square seal characters. My efforts to ascertain whether this was a picture of an actual note or if it was from a drawing, in which the artist had substituted a translation for the ancient seal characters proved fruitless. I was, however, told from the Sun office that the original came from and was returned to the American Bank Note Company. The photograph of the note in possession of that company shows that their note does not differ in any way from the ordinary one kwan Ming note described by Morse; depicted by Chaudoir; a sketch of which is given by Du Halde; and of which so many photographic copies can readily be obtained. The probability that the substitution of ordinary characters for square seal characters in this inscription was the work of a draughtsman and that there was no note bearing the inscription in ordinary characters is strongly reinforced by the fact that Chaudoir says that "in all posterior emissions" — that is, after the one kwan note was put forth by Hung Wu — the form of that note was preserved, "for," says he, "the minister of finance having asked the Emperor Tching Tsou in 1403 to change the form of the eight tchouan characters [the square seal] could not obtain permission."

In the summer of 1914, I met at York Harbor, an English gentleman, Mr. James Orange, who had spent many years in China, who had collaborated with a friend in the publication of an illustrated work on certain specimens of chinese porcelain and who himself was evidently an authority on matters which interest the ordinary collector. My interest being keen on the subject of the notes, I asked him if he had any. He told me that he had not, but that a friend of his in London, Mr. A. W. Bahr, had a number, some of which were very old. Mr. Bahr had himself lived many years in China, was in 1910

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Hon. Secretary of the North China Branch of the Royal Asiatic Society, and had in 1911 published a volume entitled "Old Chinese Porcelain and Works of Art in China," a work which was beautifully illustrated and which was in substance a catalogue of an exhibition held in Shanghai, November, 1908. In his preface to this work Mr. Bahr acknowledges his indebtedness to Mr. James Orange for aid in running the volume through the press.

It will readily be understood that the mere gain of knowledge where some of these notes were lodged, was an advance for me and that I gladly availed myself of the opportunity thus afforded to secure through Mr. Orange's intermediation some knowledge of what the notes owned by Mr. Bahr actually were. On his part Mr. Orange readily undertook the correspondence with the result that he afterward furnished me with a list of the notes in Mr. Bahr's possession and ultimately offered them to me at a price which he regarded as reasonable. Having full confidence in both of these gentlemen I purchased the notes.

In the course of my correspondence with Mr. Orange he quoted from Mr. Bahr's letter to him as follows:

"I may remark that the two Tang notes are very rare and scarce. I know of no others in existence; as well as the one of the West Liao Tartar dynasty. For anyone interested in these things they will realize it is a unique opportunity..... I guarantee them to be absolutely authentic and genuine. You will remember the paper of the Tang notes is quite light in colour, soft to the touch, and as a paper perfectly made; the Sung and Liao are also of nearly a similar quality, but have been darkened by age, or they may originally have been of a grayish colour. Later ones are of a stiffer paper and more resembling the Chinese paper used now; for instance, the earliest notes can be crumpled into a ball in the hand and hardly a crease found when stretched."

Concerning Mr. Bahr's capacity to judge of the notes, Mr. Orange wrote, "I consider Mr. Bahr one of the best, if not *the* [best], experts on chinese matters, speaking and reading the language perfectly, he has the power of acquiring which others must lack."

Mr. Bahr himself in writing to me said, "I fully guarantee these eighteen notes to be quite authentic as to the periods they are described, as I have had good opportunities during my thirty-two years' residence in China to differentiate the right from the wrong ones....."

Thinking that there might possibly be something of interest in the story of the acquisition of the notes in China, I wrote Mr. Bahr asking him to tell me where and how he happened upon them. He replied:

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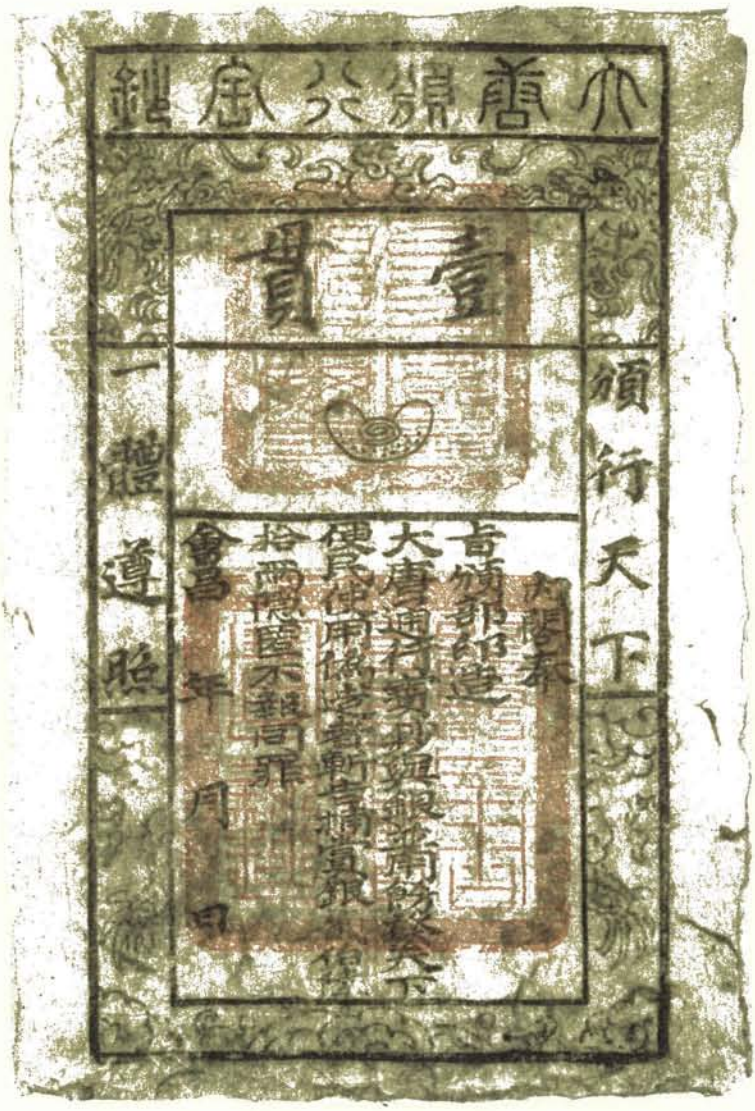

One kwan note of the Tang Dynasty. Emperor, Wu Tsung; Period, Hwei Ch'ang; A.D. 841–847. Reproduced from the original, the paper of which measures 9¾ by 6¼ inches; the impression, 9¼ by 5¼ inches. The red seals are not photographically accurate.

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"Regarding how these Notes came into my possession, I am sorry to say that it was not in a very romantic manner that they were secured, as they were purchased by a dealer in antique coins, etc. in Shanghai from a family at Soo-Chow (a provincial city about 200 miles from Shanghai) who had them in their possession, they say, for two or three generations. This refers to the Tang, Sung, and Liao Notes. The Ming notes, as well as those of the Ching period, were purchased from separate sources, from an old bookseller in Peking. I had these Notes, at the time of the purchase, carefully looked over by most of the old and rare book experts in Shanghai and they were all unanimous that the Notes were absolutely genuine of the periods represented. These men of course, understand the papers that were used in the various periods, the printing, etc. through their intercourse with the early books of China.....

I may remark before closing that a Mr. P. Petrucci, an authority in Paris of Chinese Painting, and also a Chinese scholar, who saw these notes, after carefully examining them, expressed them to be in his opinion quite authentic and said he thought the Tang paper was manufactured very probably from bamboo pulp."

The following is a list of the notes furnished by Mr. Orange:

Particulars of old Chinese Bank Notes:

Tang Dynasty	Emperor Hing Chang	A.D. 841/6	2 notes
West Liao Tartar Dynasty	Emperor Hsian Ching	A.D. 1136/41	1 note
Sung Dynasty	Emperor Chien Tao	A.D. 1165/73	2 notes
Ming Dynasty	Emperor Hung Wu	A.D. 1368/98	3 notes
Ming Dynasty	Emperor Hung Tsi	A.D. 1425/26	6 notes
Ching Dynasty	Emperor Hien Feng During Tae Ping rebellion.	A.D. 1851/61	4 notes

About the Notes Themselves.

The two Tang notes are both of them yellow in color. The paper is light and flexible, having little or no sizing. The denominations are expressed pictorially in shoes of silver, but the chinese characters indicating the denomination are interpreted to mean "kwans." The kwan was the equivalent of 10 strings of copper cash, or of one tael or ounce of silver. The term "sycee" which is used by Ramsden as the equivalent of "shoe of silver" is used by Morse to express silver itself — thus he says "shoe of sycee." The shoes of which Morse spoke weighed about 50 taels, but he also referred to "obvoid lumps" of silver in circulation, weighing up to two or three taels.

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The pictorial representation of value must refer to these rather than to the 50 tael shoes.

The vermilion seals on these notes were apparently printed on the paper before the impression of the note itself was taken in black ink. They are in fairly good condition, being in fact so well preserved that by making use of both notes they could readily be reconstructed by an engraver. There would be little or no call for him to use his imagination. The paper of the two notes is not of exactly the same size. The exterior lines of the impressions measure $9\frac{1}{4} \times 5\frac{1}{4}$ inches.

Ramsden translates the heading to the notes: "Great T'ang General Use Treasure Paper Money." The interpretation of these seal characters by Mr. Drew "Great Tang dynasty circulating precious note" is identical in substance. The characters at the right side of the note in the border panel read "For universal circulation throughout the Empire"; those on the left "To be universally accepted." The six vertical rows of characters at the bottom, contain first an announcement of the dynasty and the department of the government authorized by imperial authority to issue the note; second, a statement that the note is to circulate on the same footing as silver all over the country for the convenience of the people; third, that the penalty for counterfeiting is death by beheading; fourth, that a reward will be given the person who brings in the counterfeiter, in the case of the one kwan note, of two hundred and sixty taels, in the case of the nine kwan note of seven hundred and fifty taels; and, finally, if any person knowingly conceals the counterfeiting he shall be punished the same as the counterfeiter. Then follows the name of the emperor, or the particular period of the reign,— if the reign was thus divided,— followed by the characters representing — year,— month,— day, the particular year, month and day being filled in with a brush when the note was issued. These specific dates put on by a brush, very soon wore off when the note was in use, but the custom that prevailed for an emperor to break his reign up into periods and designate every few years a new group of consecutive years by a new title, enables us to place the emission of any of the old chinese notes within a few years. The entire reign of an emperor is indeed near enough for our purposes. In this particular case the period Hwei Ch'ang is given on the notes. It happens however that the reign of the Emperor Wu Tsung covered the same period, namely 841–847. It will thus be seen that we can fix within six years the date of the emission of these notes.

The decorative pattern of the border at the top is made up of the dragon pattern; at the bottom of conventional waves; and at the sides on the lower half of conventional representations of clouds.

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Three kwan note, Western Liao Dynasty. Emperor, Kan T'ien How; Period, Hien Ts'ing; A.D. 1136–1142. Reproduced from the original, which is, however, somewhat darker than this half-tone.

The paper measures $9\frac{3}{8}$ by $7\frac{5}{8}$ inches; the impression, $7\frac{5}{8}$ by $4\frac{5}{8}$ inches.

The vestiges of the impressions of two government seals in vermilion can be traced on the face of the note.

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The note of 650 A.D., of which we have a pictorial representation in *Chuan Pu Tung Chih,* was emitted at a period so far back in historical period that there was no England in existence under that name. The two Tang notes that we have just considered were in circulation in China during the childhood of Alfred the Great, at a period in history so far back that Scott did not venture to seek in those times for a hero for one of his romances. The contrast between the cultivated conditions of a society which could make use of paper money, in any form or in any manner, and which could also have produced such admirable specimens of workmanship and art, as are preserved for us in these notes, with English life at the same date, when the government then being evolved was in its infancy; when there was neither paper nor block printing in use; nor could there be found anywhere save possibly in some monastery any person who could read or write, does not need that we should dwell upon it. The more one thinks of it, the more the wonder grows. It seems incredible.

The Western Liao three kwan note was issued in the period 1136–1142. The decorations of the border are perfunctory in the extreme. It was emitted through the Board of Revenue to be used for general military purposes. The penalty for counterfeiting was decapitation and the reward for the arrest of the counterfeiter 500 taels silver. Mr. Bahr expressed some doubt as to whether the color of the note might not have changed. Ramsden gives the prescribed color for some of the emissions; in the case of this particular emission the designated color was blue. It is a fair presumption that some effort was made to have the notes of a given issue uniform in appearance. The distinctive dark color of the paper of this note makes it quite effective but it would not seem that it could ever have been of a blue tint. The marks of the two seals on the face of the note are still apparent though the separate seal characters of the inscription on the seals are totally obliterated. The paper of the notes measures $9\frac{3}{8}$ by $7\frac{5}{8}$ inches. The impression of the note itself, $7\frac{5}{8}$ by $4\frac{5}{8}$ inches.

The two Sung notes are of the same color as the Liao note. Ramsden says of these also that the prescribed color was blue. The period of the emission was 1165–1173. The impressions of the wood cut on the notes measure in the one case $8\frac{7}{8}$ by $5\frac{1}{4}$ inches, in the other 9 by $5\frac{1}{4}$ inches. The paper has about three eighths of an inch margin all around. The decoration of the border is inartistic and conventional. The notes were emitted by the Board of Civil Offices. They were for general circulation and were to be of equal value with copper cash. The counterfeiter was to be punished by decapitation and the person who arrested a counterfeiter was to be rewarded, in the case of

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the 70 kwan note with eleven hundred taels of silver; in the case of the 100 kwan note, with fourteen hundred taels. If local officers condoned or concealed counterfeiters they would be subject to punishment. The traces of the two government seals on the face of the notes are quite evident.

The Ming one kwan note of the period 1368–1398 has already been fully described, it being the note translated by Morse.

The six notes of the Ming dynasty and of the period 1425–1426 are printed on paper of substantially the same color as the Liao and Sung notes. Ramsden says that the prescribed color was blue black. The denominations are 10, 20, 30, 70, 90 and 100 wen — the wen being the copper cash. They were therefore of small value and were intended for general circulation. The paper is uniformly four inches in width, but the notes vary from 10 to $10\frac{1}{4}$ inches in length. The impressions on the notes vary slightly in length, running from 9 to $8\frac{3}{4}$ inches in length, but having a uniform width of $3\frac{3}{8}$ inches. They were defined to be War period notes and were emitted on petition of the Grand Council, for military purposes. The counterfeiter was to be decapitated. The reward for the informer or arrester was for the 10 cash note 11 taels silver; for the 20 cash note 13 taels silver; for the 30 cash note 15 taels silver; for the 70 cash note, 23 taels; for the 90 cash note, 27 taels; and for the 100 cash note, 29 taels. The decorations of the border are of the same inferior character as those of the Liao and Sung notes.

The four notes emitted under the Ching dynasty during the Tae Ping troubles (1851–1862) are curiosities but cannot be classed as antiquities. The one tael and the five tael notes each have five dragons in the border ornamentation. The 500 wen and 2000 wen notes each make use of two dragons in the decorative treatment of the border. The paper is white and is sized. The notes are printed in blue, although some of the characters are stamped in black ink and in the dates the brush is used. There are several government seals. Morse, in his article in the Journal of the North China Branch of the Royal Asiatic Society, Vol. XXXVIII, 1907, says of these notes: "From A.D. 1403, it may be said, or at any rate from some time in the reign of Yung-lo (A.D. 1403–1425) there were no fiduciary issues by the government, either of the Ming or the Tsing, until we come to the troubled times of Hien-feng (A.D. 1851–1861) when the necessities of the Treasury drove it to this method of replenishing its depleted reserves." He then quotes from Bushell, a detailed description of the notes, and a statement that they depreciated so rapidly that in

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Seventy kwan note, Sung Dynasty. Emperor, Hiao Tsung; Period, K'ien Tao; A.D. 1165–1174. Reproduced from the original, which is darker than this half-tone.

The paper measures 9½ by 5¾ inches; the impression, 8⅞ by 5¼ inches.

The marks of two vermilion seals can be seen on the face of the note.

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1861 they were sold by Dutch auction in the streets of Peking at a discount of 97 per cent.

What Chinese Historians say of the Notes.

Our examination of the notes themselves has brought to our attention certain features connected with the respective emissions of which they are representative. There remains, however, to be considered, the information which may be acquired from the accounts of the chinese historians, as to the functions of the notes, as to their fluctuations, and as to the character of the support received by them from the government from time to time.

The numismatic manual of Ramsden is the only one among all the works cited, so arranged as to give off-hand any idea of the chronology of the emissions. The fact that the empire was at times administered as a whole and at other times broken up into two or more governments, all of which simultaneously emitted notes, is likely to produce much confusion in the mind of an investigator, who is not familiar with the eras of the dynasties and the areas under their respective control. The dynastic arrangement by Ramsden, of his manual, relieves this confusion somewhat and the simultaneous parallelism of the emission of different dynasties thereby disclosed accounts for some of the perplexities occasioned by the statements of chinese writers. It is obvious that the student who found in Chaudoir that in 1168 copper plates were first made use of for printing notes, wood cuts alone up to that time having served that purpose, would be surprised when he found on the third page thereafter, the statement made that in 1277, the Mongols first made use of copper plates in substitution for the wood cuts from which they had previously printed the notes. If however he were familiar with chinese history, he would realize that the Mongol conquerors had merely adopted at that date a practice that they found in use by their more cultivated neighbors whose territory they had invaded. This single illustration will show that one who undertakes to examine this subject even if he follows in the footsteps of European interpreters, needs to have some knowledge of chinese history, would he comprehend clearly what he is dealing with.

Again, it must not be forgotten that in attempting to discover through chinese records the various experiences of China during this long period of the use of paper money, we are compelled to struggle against the difficulties interposed by a language which is not easy to

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translate. If we turn to one of the chinese titles already quoted, we find that the same characters are interpreted by different scholars in one case, to mean "Antiquarian Researches" and in another to read "General Examination of Records and Scholars."

In the January number of the Atlantic Monthly there is an article entitled "Secret Annals of the Manchu Court," contributed by two well known English sinologues, Messrs. Backhouse and Bland. In this article they quote the title of a work, as "Reminiscences of a Time of Suspicion and Panic." A foot-note announces that the literal translation of this title is "Monkey-like Suspicions and Panic at the Cry of a Bird." Periphrasis of this sort may in a general way be typical, but such a wide variation as this from the literal must be uncommon, and is introduced merely to show one of the difficulties met by the translator.

We have seen that Chaudoir says that copper plates were first used by the Mongols in 1277. Saburo, writing about the Mongol notes says: "At first the printing machines were made of wood, but in the 13th year [1276] it [sic] was replaced by machines made of copper or brass." It is to be presumed that the wooden printing machines of Saburo were the wood cuts from which the notes were printed, while the copper machines were the plates which we have learned elsewhere were introduced by the Mongols about this time.

It is stated that Edkins the author of "Banking and Prices in China," was called upon by Sir Robert Hart to translate into chinese a regulation concerning goods passing over the bar at the mouth of a river. He effected the translation, but made use of the chinese character for bar which is associated with cocktails, thus putting in force a customs regulation on goods of quite another character from what was intended.

These illustrations bring fairly before us the obstacles encountered by one who seeks to render in English an equivalent for sentences expressed by chinese characters, whether arising from mentality, from the necessity for periphrastic rendering, from lack of acute perceptive faculties or from deficiency of analytical capacity.

Still another obvious difficulty lies in the records themselves. Some of these chinese histories upon which we rely were written hundreds of years ago and they in turn were based upon records and official documents, many of them dating back several centuries before the day of the historians. Now, it would be unreasonable to expect to find among these officials and historians, experts so trained in the analysis of events as to seize in their narrative, upon the exact points

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which are of value to the economist to-day, and to express themselves so as to satisfy fully the demands of the modern investigator. We are compelled to take the records as we find them, and interpret them, as best we may, through the test of subsequent historical experiences. We may feel reasonably sure that the translators have given us a fairly accurate version of the chinese text, especially when we find a consensus of meaning in the papers of different translators. If we reflect that the records with which we are dealing at the beginning of the chinese paper money experience cover dates six and a half centuries and perhaps more before Columbus made his first voyage across the Atlantic, we shall not be disposed to demand too much of these chinese historians and officials in the way of exact statements.

The several writers who have undertaken to furnish us with knowledge of these chinese records unite with a single exception, in saying that the first experience of the chinese government in the emission of paper money occurred about 806 A.D. That exception throws the date of the inception of the experience back about one hundred and fifty years. The question whether the true date is 650 A.D. or 806 A.D. is of slight consequence to us. There is nothing improbable in the statement that the earlier is the true date, but the later has proved to be the one usually accepted as authentic, for although very little is known of the notes then emitted, the causes which led to the emission have been fully portrayed. Interior troubles, continuous wars, a reduction in the output of the mines, and the amount of copper used by the Buddhists in casting statues, led to the total disparition of the metal itself and of the copper coins. Edicts were issued against the use of copper in the manufacture of domestic utensils. Merchants were ordered to bring their metallic currency to the treasury, in exchange for which so-called bonds or letters of exchange payable at short terms in the principal districts of the empire were given them.

These obligations were called "light money" or as Klaproth more picturesquely translates it "flying money." They apparently had but a brief life at the capital, but in the provinces continued to circulate for some time. Other emissions followed under the different emperors, some of which were for the nominal purpose of saving to merchants the transportation from place to place, in settlement of debts, of the copper currency on which the country depended as a measure of value. Most, if not all, of these emissions had names. In one case we have seen that the appropriate designation of "flying money" was given them. In another they were called "convenient money." The fitness of this title will be appreciated when it is stated

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that at times and especially in the outlying provinces, where there was a scarcity of copper money, iron money was also made use of as a circulating medium.

It is evident that at first there was no conception that the government could maintain a purely credit currency. There was apparently no thought of gain in the transaction. The alleged purpose was to save merchants the expense of transporting the cumbrous copper or iron money, which lay at the base of the currency. Nor had they conceived of a demand note. The first notes were said to be for "short terms." Another emission at a latter date was said to be for twenty-two three-year terms redeemable at the end of 65 years. Apparently, at the end of each three years, the notes could be presented for redemption. It is said that this system was originally inaugurated by merchants, who furnished a private currency of their own, on these terms, and it is stated that after a few years the accumulation of outstanding notes which had not been presented for redemption, for the protection of which they had not maintained an adequate reserve, proved too much for the merchants, and being unable to protect the notes, they became bankrupt. Thereupon the government stepped in, forbade the emission of notes by private citizens, itself emitted notes on the same basis as that devised by the merchants and it is asserted with the same result. It would seem as though the failure to provide the funds for the redemption of these notes might better be attributed to lack of business foresight than to inflation. The notes were a convenience and so long as the people had confidence in them they were not presented for redemption at the end of the terms. Statistics are furnished from time to time which serve to show that this was the case. There were for instance afloat in 1032 more than 1.256.340 kwan of these 65 year notes. In 1072, Chaudoir says the 17th term of payment had arrived and only notes to the extent of 6540 kwan had been redeemed. The final redemption was not far away. The government was absolutely unable to procure enough metallic money to effect this redemption. New notes were therefore prepared, running for twenty-five terms of three years each, and holders of the old were ordered to exchange them for the new, the option of a redemption being, nominally at least, offered them.

These three-year notes running from twenty-two to twenty-six terms held the field for some time, but in the course of events experiments were made with notes having other terms. We hear of one year notes, of those having seven-year terms, of ten-year terms, and of notes of forty-three one-year terms whose value was expressed in

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Ten wen note, Ming Dynasty. Emperor, Jên Tsung; Period, Hung Hi; A.D. 1425, 1426. Reproduced from the original, which is darker than this half-tone.

The paper measures 10¼ by 4 inches; the impression, 8⅞ by 3⅜ inches.

The marks of two vermilion seals faintly to be seen on face of note.

From the collection of Andrew McFarland Davis.

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silver. To further the circulation of these latter notes, which were of large denominations, it was ordained that in all payments exceeding ten kwans, use should be made of notes to the extent of at least one half. The career of these silver notes was short, the depreciation rapid, and the total disappearance from circulation a matter of but a few years. Failure of the government to provide for their redemption is the alleged cause, but the rapidity of the decline suggests inflation.

So far as it might be revealed by an examination of the phraseology of the inscriptions upon certain actual notes, and also upon a few of the representations of notes in the chinese numismatical work which has been referred to, there was no effort made to inform the holder of the note, whether it was a seven-year, a three-year, or a one-year note, or indeed whether it was redeemable at any time or at any place. It is difficult to conceive, how the currency of notes could have been affected by failure on the part of the government to provide redemptions if no such agreement was to be found on the face of the note. The specimens and the pictorial representations, with their inscriptions to the effect that they were to be received as so much copper or so much silver, closely resemble in substance, though not of course in appearance, the fiat-money made use of in Massachusetts during the first half of the eighteenth century.

The temptation is strong to assert on the strength of this examination that there were no notes containing a redemption clause. Bushell, however, translates an inscription on a Kin note of 1214, to the effect that it was redeemable at certain offices, and Saburo quoting from a chinese historian also asserts that certain of the Kin notes were redeemable on demand. These notes were emitted by the Tartars during their participation in the contest with the Southern Sung dynasty.

It is not of importance to follow the meagre details of the various emissions placed at our command by chinese historians. It is not possible to fix the time when the government awoke to the fact that they could in an emergency emit an unsecured and unprotected paper currency at will. A versatile nation would have made the experiment long before the chinese did, but this people were fixed in their habits and followed year after year and century after century the customs of their ancestors. It is evident, however, that under pressure of wars and the need of money to purchase supplies and to pay the soldiers, fiat-money was finally issued, only to be followed by depreciation and monetary troubles. Morse says, "For the twelfth and the first half of

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the thirteenth centuries the country was divided between the Southern Sung and the Golden Dynasty of Nüchen Tartars, and both ran a mad race in the issue of assignats." He then quotes from Klaproth certain details concerning an emission in 1131 A.D. for the purpose of furnishing money to the soldiers, which was evidently a fiat-currency. Concerning the details of these emissions and the description of the notes emitted there is some fragmentary information, but not enough for a continuous, intelligent narrative.

From 1260 to 1329, inclusive, Chaudoir gives a table of the emissions. The table, however, does not furnish details as to the reduction in the amount in circulation during the same period through redemption. The statement is made that the last emission of notes during this long protracted period of the use of paper money was made in 1455, but if so the notes then emitted must have remained in circulation for years after this date, as we find provision made for their reception in payments to the government as late as 1489.

Saburo gives some details concerning an experiment made by the Manchus in 1651 in the way of emitting notes, which was continued for ten years and then abandoned. So far as it goes this seems to corroborate the idea that by that time paper money had actually become forgotten so that the attempt at its revival by the invaders proved to be ineffectual.

Following the slender narrative of events during these centuries it is clear that from time to time reckless rulers acting under bad advice sought to provide for temporary financial emergencies by such overwhelming emissions of government notes that they were brought into disrepute and in some instances were depreciated to such an extent as to become almost valueless. From these pitfalls the government was occasionally released by the conservatism of some intelligent emperor who listened to those advisers who insisted that the trouble lay with the government and not with the people and that a reduction of the amount of notes in circulation would relieve the situation. Matwan-lin, the old chinese author already referred to, occasionally indulged in moralizing upon the situation. Vissering was struck with the value of his observations and what Benadakis said of him: "He knows the true theory of money and of paper and what he has written on the subject is striking in its precision and good sense," has already been quoted, but is worthy of repetition.

Chinese historians apparently relied for their facts upon documents filed in the archives, and these records were made up very largely of discussions carried on by the advisers of the throne — cabinet

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meetings we should call them. A petition is presented, let us say for the emission of more notes or perhaps for the extension of the area within which certain of the notes were allowed to circulate; or, again, perhaps the emperor may call attention to the evil condition of the currency. After this there follows a discussion of the subject thus raised. The opinion of each of the advisers present and speaking is preserved in the archives and if the historian thinks it worth while is handed down to posterity in his book.

Saburo quotes from a petition in the twelfth century, in which the petitioner sets forth his opinions as to the causes of the depreciation of the notes and points out the fact that the notes being easily torn and defaced, the people prefer to keep in their possession the copper cash, and consequently get rid of the paper money and hoard the metallic coins. He then adds: "Now that the government has known only the necessity to issue the note and has not known the advisability of redeeming them, is the reason why the value of cash is daily enhanced and the note is depreciated proportionally. Hence it is clear that the depreciation of the notes is not created by the people, but it is the government themselves who have created it, and nothing is more consistent, in the opinion of this petitioner, than to withdraw part of the notes from circulation, the amount of notes corresponding to the excess in issue, so that in the alternate process of issue and withdrawal, the people may become trained to perceive the necessity of the use of paper and attach value to it, but if it be attempted to check the depreciation of the notes by means of a new issue, not only will it never accomplish the object but it is much to be feared that the notes newly issued will soon share in the same lot as the old ones." Whether the confusion of thought and expression in the foregoing is to be attributed to Saburo or to the petitioner, it is at any rate evident that the underlying opinion of the latter was that inflation was the cause of the depreciation, and that the remedy was a reduction of the amount in circulation, and a restoration of confidence by redemption.

Another writer quoted by Saburo says that "the greatest evil of the administration of the paper currency consists in too much inflation and the absence of contraction. This is so much the case that unlimited issue coupled with no withdrawal is often the result. It is too late to discuss the renewing of the issue when the paper is already beginning to fall. The people are choked with it and the paper is speedily depreciated."

There are numerous recorded opinions indicating that there were from time to time chinese statesmen of the class of those just quoted,

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who appreciated the dangers incident to the emission of fiat money by the government, and who announced the opinion not only that the amount must be proportioned to the needs of the people but that provision must be made for redemptions at promised periods and places.

Arbitrary methods were resorted to in order to maintain the currency in circulation. Edicts were issued making it compulsory to receive paper currency in payment of debts, and forbidding the hoarding of the metallic currency. At other times it was provided that in making payments a certain proportion should be paid in paper money. An avenue for the use of the paper currency was at times afforded through their reception by the government for taxes, but the policy in this regard was not uniform. They were sometimes received in full payment, sometimes in part payment and sometimes not at all.

As far back as the first note of which we have any representation, there was on the face of the note an announcement that he who counterfeited the note would be subject to the penalty of decapitation. There was also a statement that a reward would be given for the arrest of the counterfeiter. The penalty of decapitation for the counterfeiter seems to have been constant, but the reward for the informer varied with the size of the note, and was doubtless influenced by the temper of the ruler, as well as by the activity of the counterfeiters. On some of the notes threats are hurled against officials who conceal or condone such offenders. At other times, the efforts seemed to be directed towards finding out the real offenders rather than punishing accessories and it was announced in connection with one emission that "accomplices as well as those who attempt to conceal the act will on confessing the fact be pardoned and be allowed to hold official employment."

Perhaps the most original suggestion in connection with the various discussions concerning counterfeits, occurred during the Sung dynasty after a large seizure of counterfeit money had been made, during the discussion as to what should be done with the counterfeiters. One adviser said that the customary policy of beheading the criminal and destroying the counterfeits was a mistake. "If," said he, "you put the official stamp on that counterfeited paper, it will be just as good as genuine paper, and if you punish these men only with tattooing, and circulate these notes, it is exactly as if you saved each day 300000 copper cash together with fifty lives." The writer says the suggestion was adopted.

The change from wood-cut to stereotype plate could never have

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meant much. The one kwan Ming notes of the period 1368–1398 are the only notes which permit of comparison, one with another, and hence furnish foundation for an opinion as to their thorough uniformity. There are enough of these, however, accessible for such a purpose, both by actual comparison of the notes themselves with each other, and also with the photographic process pictures made use of in illustration of books, to permit the expression of an opinion that there are well established variations in the different specimens.

We have already seen what books contain pictorial representations of these one kwan Ming notes of the Hung Wu period. Further we have seen that in 1842 there were three of these notes at St. Petersburg and that the British Museum has two, one acquired in 1890, the other in 1902. Besides these the note that furnished Morse his lithograph is in the Museum of St. John's University, Shanghai. There is one at the Essex Institute, Salem, Massachusetts. There is, or was, one in the Knickerbocker Trust Co., New York, and there is one in the possession of the American Bank Note Co. Besides these there are at least the same number of private individuals known to have specimens of this particular note, and some of them have more than one specimen. It may reasonably be inferred that this scattered ownership would be found to be much larger if persistent attempts were made to run down the location of these notes.

The appearance of the notes justifies the belief that they were printed from wood cuts and the differences which can be established between them lead to the conclusion that there must have been numerous places of issue at which the blocks were cut for that purpose. The first place on the face of the notes which one would naturally select for purposes of comparison would probably be the ornamentation of the border. Ramsden, quoting, it may be inferred, from the chinese numismatical work, upon which he relied for information as to these old notes, gives their prescribed color as blue black and describes the ornamentation of the border, thus: "design: dragons on border." Even though the worn condition of the specimens interferes with the mechanical comparison of the several notes, yet it does not absolutely prevent it. On some of the notes this dragon pattern is obvious, on others it is hard to find. The statement may be made, not only that the notes are not absolutely uniform, but that they must have come from several independent sources. It may indeed be asserted that the chinese characters on the face of the notes are not themselves uniform. My attention was called to this point while examining the question of penalties for counterfeiting and rewards

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for informing. I noticed that the translation of the inscriptions on the note, furnished by Du Halde gave the reward to the person who should inform and bring in or produce the culprit, while Morse in his translation gave it to the informer, saying nothing about the arrest. At my request Mr. Drew kindly examined the chinese characters in the representations of notes furnished by Du Halde and by Morse, and assured me that Du Halde's translation was undoubtedly correct, that the informer had to bring in his prisoner to secure the reward, and further that on the Morse note, while there was required of the informer in order that he should reap his reward, the performance of a similar service, the character used to express this service was different from the one used on the Du Halde note and involved the idea of sending the prisoner in rather than of bringing him in.

Edkins says of the Mongol notes, that each province had a bank for their manufacture. It probably was true of the Ming notes, that they were emitted from various provincial sources.

The quality of the paper on which the notes were printed had to do with the facility with which they could be counterfeited. There were times when the government left the manufacture of the paper in the hands of private individuals, apparently without supervision, with the natural consequences of a degradation of quality which finally compelled intervention. Marco Polo tells us that the Mongol notes were made from the inner bark of the mulberry tree. There has been some discussion as to the probability of this being true, since mulberry trees were of so much value to those who cared for the silk worms, and the conclusion generally reached was that the tree must have been some other species of mulberry than the one which furnished food for the silk worm. In 1217, Edkins says there was a discussion as to the emission of notes. It was stated that there was a scarcity of mulberry tree bark and the question was asked, "How could the notes be made without it?" Certain notes then in circulation were known as "the mulberry bark old paper money." The testimony as to the use of the bark of some mulberry tree for the manufacture of the notes seems conclusive of that fact, but whether or not, some other bark may afterward have been availed of does not appear. It seems, however, quite clear that the paper of the notes was made from some vegetable fibre pulp. In addition to this there are records of attempts to circulate notes printed on silk. They do not appear to have been successful.

The value of the notes was as a rule stated to be in copper although there were some that were based on silver. The dynasty is conspicuously set forth on the face of the note, oftentimes more than once,

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and the characters designating the emperor or the period, enable us to determine within a few years the date of the emission. There is a year character, a month character and a day character on each note, and when the note was issued the blanks connected with these characters were filled in with a brush, so that until the brush marks wore off in use, the actual day of the emission of the note could be ascertained.

The circulation of the notes was primarily determined by the area under control of the dynasty at the time of the emission. There were, however, other factors than the limits of the power of the dynasty, which operated to restrict the circulation of some of the emissions. They were sometimes emitted for a particular purpose and for use only within a restricted area. Such restrictions were found embarrassing and naturally gave rise to complaints.

There were times during the depreciation of the notes when efforts were made to redeem the outstanding notes with new notes, the latter being emitted perhaps on a par with metallic currency, thus temporarily furnishing two concurrent paper currencies, the one of which was worth three, four, five, or whatever the proportionate value might be, times the other. To a certain extent, such discrepant values could be maintained through provision for the reception of the notes by government officials on this basis. As a rule, however, experiments of this sort did not relieve the situation.

Two things have operated to reduce the value of this chinese experience as a lesson to other peoples. One is the sluggishness of the chinese temperament, its inflexibility under ordinary pressure, its resistance through inertia to change of any sort. The other is the subordination of the people to authority, their readiness to accept and obey the orders of their constituted government, and the difficulty of organizing opposition except on the basis of war. Yet in spite of the passive endurance of the people and notwithstanding the arbitrary efforts of the government to check depreciation, the law of supply and demand overrode edicts and orders and when inflation prevailed prices went up. The total abandonment of paper money, at the end of this protracted experience was thoroughly unscientific, but in this case quite natural. China had no contact with the outer world and but few industries other than agriculture. There being no field for the development of industrial enterprise there was no special occasion to make use of the agency of credit to multiply the resources of the empire. Had the conditions been different, had there been commercial and industrial activity, chinese intellect would probably have found some

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other way of curing the abuse than that of abolishing the use, and we should not have been reminded by this throwing away of the enormous advantages possibly to be derived from a well handled government credit, of the wasteful process described by Charles Lamb, through which the delicacy of roast pig was provided in China.

Paper Money in Europe and America

A word ought perhaps to be said, in closing, concerning the experience in the use of paper money in Christendom which most closely parallels that of the chinese. It will help us to appreciate the indefinite past with which we have been dealing in China, if we recall the fact that the Bank of England was not chartered until 1694, A.D., and while claims are made that continental banks had anticipated that date in furnishing a paper currency to the people, these claims if proved would not carry back the birth of paper money in Europe by many years. Two hundred and forty years had intervened between the time set by Klaproth for the chinese abandonment of paper money and the attempt in Europe to inaugurate the use of a representative denominational paper currency. It was not, therefore, until about a thousand years after emission of the oldest note of which we have heard in China, that the conditions of life in Europe led to the development of a system of paper money. Experiences in the imprudent use of the power to furnish a circulating medium through the means of the printing press, led to catastrophes like Law's Mississippi bubble and later to the depreciation of the government paper of some of the continental powers, but the one experience which most closely resembles that of the chinese was that of the Province of Massachusetts Bay in the first half of the eighteenth century. Our ancestors compressed within the brief space of about fifty years, practically coincident with the first half of the eighteenth century, an experience in the use of fiat money, which rivals the chinese experiment in the variety of the lessons which it can teach, and is in reality far more instructive than its prototype, since we have complete records of the details of action, the methods of emission and the surrounding conditions of life. We can see here, the timid efforts of a government, having no borrowing power, to meet a debt, relatively very large and utterly unexpected, by the emission of certificates of indebtedness, in the form of a denominational currency, capable of being made use

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of as a circulating medium. We can see that through the prompt retirement of the earlier emissions by means of taxation, confidence in the bills of public credit was created, and thus the government was enabled to float enough of them, to cover the expenses of administration at first for a single year and ultimately for several years. We can watch the gradual disappearance of the metallic currency, concurrent with the increase of the quantity of the bills in circulation, and after silver had been entirely withdrawn from general circulation the continuous rise in prices which paralleled the steadily increasing amount of bills emitted by the province. We can see two sorts of bills circulating side by side, both emitted by the same government and both dependent for the value at which they would circulate upon the language used in the statement of value printed on the face of the note, one of which was worth three times as much as the other, and we can see that this circulation of notes bearing the same denominational values, of which three of the one were required to perform the functions of one of the other, was made possible by the terms on which the government would receive the notes. We can see that while the government could force this unnatural circulation of the two kinds of notes, it did not have the power to retain them or any of them on a par with specie, but that they declined in proportion as the needs of the people for trade were exceeded in the supply furnished for circulation.

The closure of this paper money experience in Massachusetts was like that in China effected by the total abandonment of paper money and the return to an absolute specie basis. There were, however, many reasons which operated to prevent the colonists of Massachusetts from drawing correct inferences as to the economic laws which governed the circulating medium. They had seen prices go up while the amount of notes emitted by the province was being steadily reduced. The reason for this apparent violation of an economic law is easy to find, the notes of adjacent colonies more than filled the gap in the circulating medium. To a certain extent this was understood but the rebuff that the province had met with in attempting to reduce the amount of the circulating medium, will explain why they felt that the only relief was not alone to abandon the use of their own paper money but also to do it in such a way as would cut off the circulation of emissions by neighboring governments.

It will thus be seen that the lessons to be learned from this fifty year experience closely resembled on the whole that which was to be derived from the incomplete accounts of what took place in the same

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line in China during centuries of experience. Neither of these experiments has affected economic thought to any measurable extent. One of them has been practically unknown until very recently. The other has been buried in the oblivion of provincial archives.

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A CRITICAL DISCUSSION OF THE CRYSTAL FORMS OF CALCITE.

By H. P. Whitlock.

With One Plate.

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A CRITICAL DISCUSSION OF THE CRYSTAL FORMS OF CALCITE.

By H. P. Whitlock.

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The enormous mass of crystallographic data which have been furnished by investigators of the species calcite since the time of Zippe and Haidinger has resulted in the addition of many new forms to the list available to those critical monographers. Whereas the list of Irby, published in 1878, recognized 156 well established forms, a similar list brought up to date will contain more than twice that number. Faced with this multiplicity of forms, thickly studding the field of gnomonic projection, the modern crystallographic investigator of calcite is ordinarily at a loss to distinguish the common forms from the less common and these again from the rare or unusual ones. It is the purpose of the present paper to furnish a criterion by which the probability of occurrence may be determined for any form, and by an extension of the critical methods, new forms tested.

In the following discussion the method of developing normal series proposed by Dr. Goldschmidt [1] has been adopted as a primary basis of comparison. A brief exposition of Goldschmidt's method of developing normal series is here appended with the object of rendering the analysis more intelligible to the reader.[2]

The mechanical basis of the method of normal series is to be found in the preliminary hypothesis that every face is crystallonomically possible which is perpendicular to a direction of particle attraction. In this way a possible crystal face may be assumed perpendicular to the resultant of any two directions of particle attraction, which is equivalent to saying that every crystal edge may be replaced by a possible crystal face. But, in as much as each mechanical resolution or redivision of the crystallizing forces is accompanied by a weaker

1 Goldschmidt, V., Zeit. f. Kryst., **28**, 1 and 414 (1897).

2 For a short summary of Goldschmidt's article see A. J. Moses, School of Mines Quarterly, **25**, 415 (1904).

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tendency toward the production of corresponding crystal faces, the degree of probability for the production of a crystal face in any zone depends directly on the number of times the primary crystallizing forces for that zone have been resolved in order to produce a secondary, tertiary, quaternary, etc., force perpendicular to the given crystal face. If we represent two primary directions of particle-attraction by 0 and ∞ and assume this for the first normal series (N_0) the subsequent results of division and redivision expressed in descending normal series will be:

N_0 0, ∞

N_1 0, 1, ∞

N_2 0, $\frac{1}{2}$, 1, 2, ∞

N_3 0, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, 1, $\frac{3}{2}$, 2, 3, ∞

N_4 0, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, 1, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, 2, $\frac{5}{2}$, 3, 4, ∞

The principles of this method of developing normal series may be then stated as:

1° The probability and therewith the frequency of a number is the greater the nearer the number stands in development to the origin, i. e., the lower the order of the series in which it first occurs.

2° For each derived number and the corresponding form there is a certain intensity. The probability and frequency of a form is the greater the greater its intensity.

Any series representing forms lying in a zone and derived from the crystallographic indices of those forms may be reduced to the series form $0 \ldots \infty$ by the addition of a constant to each number of the series, by multiplication by a constant, by inversion or by remodeling by the formula $\frac{v - v_1}{v_2 - v}$ in which v is any intermediate member in a series, v_1 the first member and v_2 the last member of the series. A zone or zone section when developed by these rules and compared with some member of the normal series N_n may be identical with its normal type, in which case it is probable that this zone or zone section is of normal development and has suffered little or no disturbance. If, on the other hand, such a zone contains terms which do not accord with the normal series such terms invite criticism and should be critically considered from all other points of view, such as zonal intersections, size and definiteness of the faces observed by the author describing them, etc. Such terms are designated "extra" forms and are enclosed in parentheses. Where terms of the normal series N_n are

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missing from the developed zone their position is indicated by dots.[3]

It is frequently advisable to divide a zone into fragments, in which case certain points, fixed in the projection by zonal intersections and occupied by the poles of notably frequent and important forms, are taken as the end forms of a series, and are known as "knot points."

No attempt has been made in the discussion to include forms which have been considered by former monographers as doubtful or vicinal as, in the writer's opinion, such an extension of the discussion would not materially add any new light and would unduly cumber the criticism with useless matter.

In many instances throughout the discussion it will be found that forms of comparatively simple indices and reducible to numbers of low normal series are missing from the hitherto observed forms. Where such an obviously probable form occurs in the series, it will be set aside as a hypothetical form, that is, one which will in all probability be ultimately added to the list of observed forms. It has not been considered wise to develop any portion of a zone further than normal series 4 — N4, although in tabulating the results at the end of each discussion, a column headed "N5" has been included.

Knot-points have been assumed at the poles of such obviously common and important forms as φ. $(02\bar{2}1)$, K : $(21\bar{3}1)$, $\mathfrak{p}$: $(13\bar{4}1)$ etc. Using the Knot-points of established zones for poles through which to pass zones subsequently to be discussed and numbering the zones approximately in the order in which they are thus established we have the network of zone intersections shown in Plate 1, which is a gnomonic projection of the field covered by about 80° from the pole of *O* (0001). The zones here shown will be designated throughout the discussion by the letter Z followed by an index; thus zone 6 will be designated by Z_6. The zones shown on Plate 1 are as follows:

3 A brief summary of the critical methods here employed will be found in W. F. Hillebrand and W. T. Schaller, Bull. 405, U. S. Geol. Surv., p. 14 (1909).

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Zone	Designation	Limiting poles	Zone equation
Zone 1	Z_1	$[10\bar{1}0 \cdot 11\bar{2}0]$	$[001]$
" 2	Z_2	$[0001 \cdot 11\bar{2}0]$	$[\bar{1}10]$
" 3	Z_3	$[0001 \cdot 10\bar{1}0] \cdot [0001 \cdot 01\bar{1}0]$	$[010]$ $[100]$
" 4	Z_4	$[01\bar{1}2 \cdot 10\bar{1}1] \cdot [10\bar{1}1 \cdot 11\bar{2}0]$	$[\bar{1}\bar{2}1]$ $[\bar{1}11]$
" 5	Z_5	$[10\bar{1}1 \cdot 02\bar{2}1]$ $[02\bar{2}1 \cdot 11\bar{2}0]$	$[\bar{2}\bar{1}2]$ $[\bar{1}1\bar{2}]$
" 6	Z_6	$[02\bar{2}1 \cdot 40\bar{4}1]$ $[40\bar{4}1 \cdot 11\bar{2}0]$	$[\bar{1}\bar{2}4]$ $[\bar{1}14]$
" 7	Z_7	$[40\bar{4}1 \cdot 08\bar{8}1]$	$[\bar{2}\bar{1}8]$
" 8	Z_8	$[10\bar{1}4 \cdot 01\bar{1}2]$ $[01\bar{1}2 \cdot 11\bar{2}0]$	$[\bar{4}\bar{2}1]$ $[\bar{2}2\bar{1}]$
" 9	Z_9	$[10\bar{1}2 \cdot 01\bar{1}1]$ $[01\bar{1}1 \cdot 11\bar{2}0]$	$[\bar{2}\bar{1}1]$ $[\bar{1}1\bar{1}]$
" 10	Z_{10}	$[0\bar{5}\bar{5}1 \cdot 10 \cdot 0 \cdot \bar{1}\bar{0} \cdot 1]$	$[1 \cdot 2 \cdot \bar{1}\bar{0}]$
" 11	Z_{11}	$[50\bar{5}2 \cdot 05\bar{5}1]$	$[21\bar{5}]$
" 12	Z_{12}	$[05\bar{5}4 \cdot 50\bar{5}2]$	$[24\bar{5}]$
" 13	Z_{13}	$[07\bar{7}2 \cdot 70\bar{7}1]$	$[12\bar{7}]$

The arbitrary rulings by which "extra" forms are adjudged "well established," "probable," "doubtful" etc., are briefly:

1° The normal series in which the form first appears.

2° Its relation to other and well established forms of the calcite form system as established by zonal relations.

3° The author's description of the form, whether curved, dull, sharply defined, etc.

Zone 1 — Dihexagonal Prisms. Symbol $\frac{k}{h}$.

Using Bravais-Miller indices and suppressing the third term, the prism zone is as follows:

Form	b 100	Ψ 10·1·0	σ 710	ξ 310	θ 210	μ 530	j 760	a 110
Symbol	0	$\frac{1}{10}$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{6}{7}$	1
$\frac{v}{1-v}$	0	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{2}$	1	$\frac{3}{2}$	6	∞
N_3	0	$(\frac{1}{9})$	$(\frac{1}{6}) \cdot \frac{1}{2} \cdot$		1	$\frac{2}{3} \cdot\cdot$	(6)	∞

Referred to N_3 the zone is incomplete and contains the extra forms $\frac{1}{10}$, $\frac{1}{7}$ and $\frac{6}{7}$. Of these the two last belong in normal series $6 = N_6$ and the first, $\frac{1}{10}$, seems to be doubtful.

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Ψ $(10\cdot1\cdot\bar{1}\bar{1}\cdot0)$. This prism was observed by Schnorr [4] on material from Neumark, Germany. It lies in zone $[0001{:}10\cdot1\cdot\bar{1}\bar{1}\cdot3{:}40\cdot4\cdot\bar{4}\bar{4}\cdot9{:}10\cdot1\cdot\bar{1}\bar{1}\cdot0]$; the two intermediate forms $(10\cdot1\cdot\bar{1}\bar{1}\cdot3)$ and $(40\cdot4\cdot\bar{4}\bar{4}\cdot9)$ were observed by Palache [5] on crystals from Lake Superior and seem rational and well established.

σ $(71\bar{8}0)$. This prism was observed by Whitlock [6] on crystals from Bergen Hill, N. J. It lies in a vertical zone containing $(71\bar{8}9)$, $(71\bar{8}6)$ and $(14\cdot2\cdot\bar{1}\bar{6}\cdot3)$ and also in the very well marked zone $[1\bar{1}02\cdot17\bar{8}0]$ containing no less than 13 intermediate forms. The form is probable.

j $(7\cdot6\cdot\bar{1}\bar{3}\cdot0)$. This prism was observed by Schaller [7] on crystals from Andreasberg. It lies in the vertical zone which contains the forms $(14\cdot12\cdot\bar{2}\bar{6}\cdot5)$ and $(7\cdot6\cdot\bar{1}\bar{3}\cdot2)$. On the whole this form seems less probable than either of the two preceding ones.

Hypothetical Prism. Referring to N_2 in the above development, it will be noted that the term 2, corresponding to $\frac{1}{2}$, is absent from the series. The hypothetical prism which would yield 2 in this series would have the indices $(32\bar{5}0) = \infty$ R5. Although not strongly marked the vertical zone $[0001\cdot32\bar{5}0]$ contains the forms $(32\bar{5}7)$, $(32\bar{5}4)$ and $(32\bar{5}1)$.

Zone 2 — Pyramids of the Second Order. Symbol $\frac{h}{l}$.

Using the Bravais-Miller indices and suppressing the third term, the zone of the second order pyramids contains the following forms:

Form	o 001	π 113	χ 7·7·12	λ 223	υ 111	α 443	ω 16·16·9
Symbol	0	$\frac{1}{3}$	$\frac{7}{12}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{16}{9}$
$\frac{3-v}{4}$	0	$\frac{1}{4}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{4}{3}$
N_4	0	$\frac{1}{4}$ ·	$(\frac{7}{16})$	$\frac{1}{2}$ ..	$\frac{3}{4}$	1	$\frac{4}{3}$

Form	ξ 221	β 773	γ 883	δ 331	ϵ 441	η 551	φ 881	a 110
Symbol	2	$\frac{7}{3}$	$\frac{8}{3}$	3	4	5	8	∞
$\frac{3-v}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2	$\frac{9}{4}$	3	$\frac{15}{4}$	6	∞
N_4	$\frac{3}{2}$ ·	$(\frac{7}{4})$	2	$(\frac{9}{4})$	3	$(\frac{15}{4})$ ·	(6)	∞

[4] Schnorr, Programm d. Realgym. Z. Zwickau, p. 6 (1896).
[5] Palache, C., Geol. Surv. Mich., **6**, 161 (1900).
[6] Whitlock, H. P., N. Y. State Mus. Bull., 133, p. 217.
[7] Schaller, W. T., Zeit. f. Kryst., **44**, 321 (1908).

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Referred to N_4 the zone is incomplete and contains the extra forms of $\frac{7}{12}$, $\frac{7}{13}$, 3, 5 and 8.

χ $(7\cdot7\cdot\bar{1}\bar{4}\cdot12)$. This pyramid was noted with the indices $(5\cdot5\cdot\bar{1}\bar{0}\cdot9)$ by Hessenberg [8] on crystals from Maderaner Thal. According to Irby,[9] Hessenberg's material was later shown to be dolomite from Binnenthal. The present indices were substituted by Descloizeaux [10] for those of Hessenberg. The form must be regarded as quite doubtful.

β $(7\cdot7\cdot\bar{1}\bar{4}\cdot3)$. This is also a rare pyramid. It is shown in combination in Fig. 51 of Zippe's [11] monograph where it is mentioned as of uncertain authority. It lies, however, at the intersection of the zones Z_{13} and $[\bar{5}\bar{7}\bar{2}3\cdot10\bar{1}0]$ both of which are well marked. Moreover the term $\frac{7}{4}$ is referable to normal series N_5. The form seems to be fairly probable.

δ $(33\bar{6}1)$. This pyramid is given by Hauy [12] as occurring on crystals from Derbyshire and Andreasberg. Zippe also shows a combination (Fig. 50) from Andreasberg. On none of these figures, where the zonal relations are susceptible of being checked by the drawing, does the pyramid shown correspond to $(33\bar{6}1)$, and in fact the drawing of Zippe would make the pyramid in question $(5\cdot5\cdot\bar{1}\bar{0}\cdot1)$. $(33\bar{6}1)$ was also noted by Sansoni [13] on a combination from Andreasberg. Sansoni's measured angles do not accord well with theory, there being an average difference of $1\frac{1}{2}°$ between adjacent polar angles. The form seems to be rather unprobable.

η $(5\cdot5\cdot\bar{1}\bar{0}\cdot1)$. This pyramid was observed by Rogers [14] on crystals from Frizington, England. It lies at the intersection of the well marked zones $[5\bar{5}02\cdot3\bar{1}\bar{2}1]$ and $[50\bar{5}1\cdot5\bar{5}01]$. The form seems to be fairly probable.

φ $(8\cdot8\cdot\bar{1}\bar{6}\cdot1)$. This pyramid was recorded by Whitlock [15] on crystals from Sommerville, N. Y. The zonal relations of this form are not so good as those of $(5\cdot5\cdot\bar{1}\bar{0}\cdot1)$ although it constitutes the limiting member of the series $(11\bar{2}1)$, $(22\bar{4}1)$, $44\bar{8}1)$, $(8\cdot8\cdot\bar{1}\bar{6}\cdot1)$. On the whole this pyramid seems less probable than the preceding one and of about the same degree of probability as $(33\bar{6}1)$.

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8 Hessenberg, F., Min. Notiz., **4**, Frankfurt (1861).
9 Irby, J. R. Mc. D., Inaug-Disser., Goettingen (1878).
10 Descloizeaux, A., Manuel de Min., Paris (1874).
11 Zippe, F. X. M., Akad. d. Wiss. Wien. Math.-naturwiss. Cl., **3** (1852).
12 Hauy, Traité de Mineralogie Paris, Figs. 4, 72, 144, 152 (1822).
13 Sansoni, F., R. Accad. d. Linc., **19** (1884).
14 Rogers, A. F., Am. Jour. Sci., **12**, 44 (1901).
15 Whitlock, H. P., N. Y. State Mus. Memoir, **13**, p. 74 (1910).

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Hypothetical Pyramids. The only two members missing from normal series N_3 in the above development are $\frac{1}{3}$ and $\frac{2}{3}$ corresponding respectively to $(44\bar{8}9)$ and $(8\cdot8\cdot\bar{1}\bar{6}\cdot9)$.

$(44\bar{8}9) = (73_1)$. Parallel zones through the poles of this form do not pass through the poles of common forms or of those whose indices

Letter	Symbol	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
Zone 1 = Z_1 Dihexagonal Prisms									
Ψ	$10\cdot1\cdot\bar{1}\bar{1}\cdot0$							?	
σ	$71\bar{8}0$							?	
ξ	$31\bar{4}0$			N_2					
θ	$21\bar{3}0$		N_1						
μ	$53\bar{8}0$				N_3				
j	$7\cdot6\cdot\bar{1}\bar{3}\cdot0$							?	
Hypothetical Prism									
	$32\bar{5}0$			N_2					
Zone 2 = Z_2 Pyramids of the Second Order									
π	$11\bar{2}3$					N_4			Z_4, Z_9
χ	$7\cdot7\cdot14\cdot12$							?	
λ	$22\bar{4}3$			N_2					Z_5
υ	$11\bar{2}1$					N_4			
α	$44\bar{8}3$		N_1						Z_6
ω	$16\cdot16\cdot\bar{3}\bar{2}\cdot9$					N_4			
ξ	$22\bar{4}1$				N_3				
β	$7\cdot7\cdot1\bar{4}\cdot3$						N_5		Z_{13}
γ	$8\cdot8\cdot\bar{1}\bar{6}\cdot3$			N_2					Z_7
δ	$33\bar{6}1$							?	
ϵ	$44\bar{8}1$				N_3				
η	$5\cdot5\cdot\bar{1}\bar{0}\cdot1$							?	
φ	$8\cdot8\cdot\bar{1}\bar{6}\cdot1$							?	

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are sufficiently simple to mark such zones in any way. The form as a hypothetical one should be rejected.

$(8\cdot8\cdot\bar{1}\bar{6}\cdot9) = (11\cdot3\cdot5)$. This hypothetical pyramid like the preceding one should be rejected, for the reasons given above.

Zone 3 — Positive Rhombohedrons.

Symbol $\frac{h}{\bar{l}}$.

The most prominent forms in the zone of the positive rhombohedrons are $(10\bar{1}1)$ and $40\bar{4}1)$. These forms should certainly be regarded as knot points. Between $(40\bar{4}1)$ and $(10\bar{1}0)$ the two most prominent zone intersections are $(70\bar{7}1)$ and $(13\cdot0\cdot\bar{1}\bar{3}\cdot1)$, both fairly common forms. Dividing the zone in this way we have for the first portion:

Form	o 001	d. 104	e. 205	f. 102	g. 407	h. 203	V. 405	p. 101
Symbol	0	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{4}{7}$	$\frac{2}{3}$	$\frac{4}{5}$	1
$\frac{v}{1-v}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	2	4	∞
N_3	0	$\frac{1}{3}$ ·	$\frac{2}{3}$	1	$(\frac{4}{3})$ ·	2 ·	(4)	∞

Referring this series to N_3 the series is incomplete and contains the extra forms $\frac{4}{7}$ and $\frac{4}{5}$ both of which would fall in normal series N_4.

g. $(40\bar{4}7)$. This rhombohedron was observed by Hessenberg (loc. cit.) on crystals from Maderaner Thal; it had previously been regarded by Zippe[16] as doubtful. As pointed out under χ $(7\cdot7\cdot\bar{1}\bar{4}\cdot12)$[17] Hessenberg's specimen proved to be dolomite from Binnenthal. The form in question lies in the zone $[1\bar{1}02\cdot17\bar{8}0]$ as well as in two less well marked zones and may be regarded as fairly probable.

V $(40\bar{4}5)$. This rhombohedron was observed by Sterrett[18] on twinned crystals from the Joplin region. Although the form was noted on a number of crystals, in the instance of only one crystal could reflections be obtained. The form seems less probable than $(40\bar{4}7)$.

The forms between $(10\bar{1}1)$ and $(40\bar{4}1)$ are:

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16 Zippe, loc. cit.
17 See page 294.
18 Sterrett, D. B., Am. Jour. Sci., **18**, 73 (1904).

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Form	*p.* 101	*R.* 201	*K.* 502	*l.* 301	*m.* 401
Symbol	1	2	$\frac{5}{2}$	3	4
$\frac{v - v_1}{v_2 - v}$	0	$\frac{1}{2}$	1	2	∞
N_2	0	$\frac{1}{2}$	1	2	∞

This portion of the zone is entirely normal.

The forms between $(40\bar{4}1)$ and $(70\bar{7}1)$ are:

Form	*m.* 401	*n.* 501	*o.* 11·0·2	*y.* 601	*x.* 13·0·2	*q.* 701
Symbol	4	5	$\frac{11}{2}$	6	$\frac{13}{2}$	7
$\frac{v - v_1}{v_2 - v}$	0	$\frac{1}{2}$	1	2	5	∞
N_2	0	$\frac{1}{2}$	1	2	(5)	∞

Referred to N_2 this portion of the zone is complete and contains the extra form 7 which would fall in normal series N_5.

x. $(13\cdot0\cdot\bar{1}\bar{3}\cdot2)$. This rhombohedron was observed by Höfer [19] on crystals from Rauris in Salzburg. On the same occurrence the same author has found the negative rhombohedron $(0\cdot13\cdot\bar{1}\bar{3}\cdot1)$, thus emphasizing the zone $[13\cdot0\cdot\bar{1}\bar{3}\cdot2 : 0\cdot13\cdot\bar{1}\bar{3}\cdot1]$ which also contains the forms $(61\bar{7}1)$ and $(16\cdot7\cdot\bar{2}\bar{3}\cdot3)$. The zone $[0\cdot13\cdot\bar{1}\bar{3}\cdot4 : 13\cdot0\cdot\bar{1}\bar{3}\cdot2]$ is also well marked. The form seems quite probable.

The forms between $(70\bar{7}1)$ and $(13\cdot0\cdot\bar{1}\bar{3}\cdot1)$ are:

Form	*q.* 701	*c.* 801	*b.* 901	*r.* 10·0·1	*k.* 11·0·1	*s.* 13·0·1
Symbol	7	8	9	10	11	13
$\frac{v - v_1}{v_2 - v}$	0	$\frac{1}{5}$	$\frac{1}{2}$	1	2	∞
N_2	0	$(\frac{1}{5})$	$\frac{1}{2}$	1	2	∞

Referred to N_2 this portion of the zone is complete, the form 8 being extra and falling in normal series N_5.

c. $(80\bar{8}1)$. This rhombohedron was found by Cesàro [20] on crystals from Rhisnes, Belgium. It is included among the doubtful forms in Rogers [21] list. The fact that the zone $[04\bar{4}1\cdot80\bar{8}1]$ is exceptionally

19 Höfer, H., Min. Mitth., **12**, 487 (1891).
20 Cesàro, G., Soc. Geol. Belg. Ann., **16**, 165 (1889).
21 Rogers, A. F., S. of M. Quar., **22**, 429 (1901).

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well marked, containing at least four poles of well established forms, would seem to remove any doubt as to the validity of this form.

Between $(13\cdot0\cdot\bar{1}\bar{3}\cdot1)$ and $(10\bar{1}0)$ the forms are:

Form	$s.$ $(13\cdot0\cdot1)$	$t\cdot$ $16\cdot0\cdot1$	$v\cdot$ $18\cdot0\cdot1$	$w\cdot$ $19\cdot0\cdot1$	i· $20\cdot0\cdot1$
Symbol	13	16	18	19	20
$\frac{v-13}{9}$	0	$\frac{1}{3}$	$\frac{5}{9}$	$\frac{2}{3}$	$\frac{7}{9}$
N_4	0	$\frac{1}{3}$	$(\frac{5}{9})$	$\frac{2}{3}$	$(\frac{7}{9})$

Form	$Z\cdot$ $(22\cdot0\cdot1)$	$c\cdot$ $24\cdot0\cdot1$	$j.$ $25\cdot0\cdot1$	$z\cdot$ $28\cdot0\cdot1$	b 100
Symbol	22	24	25	28	∞
$\frac{v-13}{9}$	1	$\frac{11}{9}$	$\frac{4}{3}$	$\frac{5}{3}$	∞
N_4	1	$(\frac{11}{9})$	$\frac{4}{3}$	$\frac{5}{3}$	∞

Referred to N_4 this portion of the zone is incomplete and contains the extra forms 18, 20, and 24.

v. $(18\cdot0\cdot\bar{1}\bar{8}\cdot1)$. This rhombohedron was observed by vom Rath [22] on crystals from Lake Superior and subsequently by von Foullon [23] on crystals from Arlsberg. Goldschmidt [24] regards both observations as doubtful and substitutes the form $(19\cdot0\cdot\bar{1}\bar{9}\cdot1)$ as both conforming better with the rhombohedral series and agreeing more closely with the recorded measurements.

i. $(20\cdot0\cdot\bar{2}\bar{0}\cdot1)$. Sansoni [25] records this rhombohedron from St. Blasien, Baden. The form is very close to $(22\cdot0\cdot\bar{2}\bar{2}\cdot1)$ which is recorded by the same authority from a neighboring locality, under what appears to be similar conditions. The form is quite doubtful.

c. $(24\cdot0\cdot\bar{2}\bar{4}\cdot1)$. This rhombohedron was observed by Hobbs [26] on crystals from Mineral Point, Wis. The form was identified from one series of measurements, the difference in calculated angle between $(24\cdot0\cdot\bar{2}\bar{4}\cdot1)$ and $(22\cdot0\cdot\bar{2}\bar{2}\cdot1)$ being 14′. It is possible that the observed form should have been referred to the simpler indices.

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22 vom Rath, G., Pogg. Ann., **132**, 387 (1867).
23 von Foullon, H., Jahrb. geol. Reichs. Wein, **35**, 47.
24 Goldschmidt, V., Index der Krystallformen, **1**, 378 (1886).
25 Sansoni, F., Gior. d. Min., **1**, 299 (1890).
26 Hobbs, W. H., Univ. Wis. Sci. Ser. Bull., **1**, 115 (1895).

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Hypothetical Positive Rhombohedrons.

The zone of the positive rhombohedrons is singularly free, particularly in that position between $(10\bar{1}1)$ and $(13\cdot0\cdot\bar{1}3\cdot1)$. In the portion

Zone 3 = Z_3 Positive Rhombohedrons

Letter	Symbol	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
z·	$28\cdot0\cdot\bar{2}8\cdot1$					N_4			
j·	$25\cdot0\cdot\bar{2}5\cdot1$					N_4			
c·	$24\cdot0\cdot\bar{2}4\cdot1$							?	
Z·	$22\cdot0\cdot\bar{2}2\cdot1$		N_1						
i·	$20\cdot0\cdot\bar{2}0\cdot1$							?	
w·	$19\cdot0\cdot\bar{1}9\cdot1$				N_3				
v·	$18\cdot0\cdot\bar{1}8\cdot1$							?	
t·	$16\cdot0\cdot\bar{1}6\cdot1$				N_3				
s·	$13\cdot0\cdot\bar{1}3\cdot1$	×							
k·	$11\cdot0\cdot\bar{1}1\cdot1$			N_2					
r·	$10\cdot0\cdot\bar{1}0\cdot1$		N_1						Z_{10}
b·	$90\bar{9}1$			N_2					
c·	$80\bar{8}1$						N_5		
q·	$70\bar{7}1$	×							Z_{13}
x·	$13\cdot0\cdot\bar{1}3\cdot2$						N_5		
y·	$60\bar{6}1$			N_2					
o·	$11\cdot0\cdot\bar{1}1\cdot2$		N_1						
n·	$50\bar{5}1$			N_2					
m·	$40\bar{4}1$	×							Z_5, Z_6
l·	$30\bar{3}1$			N_2					
K·	$50\bar{5}2$		N_1						Z_{11}, Z_{12}
R·	$20\bar{2}1$			N_2					
p·	$10\bar{1}1$	×							Z_4, Z_5
V·	$40\bar{4}5$					N_4			
h·	$20\bar{2}3$			N_2					
g·	$40\bar{4}7$					N_4			
f·	$10\bar{1}2$		N_1						Z_9
e·	$20\bar{2}5$				N_3				
d·	$10\bar{1}4$				N_3				Z_8

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between $(13\cdot0\cdot\bar{1}\bar{3}\cdot1)$ and $(10\bar{1}0)$ the missing members of normal series N_2, i. e., $\frac{1}{2}$ and 2, would develop from the highly improbable hypothetical forms $(35\cdot0\cdot\bar{3}\bar{5}\cdot2)$ and $(31\cdot0\cdot\bar{3}\bar{1}\cdot1)$ respectively.

Zone 3 — Negative Rhombohedrons. Symbol $\frac{k}{l}$.

The most prominent forms of the negative rhombohedrons are unquestionably $(0\bar{1}\bar{1}2)$ and $(02\bar{2}1)$. Assuming these poles as knots we have in the portion of the zone between (0001) and $01\bar{1}2$):

Form	$\left\{\begin{matrix} o \\ 001 \end{matrix}\right.$	α 015	β. $0\cdot7\cdot20$	γ. 025	s. 012
Symbol	0	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{2}{5}$	$\frac{1}{2}$
$\frac{v}{v_2 - v}$	0	$\frac{2}{3}$	$\frac{7}{3}$	4	∞
$\frac{v^1}{2}$	0	$\frac{1}{3}$	$\frac{7}{6}$	2	∞
N_3	0	$\frac{1}{3}$ ...	$(\frac{7}{6})$.	2 .	∞

This portion of the zone is incomplete and contains the extra form $\frac{7}{20}$.

β. $(0\cdot7\cdot\bar{7}\cdot20)$. This rhombohedron was recorded by Lévy [27] on crystal from Offenbanya, Hungary, and later by Peters [28] on a combination from Kapnik, Hungary. Peters did not compare the angle observed on this form with the calculated angle corresponding to $(0\cdot7\cdot\bar{7}\cdot20)$ and a simple calculation will show that his measured angle corresponds more closely to $(02\bar{2}5)$ than to the assumed indices. The form is doubtful.

Hypothetical Rhombohedrons. Referring the above series to N_2 the terms $\frac{1}{2}$ and 1 are missing. The hypothetical negative rhombohedrons corresponding to these members of the series are $(01\bar{1}4)$ and $(01\bar{1}3)$ respectively.

$(01\bar{1}4) = (552)$. The zones $[01\bar{1}4\cdot10\bar{1}0]$ and $[01\bar{1}4\cdot11\bar{2}0]$ although not well marked contain several rational and well established forms. The form should be retained as hypothetical.

$(01\bar{1}3) = (441)$. The zone $[01\bar{1}3\cdot10\bar{1}0]$ passes through the poles of $(11\bar{2}2)$, $(52\bar{7}6)$, $(41\bar{5}3)$, $(10\cdot1\cdot\bar{1}\bar{1}\cdot3)$, etc. The form should be retained as hypothetical.

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27 Levy, A., London (1837).
28 Peters, C. F., Neues Jahrb. Min., **7861**, 434.

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The portion of the zone between $(01\bar{1}2)$ and $(02\bar{2}1)$ is extremely rich in forms, no less than 28 rhombohedrons having been recorded between these limits. Prominent zone intersections occur at $(01\bar{1}1)$ and $(03\bar{3}2)$, which suggest these poles as knot points. Splitting at these points, the forms in the first portion are:

Form	$\delta\cdot 012$	$\mathfrak{a}\cdot 0\cdot11\cdot20$	$\epsilon\cdot 035$	$\varsigma\cdot 023$	$\mathfrak{b}\cdot 0\cdot18\cdot25$	$S\cdot 0\cdot11\cdot14$
Symbol	$\frac{1}{2}$	$\frac{11}{20}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{18}{25}$	$\frac{11}{14}$
$\frac{v-v_1}{v_2-v}$	0	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{11}{14}$	$\frac{4}{3}$
N_3	·0	$(\frac{1}{9})$	$\frac{1}{4}$ ..	$\frac{1}{2}$ ...	$(\frac{11}{14})$ ·	$\frac{4}{3}$

Form	$\eta\cdot 045$	$\mathfrak{v}\cdot 0\cdot14\cdot17$	$\theta\cdot 078$	$D\cdot 0\cdot19\cdot20$	$\kappa\cdot 011$
Symbol	$\frac{4}{5}$	$\frac{14}{17}$	$\frac{7}{8}$	$\frac{19}{20}$	1
$\frac{v-v_1}{v_2-v}$	$\frac{3}{2}$	$\frac{11}{6}$	3	9	∞
N_3	$\frac{3}{2}$ ·	$(\frac{11}{6})$ ..	3 ·	(9)	∞

This portion of the zone is incomplete and contains the extra forms $\frac{1}{9}$, $\frac{11}{14}$, $\frac{11}{6}$ and 9. *Of these the first and last are obviously extremely* close to the knots and might possibly have been due to vicinal development.

$\mathfrak{a}$. $(0\cdot11\cdot\bar{1}\bar{1}\cdot20)$. This rhombohedron was observed by Hobbs[29] on calcite crystals from southern Wisconsin. Prof. Hobbs in discussing this form suggests that it may be due to corrosion.

$\mathfrak{b}$. $(0\cdot18\cdot\bar{1}\bar{8}\cdot25)$. This rhombohedron was also observed by Hobbs[29] on the afore mentioned occurrence. It is worthy of note that the hypothetical rhombohedron $(0\cdot13\cdot\bar{1}\bar{3}\cdot18)$, which reduces to $\frac{4}{5}$ in the above series comes 5′ nearer to the measured angle as given by Hobbs. The form is doubtful.

$\mathfrak{v}$. $(0\cdot14\cdot\bar{1}\bar{4}\cdot17)$. This rhombohedron was noted by Schaller[30] on crystals from Hillsboro, New Mexico. The form, which was established from 3 measurements with the two circle gonimeter, seems more rational than any other of the extra forms in the series. The hypothetical rhombohedron $(05\bar{5}6)$ which would reduce to 2 in the normal

[29] Hobbs, W. N., loc. cit.
[30] Schaller, W. T., loc. cit.

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series lies 10′ further from the pole of the form as measured than the indices $(0\cdot14\cdot\bar{1}\bar{4}\cdot17)$ assigned to it by Schaller.

D. $(0\cdot19\cdot\bar{1}\bar{9}\cdot20)$. This rhombohedron was observed by Cesàro[31] on crystals from Arquennes, Belgium. It lies close to $(01\bar{1}1)$, a form noted by the same authority on several combinations from Rhisnes. It should be referrred to the simpler indices.

Hypothetical Rhombohedrons.— The two members missing from normal series N_2 in the above development are 1 and 2 corresponding respectively to $(03\bar{3}4)$ and $(05\bar{5}6)$. Parallel zones through the poles of these hypothetical forms, do not suggest any simple zonal relations between them and common and well established forms. The forms as hypothetical ones should be rejected.

The portion of the zone between $(01\bar{1}1)$ and $(03\bar{3}2)$ contains the following forms:—

Form	$\kappa\cdot$ 011	♄· 0·17·16	$\lambda\cdot$ 087	$\mu\cdot$ 065	$\nu\cdot$ 054	$G\cdot$ 0·14·11
Symbol	1	$\frac{17}{16}$	$\frac{8}{7}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{14}{11}$
$\frac{v-v_1}{v_2-v}$	0	$\frac{1}{7}$	$\frac{2}{5}$	$\frac{2}{3}$	1	$\frac{6}{5}$
$\frac{v^1}{2}$	0	$\frac{1}{14}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$
N_4	0	$(\frac{1}{14})$	$(\frac{1}{5})$ ·	$\frac{1}{3}$ ·	$\frac{1}{2}$	$\frac{3}{5}$ ··

Form	$\xi\cdot$ 043	$H\cdot$ 0·18·13	$\pi\cdot$ 075	$P\cdot$ 0·13·9	$Y\cdot$ 0·19·13	$\rho\cdot$ 032
Symbol	$\frac{4}{3}$	$\frac{18}{13}$	$\frac{7}{5}$	$\frac{13}{9}$	$\frac{19}{13}$	$\frac{3}{2}$
$\frac{v-v_1}{v_2-v}$	2	$\frac{10}{3}$	4	8	12	∞
$\frac{v^1}{2}$	1	$\frac{5}{3}$	2	4	6	∞
N_4	1 ··	$\frac{5}{3}$	2 ··	4	(6)	∞

This portion of the zone is incomplete and contains the extra forms $\frac{17}{16}$, $\frac{8}{7}$ and $\frac{19}{13}$.

♄. $(0\cdot17\cdot\bar{1}\bar{7}\cdot16)$. This rhombohedron was observed by Cesàro[32] on crystals from Rhisnes, Belgium. It lies close to $(01\bar{1}1)$ and, like $(0\cdot19\cdot\bar{1}\bar{9}\cdot20)$ from Arquennes, should possibly be referred to the simpler indices.

31 Cesàro, G., loc. cit.
32 Cesàro, G., loc. cit.

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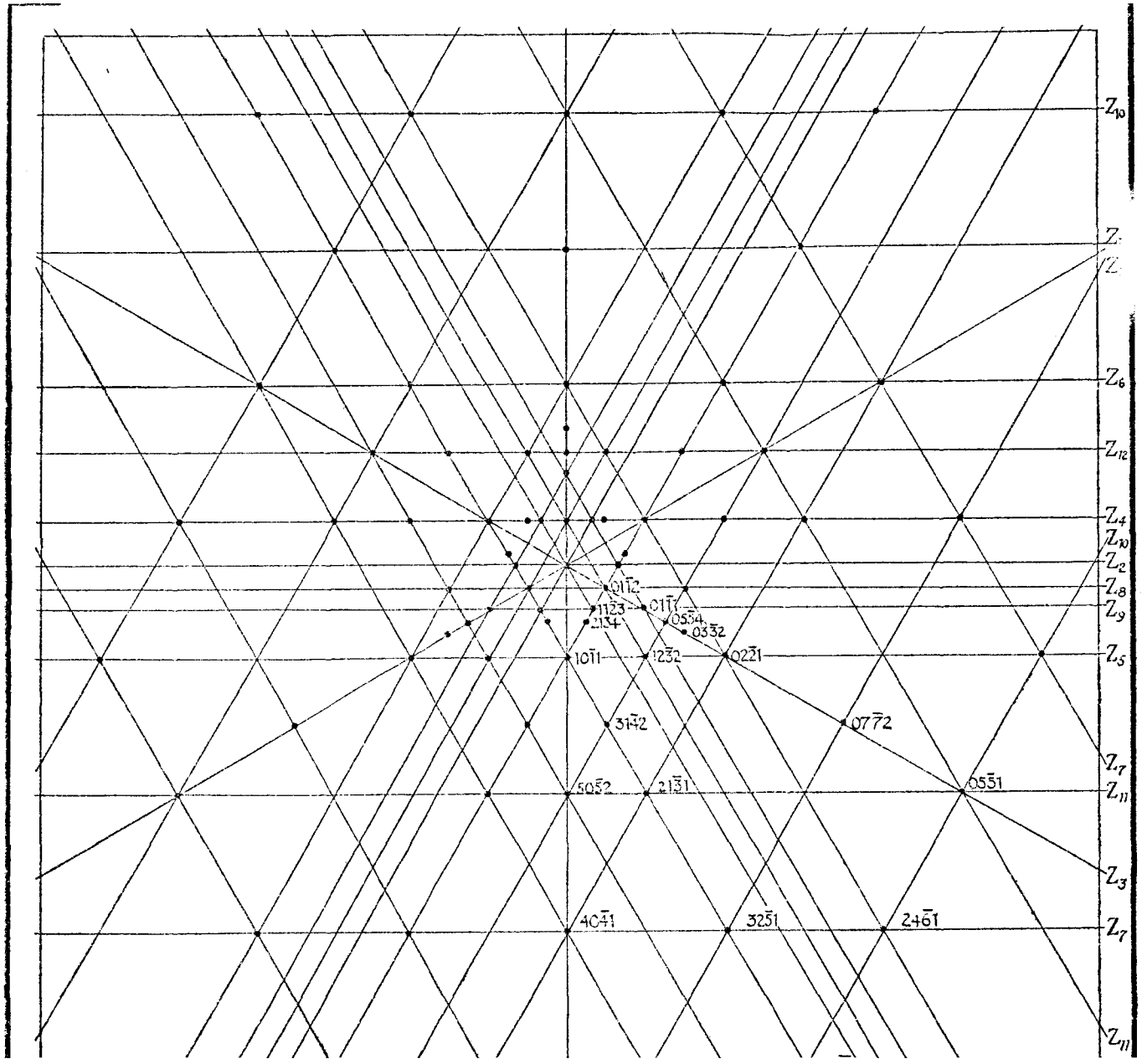

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λ. $(08\bar{8}7)$. This rhombohedron was first observed by Zippe [33] on crystals from Andreasberg. It was subsequently noted by Hessenberg [34] and Sansoni [35] on several combinations from the same locality. It also marks the intersection of the zone $[08\bar{8}7 \cdot 11\bar{2}0]$ which contains a number of well established forms. The form should be regarded as well established.

Y. $(0 \cdot 19 \cdot \bar{1}\bar{9} \cdot 13)$. This rhombohedron was observed by Whitlock [36] on crystals from Lyon Mountain, N. Y. The form occurs on two of the five types described, and on one of them (Type II) is developed to the extent of a dominant habit. It lies close to $(03\bar{3}2)$ and should be regarded as somewhat doubtful.

The following rhombohedrons lie in the portion of the zone between $(03\bar{3}2)$ and $(02\bar{2}1)$:

Form	ρ. 032	σ. 0·11·7	f. 085	τ 0·13·8	L. 053	M. 074	A. 095	N. 0·11·6	φ. 021
Symbol	$\frac{2}{3}$	$\frac{11}{7}$	$\frac{8}{5}$	$\frac{13}{8}$	$\frac{5}{3}$	$\frac{7}{4}$	$\frac{9}{5}$	$\frac{11}{6}$	2
$\frac{v - v_1}{v_2 - v}$	0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	∞
$2v'$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	2	3	4	∞
N_3	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	2	3	(4)	∞

Referred to N_3 this portion of the zone is nearly complete and contains the extra form $\frac{11}{6}$ which would fall in normal series N_4.

N. $(0 \cdot 11 \cdot \bar{1}\bar{1} \cdot 6)$. This rhombohedron was noted by Thurling [37] on crystals from Andreasberg. The form was established from good measurements in the rhombohedral zone. The zone $[0 \cdot 11 \cdot \bar{1}\bar{1} \cdot 6 : 10\bar{1}0]$ is well defined and the form, which occurs in normal series N_4, may be said to be fairly well established.

In the portion of the zone lying between $(02\bar{2}1)$ and $01\bar{1}0)$ the most prominent zone intersections occur at $(05\bar{5}1)$ and $(0 \cdot 11 \cdot \bar{1}\bar{1} \cdot 1)$ Splitting the zone at these points as knots we have for the zone fragment between $(02\bar{2}1)$ and $05\bar{5}1)$:

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33 Zippe, F. X. M., loc. cit., Fig. 90.
34 Hessenberg, F., loc. cit.
35 Sansoni, F. R., loc. cit.
36 Whitlock, H. P., N. Y. State Mus. Mem., **13**, 81 (1910).
37 Thürling, G., N. Jahrb. f. Min. B. B., **4**, 327 (1886).

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Form	$\varphi\cdot 021$	$\chi\cdot 094$	$F\cdot 0\cdot12\cdot5$	$\Psi\cdot 052$	$\omega\cdot 0\cdot11\cdot4$	$\Gamma\cdot 031$
Symbol	2	$\frac{9}{4}$	$\frac{12}{5}$	$\frac{5}{2}$	$\frac{11}{4}$	3
$\frac{v-v_1}{v_2-v}$	0	$\frac{1}{11}$	$\frac{2}{13}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{2}$
N_3	0	$(\frac{1}{11})$	$(\frac{2}{13})$	$(\frac{1}{5})$	$\frac{1}{3}$	$\frac{1}{2}$

Form	$\mathfrak{g}\cdot 0\cdot16\cdot5$	$J\cdot 0\cdot13\cdot4$	$\Delta\cdot 072$	$\Theta\cdot 041$	$\Lambda\cdot 092$	$\Xi\cdot 051$
Symbol	$\frac{16}{5}$	$\frac{13}{4}$	$\frac{7}{2}$	4	$\frac{9}{2}$	5
$\frac{v-v_1}{v_2-v}$	$\frac{2}{3}$	$\frac{5}{7}$	1	2	5	∞
N_3	$\frac{2}{3}$	$(\frac{5}{7})$	1 ·	2 ·	(5)	∞

Referred to N_3 this portion of the zone is somewhat incomplete and contains the extra forms $\frac{9}{4}$, $\frac{12}{5}$, $\frac{5}{2}$, $\frac{13}{4}$ and $\frac{9}{2}$.

χ. $(09\bar{9}4)$. This rhombohedron is recorded by Bournon[38] from Derbyshire and Cumberland. It was regarded by Zippe[39] as uncertain. Figure 199 of Bournon suggests that the rhombohedron determined by him as $(09\bar{9}4)$ may possibly be $(05\bar{5}2)$. The zone $[09\bar{9}4\cdot2\bar{1}\bar{1}0]$ contains only one unimportant scalenohedral form. The form must be considered doubtful.

F. $(0\cdot12\cdot\bar{1}\bar{2}\cdot5)$. This rhombohedron was determined by Zippe[40] from a curved face without measurements. It was also found by Sansoni[41] on crystals from Andreasberg, (Combination 189, Fig. 17). Zonal relations suggest a somewhat higher degree of probability for this form than for the preceding.

Ψ. $(05\bar{5}2)$. This rhombohedron was also recorded from Andreasberg by Zippe.[42] In the combination in which it is shown (Fig. 90), $(05\bar{5}2)$ truncate the polar edges of $(12\bar{3}1)$ in the negative sextants, a zonal relation which seems highly rational. Furthermore, the form falls in normal series N_5 and marks the intersection of three well defined zones. The form may be said to be well established.

J. $(0\cdot13\cdot\bar{1}\bar{3}\cdot4)$. This rhombohedron was observed by Rogers[43]

38 Bournon, C. de., London (1808).
39 Zippe, loc. cit.
40 Zippe, loc. cit.
41 Sansoni, loc. cit.
42 Zippe, loc. cit.
43 Rogers, A. F., S. of M. Quar., **23** (1902).

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on crystals from the trap region of New Jersey, and was identified by the fact that its planes truncated the polar edges of $(17\cdot11\cdot\bar{2}\bar{8}\cdot6)$ in the negative sextants. Like the preceding form discussed, it falls in normal series N_5. It also lies in a somewhat well marked zone $[0\cdot13\cdot\bar{1}\bar{3}\cdot4 : \bar{2}\bar{1}10]$. The form may be regarded as probable.

Λ. $(09\bar{9}2)$. This rhombohedron was noted by vom Rath[44] on crystals from Andreasberg in combination with $(11\cdot8\cdot\bar{1}\bar{9}\cdot3)$, the polar edges of which it truncates in the negative sextants. It falls in normal series N_5 and may be regarded as probable.

The zone fragment between $(05\bar{5}1)$ and $(0\cdot11\cdot\bar{1}\bar{1}\cdot1)$ contains the following forms:

Form	Ξ. 051	Q. 071	Π. 081	B. 091	Σ. 0·11·1
Symbol	5	7	8	9	11
$\frac{v-v_1}{v_2-v}$	0	$\frac{1}{2}$	1	2	∞
N_2	0	$\frac{1}{2}$	1	2	∞

Referred to N_2 this section of the zone is complete.

The zone fragment between $(0\cdot11\cdot\bar{1}\bar{1}\cdot1)$ and $(01\bar{1}0)$ contains the following forms:

Form	Σ. 0·11·1	C. 0·13·1	Φ. 0·14·1	Ψ. 0·17·1	Ω. 0·20·1	T. 0·28·1	U. 0·36·1	m 010
Symbol	11	13	14	17	20	28	36	∞
$\frac{v-11}{6}$	0	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{17}{6}$	$\frac{25}{6}$	
N_3	0	$\frac{1}{3}$	$\frac{1}{2}$ ·	1	$\frac{3}{2}$ ·	$(\frac{17}{6})$ ··	$(\frac{25}{6})$	∞

This section of the zone is incomplete and contains the extra forms 28 and 36.

T. $(0\cdot28\cdot\bar{2}\bar{8}\cdot1)$. This rhombohedron was noted by Thürling[45] on one specimen from Andreasberg. Although the observations of the author cited agree very closely with theory, it is significant that the rhombohedron $(0\cdot29\cdot\bar{2}\bar{9}\cdot1)$, the ρ value of which varies only 4′ from that of $(0\cdot28\cdot\bar{2}\bar{8}\cdot1)$, corresponds to much simpler Miller indices $[(0\cdot29\cdot\bar{2}\bar{9}\cdot1) = (10\cdot10\cdot\bar{1}\bar{9}),\ (0\cdot28\cdot\bar{2}\bar{8}\cdot1) = (29\cdot29\cdot\bar{5}\bar{5})]$ and lies in normal series N_3. The form $(0\cdot28\cdot\bar{2}\bar{8}\cdot1)$ should be regarded as doubtful.

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[44] vom Rath, G., Pogg. Ann., **132**, 517 (1867).
[45] Thürling, loc. cit.

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U. $(0\cdot36\cdot\bar{3}\bar{6}\cdot1)$. This rhombohedron was noted by Johansson [46] on crystals from Norberg, Sweden. In this instance, since the form was observed on crystals of rhombohedral habit, it was impossible to verify the identification of this rhombohedron from zonal relations with scalenohedral forms. The hypothetical rhombohedron $(0\cdot35\cdot\bar{3}\bar{5}\cdot1)$ differs from $(0\cdot36\cdot\bar{3}\bar{6}\cdot1)$ in but $2\frac{1}{2}'$ of ρ value, has much simpler Miller indices $[(0\cdot35\cdot\bar{3}\bar{5}\cdot1) = (12\cdot12\cdot\bar{2}\bar{3})$, $(0\cdot36\cdot\bar{3}\bar{6}\cdot1) = (37\cdot37\cdot\bar{7}\bar{1})]$ and falls in normal series N_4. The rhombohedron $(0\cdot36\cdot\bar{3}\bar{6}\cdot1)$ should be regarded as doubtful.

Hypothetical Rhombohedrons.— None of the forms missing from normal series N_3 as developed above suggest by zonal relations the possibility of hypothetical rhombohedrons between the limits discussed. The two hypothetical forms $(0\cdot29\cdot\bar{2}\bar{9}\cdot1)$ and $(0\cdot35\cdot\bar{3}\bar{5}\cdot1)$ suggested in the above discussion should not be retained as such.

Zone 3 = Z_3 Negative Rhombohedrons

Letter	Form	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
$\alpha\cdot$	$01\bar{1}5$				N_3				
$\beta\cdot$	$0\cdot7\cdot\bar{7}\cdot20$							?	
$\gamma\cdot$	$02\bar{2}5$			N_2					
$\delta\cdot$	$01\bar{1}2$	×							Z_4, Z_8
$\alpha\cdot$	$0\cdot11\cdot\bar{1}\bar{1}\cdot20$							?	
$\epsilon\cdot$	$03\bar{3}5$					N_4			
$s\cdot$	$02\bar{2}3$			N_2					
$\mathfrak{b}\cdot$	$0\cdot18\cdot\bar{1}\bar{8}\cdot25$							?	
$S\cdot$	$0\cdot11\cdot\bar{1}\bar{1}\cdot14$					N_4			
$\eta\cdot$	$04\bar{4}5$				N_3				
$\mathfrak{v}\cdot$	$0\cdot14\cdot\bar{1}\bar{4}\cdot17$							?	
$\theta\cdot$	$07\bar{7}8$				N_3				
$D\cdot$	$0\cdot19\cdot\bar{1}\bar{9}\cdot20$							?	
$\kappa\cdot$	$01\bar{1}1$	×							Z_9
$\mathfrak{h}\cdot$	$0\cdot17\cdot\bar{1}\bar{7}\cdot16$							?	
$\lambda\cdot$	$08\bar{8}7$						N_5		

[46] Johansson, K., Geol. Fören. Förh. **14**, 49 (1892).

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Zone 3 = Z_3 Negative Rhombohedrons (continued).

Letter	Form	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
μ·	$06\bar{6}5$				N_3				
υ·	$05\bar{5}4$			N_2					Z_{12}
G·	$0{\cdot}14{\cdot}\bar{1}\bar{4}{\cdot}11$					N_4			
ξ·	$04\bar{4}3$		N_1						
H·	$0{\cdot}18{\cdot}\bar{1}\bar{8}{\cdot}13$					N_4			
π·	$07\bar{7}5$			N_2					
P·	$0{\cdot}13{\cdot}\bar{1}\bar{3}{\cdot}9$					N_4			
Y·	$0{\cdot}19{\cdot}\bar{1}\bar{9}{\cdot}13$							?	
ρ·	$03\bar{3}2$	×							
σ·	$0{\cdot}11{\cdot}\bar{1}\bar{1}{\cdot}7$				N_3				
f·	$08\bar{8}5$			N_2					
ι·	$0{\cdot}13{\cdot}\bar{1}\bar{3}{\cdot}8$				N_3				
L·	$05\bar{5}3$		N_1						
M·	$07\bar{7}4$			N_2					
A·	$09\bar{9}5$				N_3				
N·	$0{\cdot}11{\cdot}\bar{1}\bar{1}{\cdot}6$					N_4			
φ·	$02\bar{2}1$	×							Z_4, Z_5
χ·	$09\bar{9}4$							?	
F·	$0{\cdot}12{\cdot}\bar{1}\bar{2}{\cdot}5$							?	
Ψ·	$05\bar{5}2$						N_5		
ω·	$0{\cdot}11{\cdot}\bar{1}\bar{1}{\cdot}4$				N_3				
Γ·	$03\bar{3}1$			N_2					
$\mathfrak{g}$·	$0{\cdot}16{\cdot}\bar{1}\bar{6}{\cdot}5$				N_3				
J·	$0{\cdot}13{\cdot}\bar{1}\bar{3}{\cdot}4$						N_5		
Δ·	$07\bar{7}2$		N_1						Z_{13}
Θ·	$04\bar{4}1$			N_2					
Λ·	$09\bar{9}2$						N_5		
Ξ·	$05\bar{5}1$	×							Z_{10}, Z_{11}
Q·	$07\bar{7}1$			N_2					
Π·	$08\bar{8}1$		N_1						Z_7
B·	$09\bar{9}1$			N_2					
Σ·	$0{\cdot}11{\cdot}\bar{1}\bar{1}{\cdot}1$	×							

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Zone 3 = Z_3 Negative Rhombohedrons (concluded).									
Letter	From	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
$C\cdot$	$0\cdot13\cdot\bar{1}\bar{3}\cdot1$				N_3				
$\Phi\cdot$	$0\cdot14\cdot\bar{1}\bar{4}\cdot1$			N_2					
Ψ	$0\cdot17\cdot\bar{1}\bar{7}\cdot1$		N_1						
$\Omega\cdot$	$0\cdot20\cdot\bar{2}\bar{0}\cdot1$				N_3				
$T\cdot$	$0\cdot28\cdot\bar{2}\bar{8}\cdot1$							?	
$U\cdot$	$0\cdot36\cdot\bar{3}\bar{6}\cdot1$							?	
Hypothetical Rhombohedrons									
	$01\bar{1}4$			N_2					
	$01\bar{1}3$		N_1						

In the discussion of the foregoing zones the Bravais symbols have been employed for the obvious reason that they express most clearly the relations of the forms to the zones discussed. In considering the scalenohedral forms, however, the zonal and serial relations are much better emphasized by the Miller symbols, which furthermore are more rational for the more prominent zone intersections. The Miller symbols have consequently been adopted for the subsequent analysis.

ZONE 4 — SCALENOHEDRONS OF THE ZONE $[01\bar{1}2 : 10\bar{1}1] = [110 : 100]$

SYMBOL $\frac{h}{k}$.

In this portion of the principal zone a great number of scalenohedrons have been observed in the comparatively small arc comprised between the limiting poles. Prominent zone intersections occur at $(11\bar{2}2) = (210)$ and $(21\bar{3}4) = (310)$. Splitting the zone at these poles we have for the forms in the first fragment:

Form	δ. 110	f: 760	h: 540	z: 320	i: 11·7·0	u: 850	y: 530	π 210
Symbol	1	$\frac{7}{6}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{11}{7}$	$\frac{8}{5}$	$\frac{5}{3}$	2
$\frac{v - v_1}{v_2 - v}$	0	$\frac{1}{5}$	$\frac{1}{3}$	1	$\frac{4}{3}$	$\frac{3}{2}$	2	∞
N_3	0	$(\frac{1}{5})$	$\frac{1}{3}$ ..	1	$(\frac{4}{3})$	$\frac{3}{2}$	2 .	∞

Referred to N_3 this zone fragment is incomplete and contains the extra forms $\frac{7}{6}$ and $\frac{11}{7}$.

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$f : (1\cdot6\cdot\bar{7}\cdot13) = (760)$. This scalenohedron was noted by Schaller [47] on crystals from California. It lies in the zone $[0001 : 16\bar{7}0]$ which contains the forms $(16\bar{7}4)$ and $16\bar{7}1)$, but of which the limiting prism has not as yet been recorded. The form falls in normal series N_5 and may be considered fairly probable.

$i : (4\cdot7\cdot\bar{1}\bar{1}\cdot18) = (11\cdot7\cdot0)$. This scalenohedron was recorded by Moesz [48] on crystals from Körösmezö, Hungary. Although the pole of $(4\cdot7\cdot\bar{1}\bar{1}\cdot18)$ does not mark the intersection of any well defined zone, the form falls in normal series N_4 and should be regarded as fairly probable.

Hypothetical Scalenohedrons: The hypothetical form $(13\bar{4}7) = (430)$ reduces to $\frac{1}{2}$ in the above development. The hypothetical scalenohedron $(13\bar{4}7)$ lies in the well defined vertical zone $[0001 : 13\bar{4}0]$ of which the limiting prism is a very well established form. Several other more or less well marked zones intersect at this pole. The form should be retained as hypothetical.

In the zone fragment between $(11\bar{2}3) = (210)$ and $(21\bar{3}4) = (310)$ the following forms have been observed:

Form	π 210	k: 940	x: 730	l: 520	v: 11·4·0	m: 17·6·0	t: 310
Symbol	2	$\frac{9}{4}$	$\frac{7}{3}$	$\frac{5}{2}$	$\frac{11}{4}$	$\frac{17}{6}$	3
$\frac{v - v_1}{v_2 - v}$	0	$\frac{1}{3}$	$\frac{1}{2}$	1	3	5	∞
N_3	0	$\frac{1}{3}$	$\frac{1}{2}$ ·	1	·· 3	(5)	∞

Referred to N_3 this portion of the zone is incomplete and contains the extra form $\frac{17}{6}$.

$m : (11\cdot6\cdot\bar{1}\bar{7}\cdot23) = (17\cdot6\cdot0)$. This scalenohedron was noted by Gonnard [49] on crystals from Couzon, France. The form does not fall at any zone intersection and must be regarded as doubtful.

The zone fragment between $(21\bar{3}4) = (310)$ and $(10\bar{1}1) = (100)$ contains the following forms:—

Form	t: 310	$\mathfrak{h}$: 16·5·0	n: 10·3·0	g: 720	w: 410	f: 920	e: 510	o: 11·2·0
Symbol	3	$\frac{16}{5}$	$\frac{10}{3}$	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$
$v - 3$	0	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
N_3	0	$(\frac{1}{5})$	$\frac{1}{3}$	$\frac{1}{2}$ ·	1	$\frac{3}{2}$	2	$(\frac{5}{2})$

47 Schaller, W. T., Zeit. f. Kryst., **44**, 324 (1908).
48 Moesz, G., Fold. Közlöny., **27**, 495 (1897).
49 Gonnard, F., Bull. Soc. fr. Min., **20** (1897).

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Form	q: 610	α: 13·2·0	c: 710	b: 810	a: 910	β: 10·1·0	d: 14·1·0	p· 100
Symbol	6	$\frac{13}{2}$	7	8	9	10	14	∞
$v-3$	3	$\frac{7}{2}$	4	5	6	7	11	∞
N_3	3	$(\frac{7}{2})$	(4)	(5)	(6)	(7)	(11)	∞

This fragment of the zone is somewhat incomplete with respect to N_3 and contains a great many extra forms some of which fall in normal series N_4.

$\mathfrak{h}$: $(11\cdot5\cdot\bar{16}\cdot21) = (16\cdot5\cdot0)$. This scalenohedron was observed by Schaller [50] on crystals from Terlingua, Texas. The form was determined from extremely small planes, giving weak signals. The vertical zone $[0001 : 11\cdot5\cdot\bar{16}\cdot21]$ also contains the form $(11\cdot5\cdot\bar{16}\cdot6)$ which marks several zone intersections. The form which falls in normal series N_5 seems fairly probable.

o : $(9\cdot2\cdot\bar{11}\cdot13) = (11\cdot2\cdot0)$. This scalenohedron was noted by Melczer [51] on crystals from Budapest. The pole $(9\cdot2\cdot\bar{11}\cdot13)$ does not fall in any zone other than the one under discussion and although the term $\frac{5}{2}$ is referable to normal series N_4 the form should be regarded as somewhat doubtful.

α: $(11\cdot2\cdot\bar{13}\cdot15) = (13\cdot2\cdot0)$. This scalenohedron was noted by Sansoni [52] on three crystals from Andreasberg. The measured angles as determined by him agree well with theory although the pole lies very close to $(61\bar{7}8)$, a form which agrees better in zonal relations. The scalenohedron $(11\cdot2\cdot\bar{13}\cdot15)$, however, is referable to normal series N_5 and should be regarded as somewhat probable.

c: $(61\bar{7}8) = (710)$. This scalenohedron was first noted by Hausmann [53] on crystals from Andreasberg. The form was later observed by Hessenberg [54] and is regarded by Irby as well established. The zone $[61\bar{7}8 : 01\bar{1}0]$ is, moreover very well marked and the form falls in normal series N_4. It should be regarded as well established.

b: $(71\bar{8}9) = (810)$. This scalenohedron was first recorded by Sella [55] on crystals from Sardinia. It was subsequently noted by Hessenberg from Andreasberg and by vom Rath from Lake Superior. It falls in normal series N_5 and although not as rational

50 Schaller, loc. cit.
51 Melczer, loc. cit.
52 Sansoni, F., Acad. d. Linc., **19**, Pl. 3, Fig. 20 (1884).
53 Hausmann, J. F. L., Handbuch der Mineralogie (1847).
54 Hessenberg, loc. cit.
55 Sella, Q., Mem. Ac. Turin., 2, **17**, (1856).

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from the point of view of zonal relations as the previous form should be regarded as well established.

a: $(8\cdot1\cdot\bar{9}\cdot10) = (910)$. This scalenohedron was first recorded by Zippe[56] on crystals from Andreasberg and was subsequently noted by vom Rath[57] from Lake Superior, and by Sansoni[58] from Andreasberg. The zone $[8\cdot1\cdot\bar{9}\cdot10 : 01\bar{1}0]$ is well marked and the form is probable.

β: $(9\cdot1\cdot1\bar{0}\cdot11) = (10\cdot1\cdot0)$. This scalenohedron was observed by Sansoni[59] on crystals from Andreasberg. The form occurs on combination 249, Figure 19, in combination with $(01\bar{1}2)$ and $(8\cdot1\cdot\bar{9}\cdot10)$. It marks the intersection of two minor zones containing several prominent forms. The form may be ranked as well established.

d: $(13\cdot1\cdot\bar{14}\cdot15) = (14\cdot1\cdot0)$. This scalenohedron was observed by vom Rath[60] on crystals from the trap of the Nahe basin. The form was determined by approximate measurements on corroded faces. Furthermore it lies very close to $(10\bar{1}1)$ and falls in a high normal series. The form should be classed as doubtful.

Zone 4. Scalenohedrons of the Zone $[10\bar{1}1 : 11\bar{2}0] = [100 : 10\bar{1}]$ Symbol $\frac{l}{h}$.

In this portion of the principal zone the prominent zone intersections occur at the poles of the common scalenohedrons $H : (31\bar{4}2) = (30\bar{1})$, $K : (21\bar{3}1) = (20\bar{1})$, $P : (32\bar{5}1) = (30\bar{2})$ and $T : (32\bar{7}1) = (40\bar{3})$. Splitting the zone at these points we have for the forms in the fragment between (100) and $30\bar{1}$):

Form	$p\cdot$ 100	A: $11\cdot0\cdot\bar{1}$	B: $17\cdot0\cdot\bar{2}$	C: $70\bar{1}$	γ: $19\cdot0\cdot\bar{3}$	D: $60\bar{1}$	E: $50\bar{1}$	I: $13\cdot0\cdot\bar{3}$
Symbol	0	$\frac{1}{11}$	$\frac{2}{17}$	$\frac{1}{7}$	$\frac{3}{19}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{3}{13}$
$\frac{v - v_1}{v_2 - v}$	0	$\frac{3}{8}$	$\frac{6}{11}$	$\frac{3}{4}$	$\frac{9}{10}$	1	$\frac{3}{2}$	$\frac{9}{4}$
$\frac{2v^1}{3}$	0	$\frac{1}{4}$	$\frac{4}{11}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	1	$\frac{3}{2}$
N_4	0	$\frac{1}{4}$ ·	$(\frac{4}{11})$ ·	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$ ·	1 ·	$\frac{3}{2}$

56 Zippe, loc. cit.
57 vom Rath, G., Pogg. Ann., **132**, 137.
58 Sansoni, loc. cit.
59 Sansoni, loc. cit.
60 vom Rath, G., Pogg. Ann., **135**, 572.

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Form	Z: $21\cdot0\cdot\bar{5}$	F: $40\bar{1}$	δ: $19\cdot0\cdot\bar{5}$	μ: $11\cdot0\cdot\bar{3}$	G: $70\bar{2}$	λ: $17\cdot0\cdot\bar{5}$	H: $30\bar{1}$
Symbol	$\frac{5}{21}$	$\frac{1}{4}$	$\frac{5}{19}$	$\frac{3}{11}$	$\frac{2}{7}$	$\frac{5}{17}$	$\frac{1}{3}$
$\frac{v-v_1}{v_2-v}$	$\frac{5}{2}$	3	$\frac{15}{4}$	$\frac{9}{2}$	6	$\frac{15}{2}$	∞
$\frac{2v^1}{3}$	$\frac{5}{3}$	2	$\frac{5}{2}$	3	4	5	∞
N_4	$\frac{5}{3}$	2	$\frac{5}{2}$	3	4	(5)	∞

Referred to N_4 this fragment of the zone is nearly complete and contains the extra forms $\frac{2}{17}$ and $\frac{5}{17}$.

B: $(17\cdot2\cdot\bar{1}\bar{9}\cdot15) = (17\cdot0\cdot\bar{2})$. This scalenohedron according to Irby has been observed a number of times.[61] Although not a prominent zone intersection, the form seems to be fairly probable.

λ: $(17\cdot5\cdot\bar{2}\bar{2}\cdot8) = (15\cdot0\cdot\bar{7})$. This scalenohedron was noted by Rogers [62] as occurring on calcite crystals from Great Notch, N. J. Although somewhat striated parallel to the zone the form is said to have given good reflections. It should be classed as fairly probable.

In the zone fragment between H: and K: the following forms have been recorded:

Form	H: $30\bar{1}$	J: $50\bar{2}$	ξ: $70\bar{3}$	Π: $11\cdot0\cdot\bar{5}$	Ψ: $15\cdot0\cdot\bar{7}$	K: $20\bar{1}$
Symbol	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{5}{11}$	$\frac{7}{15}$	$\frac{1}{2}$
$\frac{v-v_1}{v_2-v}$	0	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{8}{3}$	4	∞
$\frac{3v^1}{4}$	0	$\frac{1}{2}$	1	2	3	∞
N_2	0	$\frac{1}{2}$	1	2	(3)	∞

Referred to N_2 this fragment of the zone is complete and contains the extra form $\frac{7}{15}$.

Ψ: $(15\cdot7\cdot\bar{2}\bar{2}\cdot8) = (15\cdot0\cdot\bar{7})$. This scalenohedron was observed by Schnorr [63] on crystals from Neumark, Prussia. Although the pole

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61 von Zepharovich, V. R., on calcite from Bleiburg, Lotos (1878).
Hessenberg, F., on calcite from Bleiburg, Min. Not., **8**.
Schnorr, on calcite from Zwickau (1874).
vom Rath, G., on calcite from Ahrenthal, Pogg. Ann., **155**.

62 Rogers A. F., S. of M. Quar., **23**, 336 (1902).

63 Schnorr, Wissensch. Beil. z. Programme des Realgym. zu Zwickau, **16** (1896).

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of $(15\cdot7\cdot\bar{2}\bar{2}\cdot8)$ does not fall at any important zone intersection, the form occurs in normal series N_3 in the above development and may be considered fairly probable.

In the zone fragment between K: and P: the following scalenohedrons have been observed:

Form	K: $20\bar{1}$	σ: $41\cdot0\cdot\bar{2}\bar{1}$	τ: $25\cdot0\cdot\bar{1}\bar{3}$	L: $17\cdot0\cdot\bar{9}$	ϵ: $90\bar{5}$	M: $70\bar{4}$	φ: $19\cdot0\cdot\bar{1}\bar{1}$
Symbol	$\frac{1}{2}$	$\frac{21}{41}$	$\frac{13}{25}$	$\frac{9}{17}$	$\frac{5}{9}$	$\frac{4}{7}$	$\frac{11}{19}$
$\frac{v-v_1}{v_2-v}$	0	$\frac{3}{38}$	$\frac{3}{22}$	$\frac{3}{14}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{9}{10}$
$\frac{2v^1}{3}$	0	$\frac{1}{19}$	$\frac{1}{11}$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$
N_3	0	$(\frac{1}{19})$	$(\frac{1}{11})$	$(\frac{1}{7})$	$\frac{1}{3}$	$\frac{1}{2}$	$(\frac{3}{5})$

Form	χ: $29\cdot0\cdot\bar{1}\bar{7}$	N: $50\bar{3}$	$13\cdot0\cdot\bar{8}$	o: $80\bar{5}$	p: $17\cdot0\cdot\bar{1}\bar{1}$	P: $30\bar{2}$
Symbol	$\frac{17}{29}$	$\frac{3}{5}$	$\frac{8}{13}$	$\frac{5}{8}$	$\frac{11}{17}$	$\frac{2}{3}$
$\frac{v-v_1}{v_2-v}$	$\frac{15}{14}$	$\frac{3}{2}$	$\frac{9}{4}$	3	$\frac{15}{2}$	∞
$\frac{2v^1}{3}$	$\frac{5}{7}$	1	$\frac{3}{2}$	2	5	∞
N_3	$(\frac{5}{7})$	1	$\frac{3}{2}$	2	(5)	∞

Referred to N_3 this zone fragment is somewhat incomplete and contains the extra forms $\frac{21}{41}$, $\frac{13}{25}$, $\frac{9}{17}$, $\frac{11}{19}$, $\frac{17}{29}$, and $\frac{11}{17}$, three of which are obviously close to the pole of the common form $(21\bar{3}1) = (20\bar{1})$.

σ: $(41\cdot21\cdot\bar{6}\bar{2}\cdot20) = (41\cdot0\cdot\bar{2}\bar{1})$
τ: $(25\cdot13\cdot\bar{3}\bar{8}\cdot12) = (25\cdot0\cdot\bar{1}\bar{3})$ } These two scalenohedrons were reported by Sansoni [64] from Schapbachthal, Baden. They were found on one crystal and are evidently vicinal to K: $(21\bar{3}1)$. Both forms, although included in the lists of Rogers [65], Bumuller [66] and Whitlock [67], should be classed as doubtful.

L: $(17\cdot9\cdot\bar{2}6\cdot8) = (17\cdot0\cdot\bar{9})$. This scalenohedron was first noted by Zippe [68] on crystals from Hoffnungsbergbrau, near Lissnia, on

64 Sansoni, F., Giorn. d. Min., **1** (2), 299 (1900).
65 Rogers, A. F., S. of M. Quar., **22**, 429 (1901).
66 Bumuller, C. N., Jahrb. f. Min. B. B., **28**, 233.
67 Whitlock, H. P., N. Y. State Mus. Mem., **13**, 42 (1910).
68 Zippe, F. X. M., loc. cit.

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which the scalenohedron K: is not present, the only other scalenohedron of the principal zone being $V: (6\cdot5\cdot\bar{1}1\cdot1) = (60\bar{5})$. Sansoni [69] has noted the form on a crystal from Schapbachthal where he describes it as vicinal to K:. The form is somewhat doubtful but more probable than the two preceding.

$\varphi: (19\cdot11\cdot\bar{3}\bar{0}\cdot8) = (19\cdot0\cdot\bar{1}\bar{1})$. This scalenohedron was observed by Sansoni [70] on crystals from Monticatini, Italy. Although this form falls in Normal series N_4 it is noteworthy that the hypothetical scalenohedron $(12\cdot7\cdot\bar{1}\bar{9}\cdot5) = (12\cdot0\cdot\bar{7})$ which falls in normal series N_3 in the above development presents better zonal relation than $(19\cdot11\cdot\bar{3}\bar{0}\cdot8)$. The form is fairly probable.

$\chi: (29\cdot17\cdot\bar{4}\bar{6}\cdot12) = (29\cdot0\cdot\bar{1}7)$. This scalenohedron was first noted by Schnorr [71] on crystals from Neumark. As noted above the hypothetical scalenohedron $(12\cdot7\cdot\bar{1}\bar{9}\cdot5) = (12\cdot0\cdot\bar{7})$ which reduces to $\frac{2}{3}$ in normal series N_3 lies between the forms $\varphi: (19\cdot11\cdot\bar{3}0\cdot8) = (19\cdot0\cdot\bar{1}1)$ and $\chi: (29\cdot17\cdot\bar{4}\bar{6}\cdot12) = (29\cdot0\cdot\bar{1}7)$ in the zone, possesses much more rational Miller indices and lies close enough to χ: to be within the limit of error of an ordinary observation. It is possible that χ: should be referred to the simpler indices.

$p: (17\cdot11\cdot\bar{2}\bar{8}\cdot6) = (17\cdot0\cdot\bar{1}\bar{1})$. This scalenohedron was found by Palache [72] on two crystals from the Lake Superior region of Michigan. The form falls in normal series N_5 and also marks the intersection of at least two well defined secondary zones. The form is probable.

Hypothetical Scalenohedron. The hypothetical scalenohedron $(12\cdot7\cdot\bar{1}\bar{9}\cdot5)$ referred to above should be retained as a hypothetical form.

The following forms have been recorded between the poles P: $(32\bar{5}1) = (30\bar{2})$ and $T: (43\bar{7}1) = (40\bar{3})$:

Form	P: $30\bar{2}$	Q: $19\cdot0\cdot13$	$13\cdot0\cdot\bar{9}$	R: $10\cdot0\cdot\bar{7}$	s: $70\bar{5}$	s: $11\cdot0\cdot\bar{8}$	η $23\cdot0\cdot\bar{1}\bar{7}$	T: $40\bar{3}$
Symbol	$\frac{2}{3}$	$\frac{13}{19}$	$\frac{9}{13}$	$\frac{7}{10}$	$\frac{5}{7}$	$\frac{8}{11}$	$\frac{17}{23}$	$\frac{3}{4}$
$\frac{v-v_1}{v_2-v}$	0	$\frac{4}{15}$	$\frac{4}{9}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{8}{3}$	$\frac{20}{3}$	∞
$\frac{3v^1}{4}$	0	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	5	∞
N_3	0	$(\frac{1}{5})$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	(5)	∞

69 Sansoni, F., loc. cit.
70 Sansoni, F., Atti Acc. Torino, **23** (1888).
71 Schnorr, loc. cit.
72 Palache, C., Mich. Geol. Surv., **6**, 16 (1897).

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Referred to N_3 this fragment of the zone is somewhat incomplete and contains the extra forms $\frac{13}{19}$ and $\frac{17}{23}$ both of which fall in normal series N_5.

Q: $(19\cdot13\cdot\bar{3}\bar{2}\cdot6) = (19\cdot0\cdot\bar{1}\bar{3})$. This scalenohedron was noted by Hessenberg [73] on crystals from Andreasberg. Irby [74] in commenting on this form in his list, states that the only determination of it is not perfectly satisfactory. Sansoni [75] includes it in combination 242 from Andreasberg which he ascribes to Hessenberg without comment. The form must be considered as rather improbable.

η : $(23\cdot17\cdot\bar{4}\bar{0}\cdot6) = (23\cdot0\cdot\bar{1}\bar{7})$. This scalenohedron was described by Sansoni [76] on three crystals from Andreasberg. Although the pole does not lie in any well defined secondary zone, the angular values as determined by Sansoni agree so well with theory as to place the form beyond question.

The following relatively steep scalenohedral forms have been recorded in the zone fragment between T : $(43\bar{7}1) = (40\bar{3})$ and a $(11\bar{2}0) = (10\bar{1})$.

Form	T: $40\bar{3}$	Θ: $90\bar{7}$	U: $50\bar{4}$	$11\cdot0\cdot\bar{9}$	V: $60\bar{5}$	W: $13\cdot0\cdot\bar{1}\bar{1}$	X: $70\bar{6}$
Symbol	$\frac{3}{4}$	$\frac{7}{9}$	$\frac{4}{5}$	$\frac{9}{11}$	$\frac{5}{6}$	$\frac{11}{13}$	$\frac{6}{7}$
$\frac{v-v_1}{v_2-v}$	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
$2v^1$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$
N_4	0	$\frac{1}{4}$ ..	$\frac{1}{2}$ ..	$\frac{3}{4}$	1	$(\frac{5}{4})$.	$\frac{3}{2}$.

Form	ω: $15\cdot0\cdot\bar{1}\bar{3}$	ι: $17\cdot0\cdot\bar{1}\bar{5}$	Y: $90\bar{8}$	υ: $10\cdot0\cdot\bar{9}$	κ: $21\cdot0\cdot\bar{1}\bar{9}$	a $10\bar{1}$
Symbol	$\frac{13}{15}$	$\frac{15}{17}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{19}{21}$	1
$\frac{v-v_1}{v_2-v}$	$\frac{7}{8}$	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{13}{8}$	∞
$2v^1$	$\frac{7}{4}$	$\frac{9}{4}$	$\frac{5}{2}$	3	$\frac{13}{4}$	∞
N_4	$(\frac{7}{4})$.	$(\frac{9}{4})$	$\frac{5}{2}$	3	$(\frac{13}{4})$	∞

This zone fragment is quite incomplete and contains the extra forms $\frac{11}{13}$, $\frac{13}{15}$, $\frac{15}{17}$ and $\frac{19}{21}$.

W: $(13\cdot11\cdot\bar{2}\bar{4}\cdot2) = (13\cdot0\cdot\bar{1}\bar{1})$. This scalenohedron has been as-

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73 Hessenberg, F., Min. Not., **4**, Frankfort (1875).
74 Irby, loc. cit.
75 Sansoni, F., R. Accad. d. Linc. Mem., **19** (1884).
76 Sansoni, F., R. Accad. d. Linc. Mem., **19** (1884).

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signed to crystals from Andreasberg by Naumann [77] and Hausmann.[78] Irby [79] in commenting on this form throws some doubt on the correctness of the determinations and points out the difficulties in the way of distinguishing it from $(6\cdot5\cdot\bar{11}\cdot1)$ and $(7\cdot6\cdot\bar{13}\cdot1)$ both well established poles on either side of it. Sansoni [80] ascribes the form to Hausmann without comment. The form should be classed as doubtful.

ω: $(15\cdot13\cdot\bar{28}\cdot2) = (15\cdot0\cdot\bar{13})$. This scalenohedron was observed by Stöber [81] on crystals from Framont. On the combination figured it takes the place of the much commoner and more rational form U: $(54\bar{9}1) = (50\bar{4})$ shown in the three preceding figures (Figs. 9, 10 and 11).

It is possible that the form in question should be referred to $(7\cdot6\cdot\bar{13}\cdot1)$, a well established scalenohedron which lies $24\frac{1}{2}'$ from $(15\cdot13\cdot\bar{28}\cdot2)$ or to the hypothetical indices $(8\cdot7\cdot\bar{15}\cdot1)$ which lies even closer in zone.

ι : $(17\cdot15\cdot\bar{32}\cdot2)$: $(17\cdot0\cdot\bar{15})$. This scalenohedron was observed by Sansoni [82] on crystals from Andreasberg. In spite of the close agreement between measured and calculated angles recorded by Sansoni, it seems doubtful that a scalenohedron should fall in this position in the harmonic series.

κ: $(21\cdot9\cdot\bar{40}\cdot2) = (21\cdot0\cdot\bar{19})$. This scalenohedron was observed by Sansoni [83] on crystals from Arendal, Norway. The number of scalenohedrons of the principal zone present on the combination described by Sansoni suggests that the form under consideration may possibly be vicinal. It should be regarded as rather doubtful.

Hypothetical Scalenohedrons. Normal series N_4 suggests the possibility of a hypothetical scalenohedron which would reduce to 2 in the series. Such a scalenohedron would have the indices $(8\cdot7\cdot\bar{15}\cdot1) = (80\bar{7})$ and would fall between $(15\cdot13\cdot\bar{28}\cdot2)$ and $(17\cdot15\cdot\bar{32}\cdot2)$. In the opinion of the writer the angular difference between the hypothetical form and the two above mentioned lies within the limit of fairly good observation. The form should be retained as hypothetical.

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77 Naumann, C. T., Pogg. Ann., **14**, 235 (1828).
78 Hausmann, J. F. L., Handbuch der Mineralogie, **2** (1847).
79 Irby, loc. cit.
80 Sansoni, loc. cit.
81 Stöber, F., Abh. geol. Karte, Els-Lothr., **5**, Fig. 12 (1892).
82 Sansoni, loc. cit.
83 Sansoni, F., Giorn. d. Min., **1**, 2, p. 129 (1890).

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Zone 4 = Z_4 Scalenohedrons of the zone $[01\bar{1}2 : 10\bar{1}1 : 11\bar{2}0]$

Letter	Bravais	Miller	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
f:	$1\cdot6\cdot\bar{7}\cdot13$	760						N_5		
h:	$14\bar{5}9$	540				N_3				
z:	$12\bar{3}5$	320		N_1						
i:	$4\cdot7\cdot\bar{11}\cdot18$	11·7·0					N_4			
u:	$3\cdot5\cdot\bar{8}\cdot13$	850				N_3				
y:	$23\bar{5}8$	530			N_2					
k:	$5\cdot4\cdot\bar{9}\cdot13$	940				N_3				
x:	$4\cdot3\cdot\bar{7}\cdot10$	730			N_2					
l:	$32\bar{5}7$	520		N_1						
v:	$7\cdot4\cdot\bar{11}\cdot15$	11·4·0				N_3				
m:	$11\cdot6\cdot\bar{17}\cdot23$	17·6·0						N_5		
t:	$21\bar{3}4$	310	×							
ƀ:	$11\cdot5\cdot\bar{16}\cdot21$	16·5·0						N_5		
n:	$7\cdot3\cdot\bar{10}\cdot13$	10·3·0				N_3				
g:	$52\bar{7}9$	720			N_2					
w:	$31\bar{4}5$	410		N_1						
f:	$7\cdot2\cdot\bar{9}\cdot11$	920				N_3				
e:	$41\bar{5}6$	510			**N_2**					
o:	$9\cdot2\cdot\bar{11}\cdot13$	11·2·0					**N_4**			
q:	$51\bar{6}7$	610				N_3				
α:	$11\cdot2\cdot\bar{13}\cdot15$	13·2·0						N_5		
c:	$61\bar{7}8$	710					N_4			
b:	$71\bar{8}9$	810						N_5		
a:	$8\cdot1\cdot\bar{9}\cdot10$	910							?	
β:	$9\cdot1\cdot\bar{10}\cdot11$	10·1·0							?	
d:	$13\cdot1\cdot\bar{14}\cdot15$	14·1·0							?	
A:	$11\cdot1\cdot\bar{12}\cdot10$	$11\cdot0\cdot\bar{1}$					N_4			
B:	$17\cdot2\cdot\bar{19}\cdot15$	$17\cdot0\cdot\bar{2}$							?	
C:	$71\bar{8}6$	$70\bar{1}$			N_2					
γ:	$19\cdot3\cdot\bar{22}\cdot16$	$19\cdot0\cdot\bar{3}$					N_4			
D:	$61\bar{7}5$	$60\bar{1}$				N_3				
E:	$51\bar{6}4$	$50\bar{1}$		N_1						

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Zone 4 = Z_4 Scalenohedrons of the zone $[01\bar{1}2: 10\bar{1}1: 11\bar{2}0]$ (continued)

Letter	Bravais	Miller	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
I:	$13\cdot3\cdot\bar{1}\bar{6}\cdot10$	$13\cdot0\cdot\bar{3}$				N_3				
Z:	$21\cdot5\cdot\bar{2}\bar{6}\cdot16$	$21\cdot0\cdot\bar{5}$					N_4			
F:	$41\bar{5}3$	$40\bar{1}$			N_2					
δ:	$19\cdot5\cdot\bar{2}\bar{4}\cdot14$	$19\cdot0\cdot\bar{5}$					N_4			
μ:	$11\cdot3\cdot\bar{1}\bar{4}\cdot8$	$11\cdot0\cdot\bar{3}$				N_3				
G:	$72\bar{9}5$	$70\bar{2}$					N_4			
λ:	$17\cdot5\cdot\bar{2}\bar{2}\cdot12$	$17\cdot0\cdot\bar{5}$						N_5		
H:	$31\bar{4}2$	$30\bar{1}$	×							Z_{12}
J:	$52\bar{7}3$	$50\bar{2}$			N_2					
ξ:	$7\cdot3\cdot\bar{1}\bar{0}\cdot4$	$70\bar{3}$		N_1						
π:	$11\cdot5\cdot\bar{1}\bar{6}\cdot6$	$11\cdot0\cdot\bar{5}$			N_2					
Ψ:	$15\cdot7\cdot\bar{2}\bar{2}\cdot8$	$15\cdot0\cdot\bar{7}$				N_3				
K:	$21\bar{3}1$	$20\bar{1}$	×							Z_6, Z_{11}
σ:	$41\cdot21\cdot\bar{6}\bar{2}\cdot20$	$41\cdot0\cdot\bar{2}\bar{1}$							?	
τ:	$25\cdot13\cdot\bar{3}\bar{8}\cdot12$	$25\cdot0\cdot\bar{1}\bar{3}$							?	
L:	$17\cdot9\cdot\bar{2}\bar{6}\cdot8$	$17\cdot0\cdot\bar{9}$							?	
ϵ:	$9\cdot5\cdot\bar{1}\bar{4}\cdot4$	$90\bar{5}$				N_3				
M:	$7\cdot4\cdot\bar{1}\bar{1}\cdot3$	$70\bar{4}$			N_2					
φ:	$19\cdot11\cdot\bar{3}\bar{0}\cdot8$	$19\cdot0\cdot\bar{1}\bar{1}$					N_4			
χ:	$29\cdot17\cdot\bar{4}6\cdot12$	$29\cdot0\cdot\bar{1}\bar{7}$						N_5		
N:	$53\bar{8}2$	$50\bar{3}$		N_1						
	$13\cdot8\cdot\bar{2}\bar{1}\cdot5$	$13\cdot0\cdot\bar{8}$				N_3				
o:	$8\cdot5\cdot\bar{1}\bar{3}\cdot3$	$80\bar{5}$			N_2					
p:	$17\cdot11\cdot\bar{2}\bar{8}\cdot6$	$17\cdot0\cdot\bar{1}\bar{1}$						N_5		
P:	$32\bar{5}1$	$30\bar{2}$	×							Z_7, Z_{13}
Q:	$19\cdot13\cdot\bar{3}\bar{2}\cdot6$	$19\cdot0\cdot\bar{1}\bar{3}$						N_5		
	$13\cdot9\cdot\bar{2}\bar{2}\cdot4$	$13\cdot0\cdot\bar{9}$				N_3				
R:	$10\cdot7\cdot\bar{1}\bar{7}\cdot3$	$10\cdot0\cdot\bar{7}$			N_2					
s:	$7\cdot5\cdot\bar{1}\bar{2}\cdot2$	$70\bar{5}$		N_1						
s:	$11\cdot8\cdot\bar{1}\bar{9}\cdot3$	$11\cdot0\cdot\bar{8}$			N_2					
η:	$23\cdot17\cdot\bar{4}\bar{0}\cdot6$	$23\cdot0\cdot\bar{1}\bar{7}$						N_5		
T:	$43\bar{7}1$	$40\bar{3}$	×							Z_{10}

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Zone 4 = Z_4 Scalenohedrons of the zone $[01\bar{1}2:\ 10\bar{1}1:\ 11\bar{2}0]$ (concluded)

Letter	Bravais	Miller	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
θ:	$9\cdot7\cdot\bar{1}6\cdot2$	$90\bar{7}$					N_4			
U:	$54\bar{9}1$	$50\bar{4}$			N_2					
	$11\cdot9\cdot\bar{2}0\cdot2$	$11\cdot0\cdot\bar{9}$					N_4			
V:	$6\cdot5\cdot\bar{1}1\cdot1$	$60\bar{5}$		N_1						
W:	$13\cdot11\cdot\bar{2}4\cdot2$	$13\cdot0\cdot\bar{1}1$						N_5		
X:	$7\cdot6\cdot\bar{1}3\cdot1$	$70\bar{6}$				N_3				
ω:	$15\cdot13\cdot\bar{2}8\cdot2$	$15\cdot0\cdot\bar{1}3$						N_5		
ι:	$17\cdot15\cdot\bar{3}2\cdot2$	$17\cdot0\cdot\bar{1}5$							?	
Y:	$9\cdot8\cdot\bar{1}7\cdot1$	$90\bar{8}$					N_4			
υ:	$10\cdot9\cdot\bar{1}9\cdot1$	$10\cdot0\cdot\bar{9}$				N_3				
κ:	$21\cdot19\cdot40\cdot2$	$21\cdot0\cdot\bar{1}9$							?	
Hypothetical Scalenohedrons										
	$13\bar{4}7$	430			N_2					
	$12\cdot7\cdot\bar{1}9\cdot5$	$12\cdot0\cdot7$				N_3				
	$8\cdot7\cdot\bar{1}5\cdot1$	$80\bar{7}$			N_2					

ZONE 5 — SCALENOHEDRONS OF THE ZONE $[10\bar{1}1 : 02\bar{2}1] = [100 : 11\bar{1}]$

$$\text{SYMBOL} = \frac{k}{h}.$$

Considering this portion of the zone $[10\bar{1}1 : 02\bar{2}1 : 11\bar{2}0]$ the major zone intersection suggests the pole $(12\bar{3}2) = (21\bar{1})$ as a knot. Splitting the zone at this point and developing the first half, we have for the forms recorded between $(10\bar{1}1) = (100)$ and $(12\bar{3}2) = 21\bar{1})$:

Form	p. 100	$\mathfrak{v}$: $61\bar{1}$	$\mathfrak{a}$: $51\bar{1}$	$\mathfrak{b}$: $41\bar{1}$	λ $31\bar{1}$	$\mathfrak{c}$: $52\bar{2}$	$\mathfrak{d}$: $73\bar{3}$	$\mathfrak{e}$: $21\bar{1}$
Symbol	0	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{1}{2}$
$\frac{v}{v_2 - v}$	0	$\frac{1}{2}$	$\frac{2}{3}$	1	2	4	6	∞
$\frac{v^1}{2}$	0	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	∞
N_3	0	$(\frac{1}{4})$	$\frac{1}{3}$	$\frac{1}{2}$ ·	1 ·	2	3	∞

Referred to N_3 this zone fragment is somewhat incomplete and contains the extra form $\frac{1}{6}$.

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$\mathfrak{v}: (52\bar{7}6) = (61\bar{1})$. This scalenohedron was recorded by Moesz [84] on crystals from Körösmezö, Hungary. The form which is referable to normal series N_4 falls at the intersection of a number of well marked secondary zones. It should be ranked as probable.

The following scalenohedrons have been recorded in the zone fragment between the limits $(12\bar{3}2) = (21\bar{1})$ and $(02\bar{2}1) = (11\bar{1})$:

Form	$\mathfrak{e}: 21\bar{1}$	$\mathfrak{f}: 95\bar{5}$	$\mathfrak{z}: 53\bar{3}$	$\mathfrak{g}: 32\bar{2}$	$\mathfrak{h}: 75\bar{5}$	$\mathfrak{i}: 43\bar{3}$	$\mathfrak{k}: 54\bar{4}$	$\mathfrak{u}: 65\bar{5}$	$\mathfrak{m}: 98\bar{8}$	φ $11\bar{1}$
Symbol	$\frac{1}{2}$	$\frac{5}{9}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{8}{9}$	1
$\frac{v - v_1}{v_2 - v}$	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{3}{2}$	2	$\frac{7}{2}$	∞
$2v^1$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	3	4	7	∞
N_4	0	$\frac{1}{4}$ ..	$\frac{1}{2}$ ...	1 .	$\frac{3}{2}$.	2 .	3	4	(7)	∞

Referred to N_4 this zone fragment is very incomplete and contains the extra form $\frac{8}{9}$.

$\mathfrak{m}: (1\cdot16\cdot\bar{1}\bar{7}\cdot9) = (98\bar{8})$. This form, as pointed out by Irby, was substituted by Zippe [85] for $(14\bar{5}3) = (32\bar{2})$ in Lévy's [86] figure of a Derbyshire occurrence. Irby favors discarding the figure of Lévy on the ground that the zonal relations shown in the drawing do not conform to the indices assigned. The form must be regarded as doubtful.

Zone 5 — Scalenohedrons of the Zone $[02\bar{2}1 : 11\bar{2}0] = [11\bar{1} : 10\bar{1}]$

Symbol $\frac{h}{k}$.

This portion of the zone $[10\bar{1}1 : 02\bar{2}1 : 11\bar{2}0]$ can be developed as a whole without splitting:

Form	φ. $11\bar{1}$	$\mathfrak{y}: 43\bar{4}$	$\mathfrak{n}: 32\bar{3}$	$\mathfrak{a}: 53\bar{5}$	$\mathfrak{p}: 21\bar{2}$	$\mathfrak{q}: 31\bar{3}$	$\mathfrak{t}: 11\cdot3\cdot\bar{1}\bar{1}$	$\mathfrak{r}: 41\bar{4}$	$\mathfrak{f}: 51\bar{5}$	a $10\bar{1}$
Symbol	1	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	2	3	$\frac{11}{3}$	4	5	∞
$v - 1$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	2	$\frac{8}{3}$	3	4	∞
N_3	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1 .	2	$(\frac{8}{3})$	3	(4)	∞

Referred to N_3 this zone fragment is nearly complete and contains the extra forms $\frac{11}{3}$ and 5.

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84 Moesz, G., Föld. Közlöny, **27**, 495 (1897).
85 Zippe, loc. cit., Fig. 15.
86 Lévy, A., Description d'une collection des mineraux formée par M. Heuland, London, Fig. 122 (1837).

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t: $(8\cdot14\cdot\bar{2}\bar{2}\cdot3) = (11\cdot3\cdot\bar{1}\bar{1})$. This scalenohedron was observed by Whitlock[87] on crystals from Lyon Mountain, N. Y. The poles of the form lie at the intersection of several well marked zones. In the combination cited, well developed planes of the form bevel the edges $8\cdot8\cdot\bar{1}\bar{6}\cdot3 : 01\bar{1}0$. The form falls in normal series N_5 and may be regarded as fairly probable.

Zone 5 = Z_5 Scalenohedron of the zone $[10\bar{1}1 : 02\bar{2}1 : 11\bar{2}0]$

Letter	Bravais	Miller	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
v:	$52\bar{7}6$	$61\bar{1}$					N_4			
a:	$42\bar{6}5$	$51\bar{1}$				N_3				
b:	$32\bar{5}4$	$41\bar{1}$			N_2					
c:	$34\bar{7}5$	$52\bar{2}$			N_2					
d:	$4\cdot6\cdot\bar{1}\bar{0}\cdot7$	$73\bar{3}$				N_3				
e:	$12\bar{3}2$	$21\bar{1}$	×							Z_9, Z_{12}
f:	$4\cdot10\cdot\bar{1}\bar{4}\cdot9$	$95\bar{5}$					N_4			
z:	$26\bar{8}5$	$53\bar{3}$			N_2					
g:	$14\bar{5}3$	$32\bar{2}$		N_1						
h:	$2\cdot10\cdot\bar{1}\bar{2}\cdot7$	$75\bar{5}$				N_3				
i:	$16\bar{7}4$	$43\bar{3}$			N_2					
k:	$18\bar{9}5$	$54\bar{4}$				N_3				
u:	$1\cdot10\cdot\bar{1}\bar{1}\cdot6$	$65\bar{5}$					N_4			
m:	$1\cdot16\cdot\bar{1}\bar{7}\cdot9$	$98\bar{8}$							?	
y:	$17\bar{8}3$	$43\bar{4}$				N_3				
n:	$15\bar{6}2$	$32\bar{3}$			N_2					
o:	$2\cdot8\cdot\bar{1}\bar{0}\cdot3$	$53\bar{5}$				N_3				
p:	$13\bar{4}1$	$21\bar{2}$		N_1						Z_{11}, Z_{13}
q:	$24\bar{6}1$	$31\bar{3}$			N_2					Z_7, Z_{10}
t:	$8\cdot14\cdot\bar{2}\bar{2}\cdot3$	$11\cdot3\cdot\bar{1}\bar{1}$						N_5		
r:	$35\bar{8}1$	$41\bar{4}$				N_3				
f:	$4\cdot6\cdot\bar{1}\bar{0}\cdot1$	$51\bar{5}$					N_4			

87 Whitlock, H. P., N. Y. State Museum Memoir **13**, Plate X, Fig. 4 (1910).

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$\mathfrak{f}$: $(4\cdot6\cdot\bar{1}0\cdot1) = (51\bar{5})$. This scalenohedron was also described by Whitlock [88] on crystals from Rossie and Lyon Mountains, N. Y. In both of the above occurrences the form agrees well in zonal relations with other forms present on the combinations. It also falls at the intersection of several well marked zones on the general projection and is referable to normal series N_4. It may be regarded as fairly well established.

Zone 6 = Z_6 Scalenohedrons of the Zone $[02\bar{2}1 : 40\bar{4}1]$

$\frac{h+k}{l}$ (Miller) = 2. Symbol $\frac{h}{k}$.

In this section of the zone $[02\bar{2}1 : 40\bar{4}1 : 11\bar{2}0] = [11\bar{1} : 11\bar{3} : 10\bar{1}]$ a knot obviously falls at the pole $(21\bar{3}1) = (20\bar{1})$. Splitting at this point and developing the zone fragment between $(11\bar{1})$ and $(20\bar{1})$ we have:

Form	$\varphi\cdot 11\bar{1}$	$\mathfrak{A}$: $75\bar{6}$	$\mathfrak{O}$: $13\cdot9\cdot\bar{1}\bar{1}$	$\mathfrak{B}$: $64\bar{5}$	$\mathfrak{Q}$: $11\cdot7\cdot\bar{9}$	$\mathfrak{C}$: $53\bar{4}$	$\mathfrak{D}$: $95\bar{7}$
Symbol	1	$\frac{7}{5}$	$\frac{13}{9}$	$\frac{3}{2}$	$\frac{11}{7}$	$\frac{5}{3}$	$\frac{9}{5}$
$v-1$	0	$\frac{2}{5}$	$\frac{4}{9}$	$\frac{1}{2}$	$\frac{4}{7}$	$\frac{2}{3}$	$\frac{4}{5}$
N_4	0	$\frac{2}{5}$	$(\frac{4}{9})$	$\frac{1}{2}$	$(\frac{4}{7})$	$\frac{2}{3}$	$(\frac{4}{5})$

Form	$\mathfrak{E}$: $42\bar{3}$	$\mathfrak{F}$: $73\bar{5}$	$\mathfrak{G}$: $31\bar{2}$	α $51\bar{3}$	$\mathfrak{H}$: $13\cdot1\cdot\bar{7}$	K: $20\bar{1}$
Symbol	2	$\frac{7}{3}$	3	5	13	∞
$v-1$	1	$\frac{4}{3}$	2	4	12	∞
N_4	1	$\frac{4}{3}$	2	4	(12)	∞

This fragment of the zone referred to N_4 is quite incomplete and contains the extra forms, $\frac{13}{9}$, $\frac{11}{7}$, $\frac{9}{5}$ and 13.

$\mathfrak{O}$: $(4\cdot20\cdot\bar{2}\bar{4}\cdot11) = (13\cdot9\cdot\bar{1}\bar{1})$. This scalenohedron was recorded by Melczer [89] on crystals from the vicinity of Budapest. Apart from the fact that this form falls in a high normal series, the zonal relations of this pole do not indicate a high degree of probability. The form should be classed as doubtful.

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88 Whitlock, H. P., N. Y. State Museum Memoir, **13**, Plate III, Figs. 3 and 4, and Plate X, Fig. 1 (1910).

89 Melczer, G., Föld. Közlöny, **27**, 79 (1896).

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$\mathfrak{Q}$: $(4\cdot16\cdot\bar{2}\bar{0}\cdot9) = (11\cdot7\cdot\bar{9}.)$. This scalenohedron was noted by Sansoni [90] on crystals associated with the pink apophyllite of pyramidal habit from Andreasberg. The measured angles as recorded by Sansoni agree well with theory. Further, the form which falls in normal series N_5 marks the intersection of the zones $[7\bar{1}\bar{8}9 : 04\bar{4}1]$ and $[0001 : 14\bar{5}0]$ both of which are well marked. The form is quite probable.

$\mathfrak{D}$: $(4\cdot12\cdot\bar{1}\bar{6}\cdot7) = (95\bar{7})$. This scalenohedron was recorded by Cesàro [91] on two crystals from Rhisnes, Belgium. The faces measured by Cesàro were small and ill defined. The form however falls in normal series N_5 and marks the intersections of the zones $[40\bar{4}7 : 01\bar{1}0]$ and $[5\bar{1}\bar{6}7 : 08\bar{8}1]$. It is somewhat less probable than the preceding form.

$\mathfrak{H}$: $(12\cdot8\cdot\bar{2}\bar{0}\cdot7) = (13\cdot1\cdot\bar{7})$. This scalenohedron was noted by Hessenberg [92] on crystals from Matlock, Derbyshire. The form was included in Irby's list without comment. Although falling in a high normal series it marks the intersection of several fairly well marked zones. It is fairly probable.

In the zone fragment between $(20\bar{1})$ and $(11\bar{3})$ only two scalenohedrons have been recorded.

Form	K: $20\bar{1}$	O: $7\bar{1}\bar{3}$	H: $8\bar{2}\bar{3}$	m. $11\bar{3}$
Symbol	∞	7	4	1
$\frac{v-1}{3}$	∞	2	1	0
N_2	∞	2	1	0

Referred to N_2 this zone fragment lacks the term $\frac{1}{2}$ but contains no extra forms.

Zone $6 = Z_6$ Scalenohedrons of the Zone $[40\bar{4}1 : 11\bar{2}0] = [11\bar{3} : 10\bar{1}]$

$$\frac{h-l}{k} \text{ (Miller)} = 2. \quad \text{Symbol } \frac{h}{k}$$

The principal zone intersection in this section of Zone 6 is the pole of the relatively common scalenohedron $(62\bar{8}1) = (5\bar{1}\bar{3})$. Splitting

90 Sansoni, F., R. Acad. d. Linc., **19**, Plate III, Fig. 29 (1884).
91 Cesàro, G., loc. cit., Fig. 34.
92 Hessenberg, F., Min. Not., **4** (1863).

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at this point and considering the forms of the fragment between $(11\bar{3})$ and $(5\bar{1}\bar{3})$ we have:

Form	m· $11\bar{3}$	Γ: $13\cdot\bar{4}\cdot\bar{5}$	Δ: $10\cdot\bar{3}\cdot\bar{4}$	Σ: $17\cdot\bar{5}\cdot\bar{7}$	Θ: $31\cdot\bar{9}\cdot\bar{1}\bar{3}$	Φ: $7\bar{2}\bar{3}$
Symbol	3	$\frac{13}{4}$	$\frac{10}{3}$	$\frac{17}{5}$	$\frac{31}{9}$	$\frac{7}{2}$
$v-3$	0	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{4}{9}$	$\frac{1}{2}$
$\frac{v^1}{v^1{}_2 - v^1}$	0	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{1}{3}$
N_4	0	$(\frac{1}{7})$	$(\frac{1}{5})$	$\frac{1}{4}$	$(\frac{2}{7})$	$\frac{1}{3}$

Form	Λ: $25\cdot\bar{7}\cdot\bar{1}\bar{1}$	Ξ: $11\cdot\bar{3}\cdot\bar{5}$	$\mathfrak{B}$: $4\bar{1}\bar{2}$	$\mathfrak{M}$: $13\cdot\bar{3}\cdot\bar{7}$	$\mathfrak{N}$: $9\bar{2}\bar{5}$	$\mathfrak{Z}$: $5\bar{1}\bar{3}$
Symbol	$\frac{25}{7}$	$\frac{11}{3}$	4	$\frac{13}{3}$	$\frac{9}{2}$	5
$v-3$	$\frac{4}{7}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{3}{2}$	2
$\frac{v^1}{v^1{}_2 - v^1}$	$\frac{2}{5}$	$\frac{1}{2}$	1	2	3	∞
N_4	$\frac{2}{5}$	$\frac{1}{2}$ ···	1 ···	2 ·	3 ·	∞

Referred to N_4 this zone fragment is very incomplete and contains the extra forms $\frac{13}{4}$, $\frac{10}{3}$, and $\frac{31}{9}$.

Γ: $(17\cdot1\cdot\bar{1}\bar{8}\cdot4) = (13\cdot\bar{4}\cdot\bar{5})$. This scalenohedron is figured by Palache [93] on three crystals of different habit from the Lake Superior region of Michigan. Although this form falls in a high normal series its zonal relations are good, the pole lying at the intersection of several well marked zones. The form is probable.

Δ: $(13\cdot1\cdot\bar{1}\bar{4}\cdot3) = (10\cdot\bar{3}\cdot\bar{4})$. This scalenohedron was also recorded by Palache [94] on crystals from Lake Superior. It is figured as a series of very narrow planes. The form falls in normal series N_5 and shows better zonal relations than the previously discussed scalenohedron for the same occurrence. The form is probable.

Θ: $(40\cdot4\cdot\bar{4}\bar{4}\cdot9) = (31\cdot\bar{9}\cdot\bar{1}\bar{3})$. Like the two preceding forms, this scalenohedron was recorded by Palache on crystals from Lake Superior. It appears to be less probable than the two preceding forms.

The zone fragment between $(62\bar{8}1) = (5\bar{1}\bar{3})$ and $(11\bar{2}0) = (10\bar{1})$ contains the following recorded forms:

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93 Palache, C., Geol. Surv. Mich., **6**, Fig. 3, 14 and 19 (1900).
94 Palache, C., Geol. Surv. Mich., **6**, Fig. 5 (1900).

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Form	𝔍: $5\bar{1}\bar{3}$	𝔊: $13\cdot\bar{2}\cdot\bar{9}$	𝔎: $7\bar{1}\bar{5}$	𝔗: $15\cdot\bar{2}\cdot\bar{1}\bar{1}$	a $10\bar{1}$
Symbol	5	$\frac{13}{2}$	7	$\frac{15}{2}$	∞
$v-6$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	∞
N_2	0	$\frac{1}{2}$	1	$(\frac{3}{2})$	∞

Referred to N_2 this zone fragment lacks the number 2 and contains the extra form $\frac{15}{2}$.

Zone 6 = Z_6 Scalenohedrons of the zone $(02\bar{2}1 : 40\bar{4}1 : 11\bar{2}0]$

Letter	Bravais	Miller	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
𝔄:	$2\cdot11\cdot\bar{1}\bar{3}\cdot6$	$75\bar{6}$					N_4			
𝔇:	$4\cdot20\cdot\bar{2}\bar{4}\cdot11$	$13\cdot9\cdot\bar{1}\bar{1}$							?	
𝔅:	$2\cdot9\cdot\bar{1}\bar{1}\cdot5$	$64\bar{5}$			N_2					
𝔔:	$4\cdot16\cdot\bar{2}\bar{0}\cdot9$	$11\cdot7\cdot\bar{9}$						N_5		
𝔈:	$27\bar{9}4$	$53\bar{4}$				N_3				
𝔇:	$4\cdot12\cdot\bar{1}\bar{6}\cdot7$	$95\bar{7}$						N_5		
𝔈:	$25\bar{7}3$	$42\bar{3}$		N_1						Z_9
𝔉:	$4\cdot8\cdot\bar{1}\bar{2}\cdot5$	$73\bar{5}$					N_4			
𝔊:	$23\bar{5}2$	$31\bar{2}$			N_2					Z_8
𝔎:	$12\cdot8\cdot\bar{2}\bar{0}\cdot7$	$13\cdot1\cdot\bar{7}$							?	
Θ:	$8\cdot2\cdot\bar{1}\bar{0}\cdot3$	$7\bar{1}3$			N_2					
Π:	$10\cdot1\cdot\bar{1}\bar{1}\cdot3$	$8\bar{2}3$		N_1						
Γ:	$17\cdot1\cdot\bar{1}\bar{8}\cdot4$	$13\cdot\bar{4}\cdot\bar{5}$							?	
Δ:	$13\cdot1\cdot\bar{1}\bar{4}\cdot3$	$10\cdot\bar{3}\cdot\bar{4}$						N_5		
Σ:	$22\cdot2\cdot\bar{2}\bar{4}\cdot5$	$17\cdot\bar{5}\cdot\bar{7}$					N_4			
Θ:	$40\cdot4\cdot\bar{4}\bar{4}\cdot9$	$31\cdot\bar{9}\cdot\bar{1}\bar{3}$						N_5		
Φ:	$9\cdot1\cdot\bar{1}\bar{0}\cdot\bar{2}$	$7\bar{2}\bar{3}$				N_3				
Λ:	$32\cdot2\cdot\bar{3}\bar{6}\cdot7$	$25\cdot\bar{7}\cdot\bar{1}\bar{1}$					N_4			
Ξ:	$14\cdot2\cdot\bar{1}\bar{6}\cdot3$	$11\cdot\bar{3}\cdot\bar{5}$			N_2					
𝔓:	$51\bar{6}1$	$4\bar{1}\bar{2}$		N_1						Z_{13}
𝔐:	$16\cdot4\cdot\bar{2}\bar{0}\cdot3$	$13\cdot\bar{3}\cdot\bar{7}$			N_2					
𝔑:	$11\cdot3\cdot\bar{1}\bar{4}\cdot2$	$9\bar{2}\bar{5}$				N_3				
𝔍:	$62\bar{8}1$	$5\bar{1}\bar{3}$	×							Z_{10}
𝔖:	$15\cdot7\cdot\bar{2}\bar{2}\cdot2$	$13\cdot\bar{2}\cdot\bar{9}$			N_2					
𝔎:	$8\cdot4\cdot\bar{1}\bar{2}\cdot1$	$7\bar{1}\bar{5}$		N_1						
𝔗:	$17\cdot9\cdot\bar{2}\bar{6}\cdot2$	$15\cdot\bar{2}\cdot\bar{1}\bar{1}$				N_3				

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$\mathfrak{T}$: $(17\cdot9\cdot\bar{2}\bar{6}\cdot2) = (15\cdot\bar{2}\cdot\bar{1}\bar{1})$. This scalenohedron was recorded by Cesàro on two crystals from Rhisnes, Belgium. The measured angles given by Cesàro agree well with theory and the form, which falls in normal series N_3 presents good zonal relations. It should be ranked as probable.

ZONE 7 = Z_7 SCALENOHEDRONS OF THE ZONE $(40\bar{4}1 : 08\bar{8}1] = [3\bar{1}\bar{1} : 33\bar{5}]$. $\frac{k+l}{h}$ (MILLER) = $\frac{2}{3}$. SYMBOL $\frac{k}{l}$.

The most prominent zone intersection in this zone occurs at the pole $(32\bar{5}1) = (30\bar{2})$. Splitting the zone at this point we have for the recorded forms in the fragment between $(3\bar{1}\bar{1})$ and $30\bar{2})$:

Form	$\left\{\begin{matrix} m. \\ 3\bar{1}\bar{1} \end{matrix}\right.$	$6\bar{1}\bar{3}$	$\mathfrak{U}$: $21\cdot\bar{3}\cdot\bar{1}\bar{1}$	U: $9\bar{1}\bar{5}$	$\mathfrak{V}$: $15\cdot\bar{1}\cdot\bar{9}$	$\mathfrak{Y}$: $18\cdot\bar{1}\cdot\bar{1}\bar{1}$	P: $30\bar{2}$
Symbol	1	$\frac{1}{3}$	$\frac{3}{11}$	$\frac{1}{5}$	$\frac{1}{9}$	$\frac{1}{11}$	0
$\frac{v-v_1}{v}$	0	2	$\frac{8}{3}$	4	8	10	∞
$\frac{v^1}{4}$	0	$\frac{1}{2}$	$\frac{2}{3}$	1	2	$\frac{5}{2}$	∞
N_3	0 ·	$\frac{1}{2}$	$\frac{2}{3}$ ·	1 ·	2	$(\frac{5}{2})$ ·	∞

Referred to N_3 this fragment of the zone is incomplete and contains the extra form $\frac{1}{11}$.

$\mathfrak{Y}$: $(19\cdot10\cdot\bar{2}\bar{9}\cdot6) = (18\cdot\bar{1}\cdot\bar{1}\bar{1})$. This scalenohedron was recorded by Cesàro [95] on crystals from Rhisnes, Belgium. Cesàro points out the proximity of this form to $(16\cdot8\cdot\bar{2}\bar{4}\cdot5)$ which is present on the same occurrence but contends that the measured angles and zonal relations point to the given indices. The form falls in normal series N_4 and should be classed as fairly probable.

In the fragment of the zone between $(30\bar{2})$ and $(33\bar{5})$ the following forms have been recorded:

Form	$\left\{\begin{matrix} P: \\ 30\bar{2} \end{matrix}\right.$	$\mathfrak{U}$ $15\cdot1\cdot\bar{1}\bar{1}$	γ $9\bar{1}\bar{7}$	$\mathfrak{M}$ $12\cdot3\cdot\bar{1}\bar{1}$	q: $31\bar{3}$	$\mathfrak{X}$: $9\cdot5\cdot\bar{1}\bar{1}$	$\mathfrak{L}$: $32\bar{4}$	Π. $33\bar{5}$
Symbol	0	$\frac{1}{11}$	$\frac{1}{7}$	$\frac{3}{11}$	$\frac{1}{3}$	$\frac{5}{11}$	$\frac{1}{2}$	$\frac{3}{5}$
$\frac{v}{v_2-v}$	0	$\frac{5}{28}$	$\frac{5}{16}$	$\frac{5}{6}$	$\frac{5}{4}$	$\frac{25}{8}$	5	∞
$\frac{4v'}{5}$	0	$\frac{1}{7}$	$\frac{1}{4}$	$\frac{2}{3}$	1	$\frac{5}{2}$	4	∞
N_4	0	$(\frac{1}{7})$	$\frac{1}{4}$ ····	$\frac{2}{3}$ ·	1 ····	$\frac{5}{2}$ ·	4	∞

[95] Cesàro, G., loc. cit., Fig. 54.

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Referred to N_4 this zone fragment is very incomplete and contains the extra form $\frac{1}{11}$. The forms γ $(91\bar{7})$ and $\mathfrak{q}$ $(31\bar{3})$ have already been discussed under Z_2 and Z_5 respectively.

$\mathfrak{U}$ $(14\cdot12\cdot\bar{2}\bar{6}\cdot5) = (15\cdot1\cdot\bar{1}\bar{1})$. This scalenohedron was recorded by Whitlock [96] on crystals from Lyon Mountain, N. Y. Although agreeing fairly well with regard to measured and calculated angles this form falls in a rather high normal series. The form must be regarded as somewhat doubtful.

Hypothetical scalenohedrons.— In the above series the missing term 2 corresponds to the hypothetical scalenohedron $(3\cdot10\cdot\bar{1}\bar{3}\cdot2) = (63\bar{7})$. A pole corresponding to these indices would lie at the intersection of zone Z_7 with $[31\bar{4}2 : 01\bar{1}0]$, $[05\bar{5}1 : 10\bar{1}0]$ and $[10\bar{1}4 : 27\bar{9}1]$ all of which are fairly well marked with rational forms. $(3\cdot10\cdot\bar{1}\bar{3}\cdot2)$ should be retained as a hypothetical form.

Zone 8 = Z_8. — Scalenohedrons of the Zone $[01\bar{1}2 : 11\bar{2}0] = [110 : 10\bar{1}]$. $\frac{h-k}{l}$ (Miller) = 1. Symbol $\frac{h}{k}$.

This zone is susceptible of development without splitting. The recorded forms are as follows:

Form	δ. 110	e $21\bar{1}$	a $94\bar{5}$	b $52\bar{3}$	$\mathfrak{G}$ $31\bar{2}$	c $41\bar{3}$	d $51\bar{4}$	e $11\cdot2\cdot\bar{9}$	g $71\bar{6}$	a $10\bar{1}$
Symbol	1	2	$\frac{9}{4}$	$\frac{5}{2}$	3	4	5	$\frac{11}{2}$	7	∞
$\frac{v-1}{2}$	0	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	1	$\frac{3}{2}$	2	$\frac{9}{4}$	3	∞
N_4	$0\cdots$	$\frac{1}{2}$	$\cdot\ (\frac{5}{8})\ \cdot$	$\frac{3}{4}$	1	$\cdot\ \frac{3}{2}\ \cdot$	2	$(\frac{9}{4})$	$\cdot\ 3\ \cdot$	∞

Referred to N_4 this zone is incomplete and contains the extra forms $\frac{9}{4}$ and $\frac{11}{2}$.

a $(5\cdot9\cdot\bar{1}\bar{4}\cdot8) = (94\bar{5})$. This scalenohedron was recorded as Hessenberg [97] on crystals from the Canary Islands. Irby, who has reinvestigated this occurrence, regards the face to which these indices were attached by Hessenberg as utterly indeterminable. The form is very doubtful.

e $(9\cdot11\cdot\bar{2}\bar{0}\cdot4) = (11\cdot2\cdot\bar{9})$. This scalenohedron was also determined by Hessenberg [98] on crystals from Matlock, Derbyshire. Irby

96 Whitlock, H. P., Zeit. f. Kryst., **43**, Fig. 5 and 8 (1907).
97 Hessenberg, F., Min. Notiz., **7** (1866).
98 Hessenberg, F., Min. Notiz., **4** (1863).

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includes this form in his list without comment. It marks the intersection of the zones $[50\bar{5}1 : 05\bar{5}1]$, $[3\bar{1}21 : 24\bar{6}1]$ and $[13\bar{4}1 : 62\bar{8}1]$ all of which contain well established forms. The form may be considered as fairly probable.

ZONE 9 = Z_9. — SCALENOHEDRONS OF THE ZONE $[01\bar{1}1 : 11\bar{2}0]$ = $[22\bar{1} : 10\bar{1}]$. $\frac{h-l}{k}$ (MILLER) = $\frac{1}{2}$. SYMBOL $\frac{h}{k}$.

Like the preceding, this zone is susceptible of development without splitting. The following forms have been recorded:

Form	$\kappa \cdot 22\bar{1}$	$\mathfrak{g} : 32\bar{2}$	Λ $16\cdot10\cdot\bar{1}\bar{1}$	Σ $13\cdot8\cdot\bar{9}$	Q $74\bar{5}$	$\mathfrak{E} : 42\bar{3}$
Symbol	1	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{13}{8}$	$\frac{7}{4}$	2
$v-1$	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{5}{8}$	$\frac{3}{4}$	1
N_4	0 ...	$\frac{1}{2}$	$\frac{3}{5}$	$(\frac{5}{8})$ ·	$\frac{3}{4}$	1

Form	F $94\bar{7}$	Θ $52\bar{4}$	Π $62\bar{5}$	Ω $13\cdot4\cdot\bar{1}\bar{1}$	Ψ $82\bar{7}$	a $10\bar{1}$
Symbol	$\frac{9}{4}$	$\frac{5}{2}$	3	$\frac{13}{4}$	4	∞
$v-1$	$\frac{5}{4}$	$\frac{3}{2}$	2	$\frac{9}{4}$	3	∞
N_4	$(\frac{5}{4})$ ·	$\frac{3}{2}$ ·	2	$(\frac{9}{4})$ ·	3 ·	∞

Referred to N_4 this zone is incomplete and contains the extra forms $\frac{13}{8}$, $\frac{9}{4}$ and $\frac{13}{4}$.

Σ $(5\cdot17\cdot\bar{2}\bar{2}\cdot12) = (13\cdot8\cdot\bar{9})$. This scalenohedron was observed by Höfer [99] on crystals from Rauris near Salzburg. The form falls in normal series N_5 and presents some fairly good zonal relations. It is somewhat probable.

F $(5\cdot11\cdot\bar{1}\bar{6}\cdot6) = (94\bar{7})$. This scalenohedron was noted by Palache [100] on one crystal from the Lake Superior region of Michigan. Palache's figure shows this form developed to a series of planes of fair size and suggests that the form may lie in zone with $(2\cdot9\cdot\bar{1}\bar{1}\cdot5)$ and $(10\cdot1\cdot\bar{1}\bar{1}\cdot0)$, both of which are present on the combination. This zonal relation does not, however, hold. The form which falls in

99 Höfer, H., Min. Mitth., **12**, 487 (1891).
100 Palache, C., Geol. Surv. Mich., **6**, Plate 13, Fig. 11 (1900).

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normal series N_5 marks the intersection of the zones $[02\bar{2}1 : 10\cdot0\cdot\bar{1}0\cdot1]$, $[40\bar{4}7 : 17\bar{8}0]$ and $[13\bar{4}2 : 07\bar{7}2]$. It is very probable.

$\Omega\,(35\bar{8}2) = (13\cdot4\cdot\bar{1}\bar{1})$. This scalenohedron was noted by Flink [101] on crystals from Narsarsuk, southern Greenland. In spite of the high normal series to which this form is referable, it presents excellent zonal relations and should be regarded as fairly probable.

ZONE 10 = Z_{10}. — SCALENOHEDRONS OF THE ZONE $[05\bar{5}1 : 10\cdot0\cdot\bar{1}0\cdot1]$.

This zone contains only one scalenohedron which has not already been discussed in the preceding analysis, namely: $X : (16\cdot7\cdot\bar{2}\bar{3}\cdot3) = (14\cdot\bar{2}\cdot\bar{9})$. This scalenohedron was observed by Eakle on a series of crystals from the cinnabar mine at Terlingua, Texas. The form was noted on all the crystals studied and although developed as slightly rounded planes gave good agreement between measured and calculated angles. Moreover the form marks the intersection of the zones $[11\cdot0\cdot\bar{1}\bar{1}\cdot\bar{1} : 32\bar{5}1]$ and $[11\bar{2}3 : 21\bar{3}1]$ all four of which forms are present on the combination. The form is well established.

ZONE 11 = Z_{11}.— SCALENOHEDRONS OF THE ZONE $[50\bar{5}2 : 05\bar{5}1] = [11\bar{4} : 22\bar{3}]$. SYMBOL $\frac{h}{l}$.

In this portion of the zone $[50\bar{5}2 : 05\bar{5}1 : 1120]$ the most prominent pole is $(21\bar{3}1) = (20\bar{1})$. Splitting at this point and considering the fragment between $(20\bar{1})$ and $(22\bar{3})$ in as much as no forms have been recorded between $(11\bar{4})$ and $(20\bar{1})$, we have for the forms recorded:

Form	K: $20\bar{1}$	c: $41\bar{3}$	Π $62\bar{5}$	$\mathfrak{R}$: $83\bar{7}$	$\mathfrak{p}$: $21\bar{2}$	Ξ. $22\bar{3}$
Symbol	2	$\frac{4}{3}$	$\frac{6}{5}$	$\frac{8}{7}$	1	$\frac{2}{3}$
$\frac{v - v_1}{v_2 - v}$	0	1	$\frac{3}{2}$	$\frac{9}{5}$	3	∞
$\frac{v^1}{3}$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$	1	∞
N_3	0	$\frac{1}{3}$	$\frac{1}{2}$	$(\frac{3}{5})$	1 ...	∞

Referred to N_3 this zone fragment is incomplete and contains the extra form $\frac{8}{7}$.

101 Flink, G., Medd. om Grönl., **24** (1899).

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$\mathfrak{R}$: $(5\cdot10\cdot\bar{1}\bar{5}\cdot4) = (83\bar{7})$. This scalenohedron was observed by Sansoni [102] on crystals from Andreasberg. Although the angles measured by Sansoni do not agree very well with theory, the form marks the intersection of the zone $[05\bar{5}4 : 11\bar{2}0]$, a relation which was noted by Sansoni. The form moreover, falls in normal series N_4. It should be ranked as probable.

Hypothetical scalenohedrons. In the above series the missing term 2 corresponds to the hypothetical scalenohedron $(2\cdot11\cdot\bar{1}\bar{3}\cdot3) = (64\bar{7})$. This form marks the intersection of the zones $[20\bar{2}3 : 01\bar{1}0]$, $[03\bar{3}1 : 11\bar{2}0]$ and $[04\bar{4}1 : 80\bar{8}1]$ all of which are marked by rational and well established forms. The form should be retained as hypothetical.

In the portion of the zone included between $(05\bar{5}1) = (22\bar{3})$ and $(11\bar{2}0) = (10\bar{1})$ the following forms have been recorded:

Form	Ξ. $22\bar{3}$	e $54\bar{7}$	ϱ: $32\bar{4}$	υ $42\bar{5}$	a $10\bar{1}$
Symbol	$\frac{2}{3}$	$\frac{5}{7}$	$\frac{3}{4}$	$\frac{4}{5}$	1
$\frac{v - v^1}{v_2 - v}$	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	∞
$3v^1$	0	$\frac{1}{2}$	1	2	∞
N_2	0	$\frac{1}{2}$	1	2	∞

Referred to N_2 this portion of the zone is complete and contains no extra forms.

Zone 12 = Z_{12}.— Scalenohedrons of the Zone $[05\bar{5}4 : 50\bar{5}2]$.

This zone contains only one scalenohedron which has not already been discussed in the analysis of the preceding zones, namely:

$\mathfrak{W}$ $(7\cdot4\cdot\bar{1}\bar{1}\cdot6) = (81\bar{3})$. This scalenohedron was observed by Palache [103] on crystals from the Lake Superior region of Michigan. On the combination figured by Palache, the form occurs with $(20\cdot11\cdot\bar{3}\bar{1}\cdot15)$ and $(37\cdot19\cdot\bar{5}\bar{6}\cdot21)$ in a zone extending from $[11\bar{2}3 : 21\bar{3}1] = [\frac{2}{3}P_2 : R_3]$ not $[R : R_3]$ as stated by Palache. The form also falls in zones $[02\bar{2}3 : 10\bar{1}0]$ and $[41\bar{5}3 : 03\bar{3}1]$. It may be regarded as well established.

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102 Sansoni, F., Accad. d. Linc. Mem. **19** (1884).
103 Palache, C., Geol. Surv. Mich., **6**, Plate XII, Fig. 5 (1900).

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ZONE 13 = Z_{13}.— SCALENOHEDRONS OF THE ZONE $[07\bar{7}2 : 70\bar{7}1] = [33\bar{4} : 5\bar{2}\bar{2}]$. SYMBOL $\frac{h}{k}$.

Developing the recorded forms of this zone into a series we have:

	Δ.	Q:	$\mathfrak{p}$:	V:	d:	β	P:	$\mathfrak{P}$:	q.
Form	$33\bar{4}$	$17\cdot10\cdot\bar{1}\bar{8}$	$21\bar{2}$	$29\cdot10\cdot\bar{2}\bar{6}$	$51\bar{4}$	$81\bar{6}$	$30\bar{2}$	$41\bar{2}$	$5\bar{2}\bar{2}$
Symbol	1	$\frac{17}{10}$	2	$\frac{29}{10}$	5	8	∞	4	$\frac{5}{2}$
$v-1$	0	$\frac{7}{10}$	1	$\frac{19}{10}$	4	7	∞		
N_4	$0\cdots\cdots(\frac{7}{10})$		$\cdots 1 \cdots$	$(\frac{19}{10})$	$\cdots 4$	(7)	∞		

Referred to N_4 the zone fragment between $(33\bar{4})$ and $(30\bar{2})$ is very incomplete and contains the extra forms $\frac{17}{10}$, $\frac{29}{10}$ and 8, of which the latter has already been discussed under zone 2. The portion of the zone between $(30\bar{2})$ and $5\bar{2}\bar{2})$ contains no forms which have not been already discussed.

Q: $(7\cdot28\cdot\bar{3}\bar{5}\cdot9) = (17\cdot10\cdot\bar{1}\bar{8})$. This scalenohedron was noted by Hessenberg [104] on crystals from Rödefiord, Iceland. Irby includes this form in his list without comment; it is somewhat worthy of note, however, that the hypothetical form $(6\cdot25\cdot\bar{3}\bar{1}\cdot8) = (15\cdot9\cdot\bar{1}\bar{6})$ differs from $(7\cdot28\cdot\bar{3}\bar{5}\cdot9)$ by less than 20′ of φ value and by only 1′ of ρ value. This hypothetical scalenohedron would reduce to $\frac{2}{3}$ in the above series. The form $(7\cdot28\cdot\bar{3}\bar{5}\cdot9)$ is somewhat doubtful.

V: $(19\cdot36\cdot\bar{5}\bar{5}\cdot13) = (29\cdot10\cdot\bar{2}\bar{6})$. This scalenohedron was observed by Gonnard [105] on crystals from Couzon, Rhône, France. As in the previous instance it would seem that a hypothetical scalenohedron could be substituted for this form. The hypothetical form $(6\cdot11\cdot\bar{1}\bar{7}\cdot4) = (93\bar{8})$ gives the following values of φ and ρ compared with $(19\cdot36\cdot\bar{5}\bar{5}\cdot13)$:

	φ	ρ
$(6\cdot11\cdot\bar{1}\bar{7}\cdot4)$	39° 38′	74° 52′
$(19\cdot36\cdot\bar{5}\bar{5}\cdot13)$	40 7	74 46

The above hypothetical scalenohedron would reduce to 2 in the series under discussion. It is possible that the scalenohedron $(19\cdot36\cdot\bar{5}\bar{5}\cdot13)$ should be referred to the simpler indices.

104 Hessenberg, F., Min. Notiz., **8** (1872).
105 Gonnard, F., Bull. soc. fr. min., **20**, 18 (1897).

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Letter	Bravais	Miller	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
Zone 7 = Z_7 Scalenohedrons of the zone $[40\bar{4}1 : 08\bar{8}1]$										
n'	$72\bar{9}2$	$6\bar{1}\bar{3}$			N_2					Z_{12}
$\mathfrak{U}$:	$24\cdot8\cdot\bar{3}\bar{2}\cdot7$	$21\cdot\bar{3}\cdot\bar{1}\bar{1}$				N_3				
U:	$10\cdot4\cdot\bar{1}\bar{4}\cdot3$	$9\bar{1}\bar{5}$		N_1						
$\mathfrak{B}$:	$16\cdot8\cdot\bar{2}\bar{4}\cdot5$	$15\cdot\bar{1}\cdot\bar{9}$			N_2					
$\mathfrak{Y}$:	$19\cdot10\cdot\bar{2}\bar{9}\cdot6$	$18\cdot\bar{1}\cdot\bar{1}\bar{1}$					N_4			
$\mathfrak{U}$	$14\cdot12\cdot\bar{2}\bar{6}\cdot5$	$15\cdot1\cdot\bar{1}\bar{1}$							?	
$\mathfrak{M}$:	$9\cdot14\cdot\bar{2}\bar{3}\cdot4$	$12\cdot3\cdot\bar{1}\bar{1}$				N_3				
$\mathfrak{X}$:	$4\cdot16\cdot\bar{2}\bar{0}\cdot3$	$9\cdot5\cdot\bar{1}\bar{1}$					N_4			
$\mathfrak{L}$:	$16\bar{7}1$	$32\bar{4}$					N_4			Z_{11}
Hypothetical scalenohedron of the zone $[40\bar{4}1 : 08\bar{8}1]$										
	$3\cdot10\cdot\bar{1}\bar{3}\cdot2$	$63\bar{7}$			N_2					
Zone 8 = Z_8 Scalenohedrons of the zone $[01\bar{1}2 : 11\bar{2}0]$										
a:	$5\cdot9\cdot\bar{1}\bar{4}\cdot3$	$94\bar{5}$						N_5		
b:	$35\bar{8}4$	$52\bar{3}$					N_4			
c:	$34\bar{7}2$	$41\bar{3}$				N_3				Z_{11}
d:	$45\bar{9}2$	$51\bar{4}$			N_2					Z_{13}
e:	$9\cdot11\cdot\bar{2}\bar{0}\cdot4$	$11\cdot2\cdot\bar{9}$							?	
g:	$6\cdot7\cdot\bar{1}\bar{3}\cdot2$	$71\bar{6}$				N_3				
Zone 9 = Z_9 Scalenohedrons of the zone $[01\bar{1}1 : 11\bar{2}0]$										
Λ	2795	$16\cdot10\cdot\bar{1}\bar{1}$					N_4			
Σ	$5\cdot17\cdot\bar{2}\bar{2}\cdot12$	$13\cdot8\cdot\bar{9}$						N_5		
Q	$13\bar{4}2$	$74\bar{5}$					N_4			
F	$5\cdot11\cdot\bar{1}\bar{6}\cdot6$	$94\bar{7}$						N_5		
Θ	$12\bar{3}1$	$52\bar{4}$				N_3				
Π	$4\cdot7\cdot\bar{1}\bar{1}\cdot3$	$62\bar{5}$			N_2					Z_{11}
Ω	$35\bar{8}2$	$13\cdot4\cdot\bar{1}\bar{1}$							?	
Ψ	$23\bar{5}1$	$82\bar{7}$				N_3				

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Letter	Bravais	Miller	Knot	N_1	N_2	N_3	N_4	N_5	?	Zone intersections
Zone 10 = Z_{10} Scalenohedron of the zone $[05\bar{5}1 : 10\cdot0\cdot\bar{1}\bar{0}\cdot1]$										
X:	$16\cdot7\cdot\bar{2}\bar{3}\cdot3$	$14\cdot\bar{2}\cdot\bar{9}$								
Zone 11 = Z_{11} Scalenohedrons of the zone $[50\bar{5}2 : 05\bar{5}1 : 11\bar{2}0]$										
$\mathfrak{N}$:	$5\cdot10\cdot\bar{1}\bar{5}\cdot4$	$83\bar{7}$					N_4			
e	$1\cdot11\cdot\bar{1}\bar{2}\cdot2$	$54\bar{7}$			N_2					
$\mathfrak{v}$	$27\bar{9}1$	$42\bar{3}$			N_2					
Hypothetical scalenohedron of the zone $[50\bar{5}2 : 05\bar{5}1 : 11\bar{2}0]$										
	$2\cdot11\cdot\bar{1}\bar{3}\cdot3$	$64\bar{7}$			N_2					
Zone 12 = Z_{12} Scalenohedrons of the zone $[05\bar{5}4 : 50\bar{5}2]$										
$\mathfrak{w}$	$7\cdot4\cdot\bar{1}\bar{1}\cdot6$	$81\bar{3}$								
Zone 13 = Z_{13} Scalenohedrons of the zone $[07\bar{7}2 : 70\bar{7}1]$										
Q:	$7\cdot28\cdot\bar{3}\bar{5}\cdot9$	$17\cdot10\cdot\bar{1}\bar{8}$							?	
V:	$19\cdot36\cdot\bar{5}\bar{5}\cdot13$	$29\cdot10\cdot\bar{2}\bar{6}$							?	
Hypothetical scalenohedrons of the zone $[07\bar{7}2 : 70\bar{7}1]$										
	$6\cdot25\cdot\bar{3}\bar{1}\cdot8$	$15\cdot9\cdot\bar{1}\bar{6}$				N_3				
	$6\cdot11\cdot\bar{1}\bar{7}\cdot4$	$93\bar{8}$			N_2					

In order to reduce as much as possible the number of forms which fall outside the limit of the discussion by developed zonal series, a number of supplementary zones have been chosen with a view to:

1° The importance of the limiting poles.

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2° The number of forms included in the zone.

3° The number of forms not previously discussed which are included in the zone.

Accepting these conditions we pass to the discussion of these supplementary zones.

Scalenohedrons of the Zone $[20\bar{2}1 : 02\bar{2}1 : 10\bar{1}0] = [11\bar{2} : 11\bar{1} : 2\bar{1}\bar{1}]$. Symbol $\frac{k}{l}$.

In the portion of this zone between $(20\bar{2}1)$ and $(02\bar{2}1)$, an obvious knot occurs at $(31\bar{4}2) = (30\bar{1})$ which is also a knot point of zone 4.

4. Developing the fragment between $(30\bar{1})$ and $(11\bar{1})$ we have:

Form	H: $30\bar{1}$	υ $41\bar{2}$	b: $52\bar{3}$	Q $74\bar{5}$	$\mathfrak{f}$ $15\cdot9\cdot\bar{1}\bar{1}$	$\mathfrak{E}$ $85\bar{6}$	V $17\cdot11\cdot\bar{1}\bar{3}$	t $96\bar{7}$	φ $11\bar{1}$
Symbol	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{9}{11}$	$\frac{5}{6}$	$\frac{11}{13}$	$\frac{6}{7}$	1
$\frac{v}{v_2 - v}$	0	1	2	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	∞
$\frac{v^1}{2}$	0	$\frac{1}{2}$	1	2	$\frac{9}{4}$	$\frac{5}{2}$	$\frac{11}{4}$	3	∞
N_3	0 ·	$\frac{1}{2}$ ·	1 ·	2	$(\frac{9}{4})$	$(\frac{5}{2})$	$(\frac{11}{4})$	3	∞

Referred to N_3 this zone fragment is very incomplete and contains the extra forms $\frac{9}{11}$ and $\frac{11}{13}$. $\frac{5}{6}$ falls in normal series N_4.

$\mathfrak{f}$ $(6\cdot20\cdot\bar{2}\bar{6}\cdot13) = (15\cdot9\cdot\bar{1}\bar{1})$. This scalenohedron was observed by Palache [106] on three crystals from the Lake Superior region of Michigan. On the combination cited it occurs developed to a considerable habit combined with $(14\bar{5}3)$ and $(4\cdot20\cdot\bar{2}\bar{4}\cdot17)$, the three scalenohedrons lying in zone. This fact taken in conjunction with the perfect quality of the faces leaves little question as to the probability of the form.

V $(2\cdot8\cdot\bar{1}0\cdot5) = (17\cdot11\cdot\bar{1}\bar{3})$. This scalenohedron was observed by Sansoni [107] on two crystals from Andreasberg. The form as measured by Sansoni agrees well with theory and marks the intersection of the zones $[06\bar{6}5 : 11\bar{2}0]$ and $[20\bar{2}5 : 01\bar{1}0]$. It should be regarded as fairly probable.

In the portion of the zone between $(02\bar{2}1)$ and $(10\bar{1}0)$ the principal zone intersection occurs at $(32\bar{5}1) = (30\bar{2})$ which developed as a knot

106 Palache, C., loc. cit., Plate XIV, Fig. 16.
107 Sansoni, F., loc. cit., Plate III, Fig. 25.

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point in zone 4. Splitting at this point we have for the recorded forms in the first fragment:

Form	φ $11\bar{1}$	D $63\bar{5}$	A $11\cdot5\cdot\bar{9}$	Θ $52\bar{4}$	C: $41\bar{3}$	ξ $71\bar{5}$	P: $30\bar{2}$
Symbol	1	$\frac{3}{5}$	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{5}$	0
$\frac{v-v_1}{v}$	0	$\frac{2}{3}$	$\frac{4}{5}$	1	2	4	∞
N_3	0 ..	$\frac{2}{3}$	$(\frac{4}{5})$	1 .	2 .	(4)	∞

Referred to N_3 this zone fragment is incomplete and contains the extra forms $\frac{9}{5}$ and $\frac{1}{5}$, of which the latter has been discussed under zone 2.

A $(6\cdot14\cdot\bar{2}\bar{0}\cdot7) = (11\cdot5\cdot\bar{9})$. This scalenohedron was observed by Palache [108] on a single crystal from the Lake Superior region. On the combination cited it occurs in the zone under discussion which is here marked by $(02\bar{2}1)$ and $(62\bar{8}1)$. It also lies in zone with $(35\bar{8}4)$ and $(13\bar{4}1)$ both present on the combination, and falls in normal series N_5. The form is probable.

The following forms have been recorded between $(32\bar{5}1)$ and $(10\bar{1}0)$

Form	P: $30\bar{2}$	$\mathfrak{g}$ $47\cdot\bar{1}\cdot\bar{3}\bar{1}$	T $11\cdot\bar{1}\cdot\bar{7}$	$\mathfrak{H}$ $8\bar{1}\bar{5}$	$\mathfrak{m}$ $13\cdot\bar{2}\cdot\bar{8}$	$\mathfrak{I}$ $5\bar{1}\bar{3}$	b $2\bar{1}\bar{1}$
Symbol	0	$\frac{1}{31}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	1
$\frac{v}{v_2-v}$	0	$\frac{1}{30}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	∞
$2v'$	0	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	∞
N_3	0	$(\frac{1}{15})$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1 ...	∞

Referred to N_3 this zone fragment is complete up to the term 1 and contains the extra form $\frac{1}{31}$.

$\mathfrak{g}$ $(16\cdot10\cdot\bar{2}\bar{6}\cdot5) = (47\cdot\bar{1}\cdot\bar{3}\bar{1})$. This scalenohedron was observed by Johansson [109] on crystals from Norberg, Sweden. The form occurs on a somewhat complex combination (Type 3) close to the poles $(32\bar{5}1)$ and $(19\cdot13\cdot\bar{3}\bar{2}\cdot6)$ of zone 4. It should be classed as rather doubtful.

Scalenohedrons of the Zone $[1\bar{1}02 : 40\bar{4}7 : 17\bar{8}0] = [101 : 511 : 32\bar{5}]$

Symbol $\frac{k}{l}$.

Although oblique to the zone directions laid down in Plate I this zone is particularly well marked and is further emphasized by the fact

[108] Palache, C., loc. cit., Plate XII, Fig. 6.
[109] Johansson, K., Geol. Foren. Forh., **14**, 49 (1892).

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of its having been observed in considerable completeness on at least two occurrences of calcite.[110] Splitting the zone at $(31\bar{4}5) = (40\bar{1})$ its point of intersection with zone 4 and developing the portion between $(40\bar{1})$ and $(32\bar{5})$ we have:

Form	w: $40\bar{1}$	λ $31\bar{1}$	b: $52\bar{3}$	$\mathfrak{F}$: $73\bar{5}$	F $94\bar{7}$	A $11\cdot5\cdot\bar{9}$	I $13\cdot6\cdot\bar{11}$
Symbol	∞	1	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{4}{7}$	$\frac{5}{9}$	$\frac{6}{11}$
$\frac{v_2 - v}{v}$	∞	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{7}{25}$	$\frac{4}{15}$
$5v^1$	∞	3	2	$\frac{5}{3}$	$\frac{3}{2}$	$\frac{7}{5}$	$\frac{4}{3}$
N_4	∞	3	2	$\frac{5}{3}$	$\frac{3}{2}$	$(\frac{7}{5})$	$\frac{4}{3}$

Form	J $15\cdot7\cdot\bar{13}$	i $17\cdot8\cdot\bar{15}$	Y $23\cdot11\cdot\bar{21}$	$\mathfrak{p}$: $21\bar{2}$	B $25\cdot13\cdot\bar{27}$	$\mathfrak{X}$: $9\cdot5\cdot\bar{11}$	σ $32\bar{5}$
Symbol	$\frac{7}{13}$	$\frac{8}{15}$	$\frac{11}{21}$	$\frac{1}{2}$	$\frac{13}{27}$	$\frac{5}{11}$	$\frac{2}{5}$
$\frac{v_2 - v}{v}$	$\frac{9}{35}$	$\frac{1}{4}$	$\frac{13}{55}$	$\frac{1}{5}$	$\frac{11}{65}$	$\frac{3}{25}$	0
$5v^1$	$\frac{9}{7}$	$\frac{5}{4}$	$\frac{13}{11}$	1	$\frac{11}{13}$	$\frac{3}{5}$	0
N_4	$(\frac{9}{7})$	$(\frac{5}{4})$	$(\frac{13}{11})$	1	$(\frac{11}{13})$	$\frac{3}{5}$	0

Referred to N_4 this zone fragment is incomplete and contains the extra forms $\frac{5}{9}$, $\frac{7}{13}$, $\frac{8}{15}$, $\frac{11}{21}$, and $\frac{13}{27}$. Of these the first, A $(6\cdot14\cdot\bar{20}\cdot7) = (11\cdot5\cdot\bar{9})$, has been discussed under the preceding zone, where, as in this case, it falls in normal series N_5.

J $(8\cdot20\cdot\bar{28}\cdot9) = (15\cdot7\cdot\bar{13})$. This scalenohedron was observed by Pirsson [111] on a single crystal supposed to have come from Mexico. In the combination figured by Pirsson the form is present in combination with $(4\cdot16\cdot\bar{20}\cdot3)$ and $(13\bar{4}1)$ of the zone under discussion although this zonal relation was not apparently tested in the gonimeter. The angles as measured agree well with theory and the pole lies at the intersection of the zones $[20\bar{2}1 : 04\bar{4}1]$ and $[04\bar{4}3 : 11\bar{2}0]$. The form is probable.

i $(9\cdot23\cdot\bar{32}\cdot10) = (17\cdot8\cdot\bar{15})$. This scalenohedron was observed by Gonnard [112] on crystals from Couzon, Rhône, France. The form occurs in normal series N_5 and marks the intersection of the zones

110 Palache, C., loc. cit., p. 177.
Whitlock, H. P., N. Y. State Mus. Bull. **133**, p. 219 (1909).
111 Pirsson, L.V., Am. Jour. Sci., **41**, 61 (1891).
112 Gonnard, F., Bull. soc. fr. min., **20**, 18 (1897).

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$[0\cdot11\cdot\bar{1}\bar{1}\cdot4 : 11\cdot0\cdot\bar{1}\bar{1}\cdot2]$ and $[02\bar{2}1 : 31\bar{4}0]$. It should be classed as probable.

Y $(12\cdot32\cdot\bar{4}\bar{4}\cdot13) = (23\cdot11\cdot\bar{2}1)$. This scalenohedron was observed by vom Rath [113] on crystals from Bergen Hill, N. J. Irby includes it in his list without comment as does also Goldschmidt. In spite of the high normal series to which the form is referable it falls at the intersection of the zones $[02\bar{2}1 : 21\bar{3}0]$, $[\bar{1}4\bar{3}1 : 40\bar{4}1]$ and $[4\bar{1}\bar{3}2 : 05\bar{5}1]$ The form is probable.

B $(12\cdot40\cdot\bar{5}2\cdot11) = (25\cdot13\cdot\bar{2}\bar{7})$. This scalenohedron was observed by Palache [114] on two crystals from the Lake Superior region of Michigan. On the combination figured it is present as narrow face. It marks the intersection of the zone $[05\bar{5}1 : 40\bar{4}1]$ and is referable to the same normal series as the preceding form. The form is not as probable as those of this zone previously discussed.

Scalenohedrons of the Zone $[11\bar{2}3 : 13\bar{4}1] = [210 : 21\bar{2}]$. Symbol $\frac{k}{l}$.

Like the preceding, this zone is oblique to the directions of the zonal network. As limited in the present discussion it contains two knots of the zonal network and extended beyond $(11\bar{2}3)$ it includes $(0\bar{1}\bar{1}1)$ the most important pole of the system. Developing the fragment between $(11\bar{2}3)$ and $(13\bar{4}1)$ we have:

Form	π 210	e 21$\bar{1}$	w 63$\bar{4}$	$\mathfrak{E}$ 42$\bar{3}$	X $10\cdot5\cdot\bar{8}$	D 63$\bar{5}$	$\mathfrak{p}$ 21$\bar{2}$
Symbol	∞	1	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{3}{5}$	$\frac{1}{2}$
$v-\frac{1}{2}$	∞	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	0
$2v^1$	∞	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	0
N_3	∞ ···	1 ·	$\frac{1}{2}$	$\frac{1}{3}$	$(\frac{1}{4})$	$(\frac{1}{5})$	0

Referred to N_3 this zone fragment is incomplete, specially as to the higher members of the series, and contains the extra forms $\frac{5}{8}$ and $\frac{3}{5}$, the latter of which has already been discussed.

X $(5\cdot13\cdot\bar{1}\bar{8}\cdot7) = (10\cdot5\cdot\bar{8})$. This scalenohedron was recorded by Lévy.[115] In commenting on this form Irby points out that the sub-

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113 Vom Rath, G., Zeit. f. Kryst., **1**, 604 (1877).
114 Palache, C., loc. cit., Plate XIV, fig. 17.
115 A. Lévy, loc. cit., Fig. 138.

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stitution of $(4\cdot13\cdot\bar{1}\bar{7}\cdot6) = (95\bar{8})$ for it by Descloizeaux is a somewhat arbitrary one. Descloizeaux' substitution seems to have been based upon the fact that $(4\cdot13\cdot\bar{1}\bar{7}\cdot6)$ marks the intersection of the zones $[31\bar{4}2 : 07\bar{7}2]$ and $[0552 : 50\bar{5}1]$. The assumed pole is at a considerable distance from $(5\cdot13\cdot\bar{1}\bar{8}\cdot7)$ in both φ and ρ values, several observed forms being nearer. $(5\cdot13\cdot\bar{1}\bar{8}\cdot7)$ is referable to normal series N_4 and marks the intersection of the zones $[2\bar{2}01 : 04\bar{4}1]$ and $[02\bar{2}1 : 51\bar{6}1]$, which in fact is a somewhat better zonal relation than that of Descloizeaux' assumed form. The form is probable.

Scalenohedrons of the Zone $[05\bar{5}1 : 50\bar{5}1 : 01\bar{1}0] = [22\bar{3} : 11\cdot\bar{4}\cdot\bar{4} : 11\bar{2}]$.

Splitting the first portion of this zone at $(32\bar{5}1) = (30\bar{2})$ which developed a knot in zone 4 and developing the fragment between $(30\bar{2})$ and $(11\cdot\bar{4}\cdot\bar{4})$ we have:

Form	P: $30\bar{2}$	S $13\cdot\bar{2}\cdot\bar{7}$	Φ: $7\bar{2}\bar{3}$	$n\cdot$ $11\cdot\bar{4}\cdot\bar{4}$
Symbol	∞	$\frac{7}{2}$	$\frac{3}{2}$	1
$v-1$	∞	$\frac{5}{2}$	$\frac{1}{2}$	0

In this development the form $(15\cdot5\cdot\bar{2}\bar{0}\cdot4) = (13\cdot\bar{2}\cdot\bar{7})$ stands in normal series N_4.

$S\,(15\cdot5\cdot\bar{2}\bar{0}\cdot4) = (13\cdot\bar{2}\cdot\bar{7})$. This scalenohedron is credited by Zippe[116] to Haidinger[117] on crystals from Derbyshire. The figure published by Zippe presents no zonal relations which would help in establishing the rank of probability of the form. Goldschmidt regards it as uncertain. On a basis of general zonal relations it should be ranked as fairly probable.

In the zone fragment between $(50\bar{5}1) = (11\cdot\bar{4}\cdot\bar{4})$ and $(54\bar{9}1) = (50\bar{4})$ the following forms have been recorded:

Form	$n\cdot$ $11\cdot\bar{4}\cdot\bar{4}$	$\mathfrak{P}$: $4\bar{1}\bar{2}$	$\mathfrak{D}$ $37\cdot\bar{8}\cdot\bar{2}\bar{0}$	$\mathfrak{C}$ $62\cdot\bar{1}\bar{3}\cdot\bar{3}4$	$\mathfrak{b}$ $25\cdot\bar{5}\cdot\bar{1}\bar{4}$	$\mathfrak{m}$: $13\cdot\bar{2}\cdot\bar{8}$	U: $50\bar{4}$
Symbol	1	2	$\frac{5}{2}$	$\frac{34}{13}$	$\frac{14}{5}$	4	∞
$v-1$	0	1	$\frac{3}{2}$	$\frac{21}{13}$	$\frac{9}{5}$	3	∞
N_3	0 ···	1 ·	$\frac{3}{2}$	$(\frac{21}{13})$	$(\frac{9}{5})$ ·	3	∞

116 Zippe, loc. cit., Fig. 5.
117 Haidinger, W., Leipzig (1829).

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Referred to N_3 this zone fragment is incomplete and contains the extra forms $\frac{34}{13}$ and $\frac{14}{5}$.

$\mathfrak{S}$ $(25\cdot7\cdot\bar{3}\bar{2}\cdot5) = (62\cdot\bar{1}\bar{3}\cdot\bar{3}\bar{4})$. This scalenohedron was observed by Beykirch [118] on crystals from Dortmund, Prussia. This form falls very close to $(15\cdot4\cdot\bar{1}\bar{9}\cdot3) = (37\cdot\bar{8}\cdot\bar{2}\bar{0})$ and should possibly be referred to the simpler indices.

$\mathfrak{v}$ $(10\cdot3\cdot\bar{1}\bar{3}\cdot2) = (25\cdot\bar{5}\cdot\bar{1}\bar{4})$. This scalenohedron was recorded by Sansoni [119] on crystals from Freiberg. In spite of the relatively high normal series to which this form is referable, the fact that it lies in the well marked zone $[04\bar{4}1 : 80\bar{8}1]$ should class it as probable.

Scalenohedrons of the Zone $[10\bar{1}1 : 24\bar{6}1] = [100 : 31\bar{3}]$.

Symbol $\frac{h}{k}$.

Extended beyond the limits to be discussed this zone contains several important knot points. Between (100) and $31\bar{3}$) the forms are:

Form	p. 100	Z $17\cdot1\cdot\bar{3}$	N $13\cdot1\cdot\bar{3}$	$\mathfrak{w}$ $8\bar{1}\bar{3}$	α $5\bar{1}\bar{3}$	c $4\bar{1}\bar{3}$	$\mathfrak{q}$ $3\bar{1}\bar{3}$
Symbol	∞	17	13	8	5	4	3
$v-3$	∞	14	10	5	2	1	0
$\frac{v^1}{2}$	∞	7	5	$\frac{5}{2}$	1	$\frac{1}{2}$	0
N_2	∞	(7)	(5)	$(\frac{5}{2})$	1	$\frac{1}{2}$	0

Referred to N_2 the zone fragment lacks the term 2 and contains the extra forms 17, 13 and 8, the latter of which has been discussed under zone 12. It is noteworthy that these three extra forms were observed on one occurrence, i. e., crystals from Lake Superior, Mich.

Z $(16\cdot4\cdot\bar{2}\bar{0}\cdot15) = (17\cdot1\cdot\bar{3})$. This scalenohedron was observed by Palache [120] on several crystals from Lake Superior. It was described as a somewhat rounded series of planes tending to pass into N $(12\cdot4\cdot\bar{1}\bar{6}\cdot11) = (13\cdot1\cdot\bar{3})$. It marks the intersection of zones $(04\bar{4}3 : 8\bar{4}\bar{4}3]$ and $[11\bar{2}\bar{3} : 40\bar{4}1]$. The form should be regarded as fairly probable.

N $(12\cdot4\cdot\bar{1}\bar{6}\cdot11) = (13\cdot1\cdot\bar{3})$. This scalenohedron was also ob-

118 Beykirch, J., Centralblat. f. Min., **2**, 494 (1901).
119 Sansoni, F.,Giorn. d. Min., **5**, 72 (1894).
120 Palache, loc. cit., Fig. 17.

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served by Palache [121] on crystals from Lake Superior. On one combination figured it occurs with $(10\bar{1}1)$ and $(16\cdot4\cdot\bar{2}\bar{0}\cdot15)$ emphasizing the zone under discussion. The planes were described as smooth and often quite large, evidently furnishing a number of good measurements. The zonal relations are much better than in the previous case and the form should be regarded as well established.

Hypothetical Scalenohedrons. In the above series the missing terms 3 and 2 correspond to the hypothetical forms $(8\cdot4\cdot\bar{1}\bar{2}\cdot7) = (91\bar{3})$ and $(6\cdot4\cdot\bar{1}\bar{0}\cdot5) = (71\bar{3})$ respectively.

$(8\cdot4\cdot\bar{1}\bar{2}\cdot7) = (91\bar{3})$. The pole of this scalenohedron lies in excellent zonal relations with frequent and prominent forms, falling in zones $[40\bar{4}1 : 22\bar{4}3]$ and $[02\bar{2}1 : 41\bar{5}3]$ both of which are well marked. It is noteworthy that the scalenohedron recorded by Flink [122] from Långban, Sweden, falls very close to this hypothetical form. It is possible that it should be substituted for the latter.

$(6\cdot4\cdot\bar{1}\bar{0}\cdot5) = (71\bar{3})$. The pole of this form presents zonal relations which are better than those of the foregoing. It lies in the following well marked zones $[02\bar{2}1 : 31\bar{4}2 : 20\bar{2}1]$, $[40\bar{4}1 : 12\bar{3}2]$ and $[0\bar{1}\bar{1}2 : 21\bar{3}1]$.

Scalenohedrons of the Zone $[41\bar{5}3 : 03\bar{3}1 : 11\bar{2}0] = [40\bar{1} : 44\bar{5} : 10\bar{1}]$

Symbol $\frac{h}{k}$.

Developing the first portion of this zone between $(40\bar{1})$ and $(44\bar{5})$ we have:

Form	$\left\{\begin{matrix}F:\\40\bar{1}\end{matrix}\right.$	H	$\mathfrak{w}$	G	ν	$\mathrm{I}\cdot$	$\mathfrak{E}:$	$\mathfrak{h}:$	$\Gamma\cdot$
		$20\cdot2\cdot\bar{7}$	$81\bar{3}$	$12\cdot2\cdot\bar{5}$	$41\bar{2}$	$32\cdot15\cdot\bar{2}\bar{3}$	$42\bar{3}$	$43\bar{4}$	$44\bar{5}$
Symbol	∞	10	8	6	4	$\frac{32}{15}$	2	$\frac{4}{3}$	1
$v-1$	∞	9	7	5	3	$\frac{17}{15}$	1	$\frac{1}{3}$	0
N_3	∞	(9)	(7)	(5)	3 ..	$(\frac{17}{15})$	1 ..	$\frac{1}{3}$	0

Referred to N_3 this portion of the zone is incomplete and contains the extra forms 10, 8, 6, and $\frac{32}{15}$, of which the second has been discussed under zone 12.

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121 Palache, loc. cit., Figs. 6 and 19.
122 Flink, Arkiv. f. Kemi. Min. Geol., **3**, 55, Fig. 155 and 156 (1910).

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H $(63\bar{9}5) = (20\cdot2\cdot\bar{7})$. This scalenohedron was recorded by Whitlock [123] on crystals from West Paterson, N. J. In the combination cited (Type II) this form is present in zone $[0001 : 21\bar{3}0]$ with $(8\cdot4\cdot\bar{12}\cdot5)$, (0001) and $(42\bar{6}\bar{1})$. The form also falls in zone $[40\bar{4}7 : 11\bar{2}0]$. It may be regarded as fairly probable.

G $(10\cdot7\cdot\bar{17}\cdot9) = (12\cdot2\cdot\bar{5})$. This scalenohedron was simultaneously observed in 1897 by Butgenbach [124] on crystals from Cumberland, and by Moesz [125] on crystals from Körösmezö, Hungary. In addition to the evidence of two observers, this form presents better zonal relations than the preceding and is referable to normal series N_5. It should be classed as probable.

I $\cdot(17\cdot38\cdot\bar{55}\cdot24) = (32\cdot15\cdot\bar{23})$. This scalenohedron was observed by Palache [126] on crystals from the Lake Superior region of Michigan. The form which was observed twice was developed to an appreciable habit on one crystal but as observed by Palache its occurrence at this locality is exceptional and under ordinary condition the form would be vicinal, probably to $(25\bar{7}3)$.

ϖ $(3\cdot24\cdot\bar{27}\cdot7) = (37\cdot28\cdot\bar{44})$. This scalenohedron was recorded by Morton [127] on crystals from Bamle, Norway. In the combination described by Morton the form occurs with $(32\bar{5}7)$ and $(01\bar{1}0)$ in a fairly well marked zone. It is noteworthy, however, that the hypothetical indices $(4\cdot31\cdot\bar{35}\cdot9) = (19\cdot12\cdot\bar{16})$ fall very close to the position assumed by Morton and offer much better zonal relations. The form is somewhat doubtful.

In the course of the preceding analysis the following tentative substitutions have suggested themselves as offering better zonal relations, as falling in relatively low normal series and as being expressed in simpler Bravais and Miller indices. In every case the difference in φ and ρ value between the questioned and substituted pole is well within the limit of ordinary fair observation.

E $(32\cdot56\cdot\bar{88}\cdot69) = (61\cdot31\cdot\bar{25})$. This scalenohedron was substituted by Palache [128] for the form $(21\cdot35\cdot\bar{56}\cdot44) = (71\cdot58\cdot\bar{47})$, described by him on one crystal from Lake Superior, Mich. Neither form falls in any of the zones discussed, nor are they tied by zonal relations to any of the knot poles of the calcite network. The hypo-

123 Whitlock, H. P., Am. Jour. Sci., **24**, 426 (1907).
124 Butgenbach, H., Soc. Geol. Belg. Ann., **24**, 66.
125 Moesz, G., Fold. Kozlony, **27**, 495.
126 Palache, C., loc. cit.
127 Morton, C., Stockh. Ofvers., **65** (1884).
128 Palache, C., loc. cit., Fig. 5.

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Letter	Bravais	Miller	N_2	N_3	N_4	N_5	N_+	Zone intersections
			Zone $[20\bar{2}1 : 02\bar{2}1 : 10\bar{1}0]$					
f	$6\cdot20\cdot\bar{2}\bar{6}\cdot13$	$15\cdot9\cdot\bar{1}\bar{1}$					N+	$[01\bar{1}1 : 2\cdot8\cdot\bar{1}\bar{0}\cdot3]$, $[16\bar{7}4 : 32\bar{5}1]$
ℭ	$3\cdot11\cdot\bar{1}\bar{4}\cdot7$	$85\bar{6}$			N_4			$[07\bar{7}5 : 43\bar{7}1]$, $[10\bar{1}1 : 0\cdot11\cdot\bar{1}\bar{1}\cdot1]$
V	$2\cdot8\cdot\bar{1}\bar{0}\cdot5$	$17\cdot11\cdot\bar{1}\bar{3}$					N+	$[20\bar{2}5 : 01\bar{1}0]$, $[05\bar{5}3 : 23\bar{5}2]$
t	$3\cdot13\cdot\bar{1}\bar{6}\cdot8$	$96\bar{7}$		N_3				$[0\cdot13\cdot\bar{1}\bar{3}\cdot8 : 10\bar{1}0]$
D	$3\cdot8\cdot\bar{1}\bar{1}\cdot4$	$63\bar{5}$		N_3				$[11\bar{2}3 : 27\bar{9}1]$, $[04\bar{4}5 : 24\bar{6}1]$
A	$6\cdot14\cdot\bar{2}\bar{0}\cdot7$	$11\cdot5\cdot\bar{9}$				N_5		$[3\bar{1}\bar{2}1 : 44\bar{8}3]$, $[2\bar{2}01 : 05\bar{5}1]$
g	$16\cdot10\cdot\bar{2}\bar{6}\cdot5$	$47\cdot\bar{1}\cdot\bar{3}1$					N+	
T	$42\bar{6}1$	$11\cdot\bar{1}\cdot\bar{7}$		N_3				$[40\bar{4}1 : 01\bar{1}0]$, $[50\bar{5}1 : 0\cdot10\cdot\bar{1}\bar{0}\cdot1]$
ℌ	$9\cdot4\cdot\bar{1}\bar{3}\cdot2$	$8\bar{1}\bar{5}$	N_2					$[11\bar{2}3 : 21\bar{3}1]$, $[40\bar{4}1 : 54\bar{9}1]$
m	$52\bar{7}1$	$13\cdot\bar{2}\cdot\bar{8}$		N_3				$[50\bar{5}1 : 01\bar{1}0]$, $[70\bar{7}1 : 16\bar{7}1]$
			Zone $[1\bar{1}02 : 40\bar{4}7 : 17\bar{8}0]$					
I	$7\cdot17\cdot\bar{2}\bar{4}\cdot8$	$13\cdot6\cdot\bar{1}\bar{1}$			N_4			$[30\bar{3}1 : 03\bar{3}1]$, $[02\bar{2}3 : 24\bar{6}1]$
J	$8\cdot20\cdot\bar{2}\bar{8}\cdot9$	$15\cdot7\cdot\bar{1}\bar{3}$					N+	$[04\bar{4}3 : 11\bar{2}0]$, $[04\bar{4}5 : 24\bar{6}1]$
i	$9\cdot23\cdot\bar{3}\bar{2}\cdot10$	$17\cdot8\cdot\bar{1}\bar{5}$				N_5		$[0\cdot11\cdot\bar{1}\bar{1}\cdot4 : 11\cdot0\cdot\bar{1}\bar{1}\cdot2]$, $[02\bar{2}1 : 31\bar{4}0]$
Y	$12\cdot32\cdot\bar{4}\bar{4}\cdot13$	$23\cdot11\cdot\bar{2}\bar{1}$					N+	$[02\bar{2}1 : 21\bar{3}0]$, $[\bar{1}4\bar{3}1 : 40\bar{4}1]$
B	$12\cdot40\cdot52\cdot11$	$25\cdot13\cdot\bar{2}\bar{7}$					N+	$[05\bar{5}1 : 40\bar{4}1]$, $[20\bar{2}1 : 08\bar{8}1]$
			Zone $[11\bar{2}3 : 13\bar{4}1]$					
W	$3\cdot7\cdot\bar{1}\bar{0}\cdot5$	$63\bar{4}$	N_2					$[3\bar{1}\bar{2}1 : 35\bar{8}4]$
X	$5\cdot13\cdot\bar{1}\bar{8}\cdot7$	$10\cdot5\cdot\bar{8}$			N_4			$[2\bar{2}01 : 04\bar{4}1]$, $[02\bar{2}1 : 51\bar{6}1]$

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Letter	Bravais	Miller	N_2	N_3	N_4	N_5	N+	Zone intersections
Zone $[05\bar{5}1 : 50\bar{5}1 : 01\bar{1}0]$								
S	$15\cdot5\cdot\bar{2}\bar{0}\cdot4$	$13\cdot\bar{2}\cdot\bar{7}$			N_4			$[50\bar{5}2 : 11\bar{2}0]$, $[05\bar{5}4 : 10\bar{1}0]$
ℑ	$15\cdot4\cdot\bar{1}\bar{9}\cdot3$	$37\cdot\bar{8}\cdot\bar{2}\bar{0}$		N_3				$[04\bar{4}3 : 10\bar{1}0]$, $[30\bar{3}1 : 62\bar{8}1]$
ℭ	$25\cdot7\cdot\bar{3}\bar{2}\cdot5$	$62\cdot\bar{1}\bar{3}\cdot\bar{3}\bar{4}$					N+	
ɒ	$10\cdot3\cdot\bar{1}\bar{3}\cdot2$	$25\cdot\bar{5}\cdot\bar{1}\bar{4}$					N+	$[04\bar{4}1 : 80\bar{8}1]$ $[13\cdot0\cdot\bar{1}\bar{3}\cdot2 : \bar{1}100]$
Zone $[10\bar{1}1 : 24\bar{6}1]$								
Z	$16\cdot4\cdot\bar{2}\bar{0}\cdot5$	$17\cdot1\cdot\bar{3}$					N+	$[04\bar{4}3 : 8\bar{4}\bar{4}3]$ $[11\bar{2}3 : 40\bar{4}1]$
N	$12\cdot4\cdot\bar{1}\bar{6}\cdot11$	$13\cdot1\cdot\bar{3}$				N_5		$[08\bar{8}7 : 3\bar{1}\bar{2}1]$, $[20\bar{2}1 : 04\bar{4}5]$
Hypothetical scalenohedrons of the zone $[10\bar{1}1 : 24\bar{6}1]$								
	$8\cdot4\cdot\bar{1}\bar{2}\cdot7$	$91\bar{3}$		N_3				$[11\bar{2}3 : 62\bar{8}1]$ $[02\bar{2}1 : 41\bar{5}3]$
	$6\cdot4\cdot\bar{1}\bar{0}\cdot5$	$71\bar{3}$	N_2					$[01\bar{1}2 : 21\bar{3}1]$ $[40\bar{4}1 : 08\bar{8}7]$
Zone $[41\bar{5}3 : 03\bar{3}1 : 11\bar{2}0]$								
H	$63\bar{9}5$	$20\cdot2\cdot\bar{7}$					N+	$[40\bar{4}7 : 11\bar{2}0]$ $[01\bar{1}1 : 30\bar{3}1]$
G	$10\cdot7\cdot\bar{1}\bar{7}\cdot9$	$12\cdot2\cdot\bar{5}$				N_5		$[01\bar{1}1 : 50\bar{5}1]$ $[01\bar{1}2 : 21\bar{3}1]$
I.	$17\cdot38\cdot\bar{5}\bar{5}\cdot24$	$32\cdot15\cdot\bar{2}\bar{3}$					N+	
ɯ	$3\cdot24\cdot\bar{2}\bar{7}\cdot7$	$37\cdot28\cdot\bar{4}\bar{4}$					N+	$[32\bar{5}7 : 01\bar{1}0]$

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thetical scalenohedron $(4\cdot7\cdot\bar{1}\bar{1}\cdot9)$, which lies extremely close to both of the above forms, falls in zone $[11\bar{2}3\cdot13\bar{4}1]$ reducing to $\frac{5}{3}$ in normal series N_4. The latter form, moreover, lies at the intersection of zones $[01\bar{1}2 : 43\bar{7}1]$ and $[4047 : 07\bar{7}2]$. The scalenohedrons $(32\cdot56\cdot\bar{8}\bar{8}\cdot69)$ and $(4\cdot7\cdot\bar{1}\bar{1}\cdot9)$ have the same value for φ and differ in the value of the ρ angle by $1^\circ\,15'$. *In commenting on the form Palache states that* the faces were somewhat uneven.

$\mathfrak{F}$ $(12\cdot68\cdot\bar{8}\bar{0}\cdot35) = (41\cdot29\cdot\bar{3}\bar{9})$. This scalenohedron was observed by Schnoor [129] on crystals from Neumark, Germany. The form lies close to the zone $[02\bar{2}1 : 10\bar{1}0]$ and is unrelated to any of the important poles by zone lines. The nearest rational hypothetical form in the zone $[02\bar{2}1 : 10\bar{1}0]$ is $(16\bar{7}3) = (11\cdot8\cdot\bar{1}\bar{0})$ which develops to the term $\frac{1}{4}$ in normal series N_4. Although differing somewhat beyond the limits of good observation in position from $(12\cdot68\cdot\bar{8}\bar{0}\cdot35)$ this latter form presents fair zonal relations. It might possibly be substituted.

Ξ $(18\cdot49\cdot\bar{6}\bar{7}\cdot20) = (35\cdot17\cdot\bar{3}\bar{2})$. This scalenohedron was noted by vom Rath [130] on crystals from Elba. Irby in commenting on it observes that the indices are complex but is unable to find a substitute for it. The pole of the form lies close to $(12\cdot32\cdot\bar{4}\bar{4}\cdot13)$ in zone $[40\bar{4}7: 17\bar{8}0]$ a form which has been ranked as probable in the previous discussion. Another possible substitution would be the hypothetical scalenohedron $(7\cdot20\cdot\bar{2}\bar{7}\cdot8) = (14\cdot7\cdot\bar{1}\bar{3})$ in the zone $[11\bar{2}3 : 13\bar{4}1]$. The zonal relations of both these suggested substitutions are good.

I: $(17\cdot8\cdot\bar{2}\bar{5}\cdot8) = (50\cdot1\cdot\bar{2}\bar{5})$. This scalenohedron is included in Roger's list on the authority of Goldschmidt [131]. The form is listed by Rogers with the note "Observer not known." It lies very close to zone 4 and may be vicinal to $(21\bar{3}1)$, if indeed it is not identical with the latter form.

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129 Schnoor, Wissen. Beil. L., Programm d. Realgym. Z. Zwickau, 16 (1896).
130 vom Rath, G., Pogg. Ann., **158**.
131 Goldschmidt, V., Kryst. Winkeltab.

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Positive Scalenohedrons not Previously Included.

L $(22\cdot8\cdot\bar{3}\bar{0}\cdot37) = (89\cdot23\cdot\bar{1})$. This scalenohedron was observed by Thürling [132] on crystals from Andreasberg. In the combination cited by Thürling the form occurs as the inner kernal of a compound crystal of scalenohedral habit, the form in question projecting through the terminating planes $(12\bar{3}5)$ of the outer individual. The planes were described as smooth and brilliant. It is, however, noteworthy that the pole of $(22\cdot8\cdot\bar{3}\bar{0}\cdot37)$ falls very close to that of the common scalenohedron $(31\bar{4}5)$ of zone 4 and may possibly be vicinal to it.

𝔄 $(7\cdot6\cdot\bar{1}\bar{3}\cdot2) = (22\cdot1\cdot\bar{1}\bar{7})$. This scalenohedron was recorded by Johansson [133] on crystals from Norberg, Sweden. The pole of the form lies at the intersection of several well marked zones, as $[50\bar{5}1 : 8\cdot14\cdot\bar{2}\bar{2}\cdot3]$, $[03\bar{3}1 : 43\bar{7}1]$ and $[20\bar{2}1 : 44\bar{8}1]$ and should be classed as probable.

𝔟 $(20\cdot11\cdot\bar{3}\bar{1}\cdot15) = (33\cdot2\cdot\bar{9})$
𝔞 $(37\cdot19\cdot\bar{5}\bar{6}\cdot21) = (38\cdot1\cdot\bar{1}\bar{8})$ } These two scalenohedrons were observed by Palache [134] on several crystals from the Lake Superior region of Michigan. They, together with 𝔴 $(7\cdot4\cdot\bar{1}\bar{1}\cdot6)$ as previously discussed, lie in zone $[\bar{1}012 : 21\bar{3}1]$ not $[R : R_3]$ as given by Palache. The good quality of the faces which served to determine these forms and the excellent zonal relations should serve to class both forms as probable.

M $(8\cdot4\cdot\bar{1}\bar{2}\cdot5) = (25\cdot1\cdot\bar{1}\bar{1})$. This scalenohedron like the two last was observed by Palache [135] on one crystal from the Lake Superior region. The planes were relatively large and well developed and the zonal relations very good. The form is well established.

𝔅 $(10\cdot5\cdot\bar{1}\bar{5}\cdot4) = (29\cdot\bar{1}\cdot\bar{1}\bar{6})$. This scalenohedron was observed by Sansoni [136] on crystals from Blaton, Belgium, and by Cesàro [137] on crystals from Rhisnes, Belgium. Cesàro points out that the measurements obtained by Sansoni correspond better to the form $(26\cdot13\cdot\bar{3}\bar{9}\cdot10)$ but emphasizes the fact that these latter indices do not fall in the zones $[50\bar{5}1 : 05\bar{5}2]$, $[50\bar{5}2 : 53\bar{8}2]$ and $[0001 : 21\bar{3}0]$, which zonal rela-

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132 Thürling, G., N. Jahrb. f. Min. B. B., **4**, 327 (1886).
133 Johansson, K., Geol. Fören. Förh., **14**, 49 (1892).
134 Palache, C., loc. cit., Fig. 5.
135 Palache, C., loc. cit., Fig. 9.
136 Sansoni, F., Bull. Ac. Belg., (3), **9**, 287 (1885).
137 Cesàro, G., Ann. Soc. Geol. Belg., **16**, 165 (1889).

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tions hold good for the form $(10\cdot5\cdot\bar{1}\bar{5}\cdot4)$. The form should be classed as quite probable.

P $(13\cdot7\cdot\bar{2}\bar{0}\cdot3) = (12\cdot\bar{1}\cdot\bar{8})$. This scalenohedron was observed by Palache [138] on one crystal from the Lake Superior region. The form corresponds to relatively simple indices and lies at the intersection of a number of well marked zones. The form may be regarded as well established.

Φ $(17\cdot8\cdot\bar{2}\bar{5}\cdot3) = (15\cdot\bar{2}\cdot\bar{1}\bar{0})$. This scalenohedron was observed by Cesàro [139] on a number of crystals from Rhisnes. The form was determined from a number of good measurements, and presents excellent zonal relations. It may be regarded as well established.

π $(63\cdot28\cdot\bar{9}1\cdot11) = (55\cdot\bar{8}\cdot\bar{3}\bar{6})$. This scalenohedron was observed by Melczer [140] on crystals from the vicinity of Budapest. The pole of this form does not fall in any well marked zone and lies quite close to that of $(17\cdot8\cdot\bar{2}\bar{5}\cdot3)$. Again, the hypothetical scalenohedron $(23\cdot10\cdot33\cdot4) = (20\cdot\bar{3}\cdot\bar{1}\bar{3})$ lies sufficiently close to $(63\cdot28\cdot\bar{9}\bar{1}\cdot11)$ to be well within the limits of good measurements, thus:

	φ	ρ
$(63\cdot28\cdot\bar{9}1\cdot11)$	17° 29′	82° 8′
$(23\cdot10\cdot\bar{3}\bar{3}\cdot4)$	17 11	82 7½

From this it would appear that either $(17\cdot8\cdot\bar{2}\bar{5}\cdot3)$ or $(23\cdot10\cdot\bar{3}\bar{3}\cdot4)$, both of which have more rational indices and better zonal relations, could be substituted for $(63\cdot28\cdot\bar{9}\bar{1}\cdot11)$.

Δ $(70\cdot21\cdot\bar{9}\bar{1}\cdot13) = (58\cdot\bar{1}\bar{3}\cdot\bar{3}\bar{3})$. This scalenohedron was observed by Sansoni [141] on two incomplete crystals from Andreasberg. The somewhat irrational indices assigned to this form by Sansoni were evidently derived from the mean values of the measured angles by an emperical application of the derivation formulas. The form does not present good zonal relations and was suspected of being vicinal to $(62\bar{8}1)$. The form is rather doubtful.

U $(61\bar{7}1) = (14\cdot\bar{4}\cdot\bar{7})$. This scalenohedron was recorded by Buttgenbach.[142] The form presents excellent zonal relations and falls in normal series N_3 in the zone developed between $[21\bar{3}1 : 10\bar{1}0] = [20\bar{1} : 2\bar{1}\bar{1}]$. It should be classed as probable.

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138 Palache, C., loc. cit., Fig. 7.
139 Cesàro, G., loc. cit.
140 Melczer, G., Föld. Közlöny, **26**, 79 (1896).
141 Sansoni, F., Atti. d. Lincei Mem., **14** (1884).
142 Buttgenbach, H., Soc. Geol. Belg. Ann., **32**, 86 (1905).

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$\mathfrak{D}$ $(21\cdot5\cdot\bar{2}\bar{6}\cdot18) = (65\cdot2\cdot\bar{1}\bar{3})$. This scalenohedron was observed by Whitlock [143] on crystals of the second type from Kelley's Island, Ohio. This form was identified from four series of consistent measurements. It is, however, apparent from a consideration of the zonal relations that the indices $(8\cdot2\cdot\bar{1}\bar{0}\cdot7) = (25\cdot1\cdot\bar{5})$ present a much better agreement than those originally assigned. The pole of the latter form falls at the intersection of the zones $(61\bar{7}5 : 02\bar{2}1]$ and $[4\bar{2}\bar{2}3 : 11\bar{2}1]$. Moreover the calculated angles for $(8\cdot2\cdot\bar{1}\bar{0}\cdot7)$ agree almost as well with observation as do those of $(21\cdot5\cdot\bar{2}\bar{6}\cdot18)$. The indices $(8\cdot2\cdot\bar{1}\bar{0}\cdot7)$ should be substituted for $(21\cdot5\cdot\bar{2}\bar{6}\cdot18)$.

e^3 $(43\bar{7}2) = (13\cdot1\cdot\bar{8})$. This scalenohedron was recorded by Flink [144] on crystals from Långbån, Sweden. As shown on Flink's figure this form lies in the zone $[21\bar{3}1 : 24\bar{6}1 : 01\bar{1}0]$ which developed between the poles $(21\bar{3}1) = (20\bar{1})$ and $(24\bar{6}1) = (31\bar{3})$ gives:

Forms	$20\bar{1}$	$13\cdot1\cdot\bar{8}$	$71\bar{5}$	$51\bar{4}$	$23\cdot5\cdot\bar{1}9$	$82\bar{7}$	$31\bar{3}$
Symbol	2	$\frac{13}{8}$	$\frac{7}{5}$	$\frac{5}{4}$	$\frac{23}{19}$	$\frac{8}{7}$	1
$\frac{r - r_1}{r_2 - r}$	0	$\frac{3}{5}$	$\frac{3}{2}$	3	$\frac{15}{4}$	6	∞
$\frac{r^1}{3}$	0	$\frac{1}{5}$	$\frac{1}{2}$	1	$\frac{3}{4}$	2	∞

From which development it will be seen that $(43\bar{7}2)$ fall in normal series N_5. It also lies in zones $[03\bar{3}2 : 10\bar{1}0]$ and $[05\bar{5}2 : 50\bar{5}1]$ both of which are well marked. The form is probable.

e^1 $(14\cdot7\cdot\bar{2}\bar{1}\cdot12) = (47\cdot9\cdot\bar{1}\bar{6})$. This scalenohedron was also recorded by Flink [144] on crystals from the same occurrence as the preceding. The zonal relations of this form are very poor, which coupled with the high Miller indices seem to render the form somewhat doubtful. It is noteworthy that the hypothetical form $(8\cdot4\cdot\bar{1}\bar{2}\cdot7) = (91\bar{3})$ meets the critical requirements of zonal relations and rational Miller indices much better than $(14\cdot7\cdot\bar{2}\bar{1}\cdot12)$, has the same value for the φ angle and differs in ρ value from the latter form by only 12′. It is possible that $(8\cdot4\cdot\bar{1}\bar{2}\cdot7)$ should be substituted.

h' $(16\cdot14\cdot\bar{3}\bar{0}\cdot5) = (12\cdot1\cdot\bar{8})$. This scalenohedron was observed by Flink [145] on crystals from Långbån, Sweden. The form lies in zones $[52\bar{6}1 : 24\bar{6}1]$ and $[50\bar{5}2 : 32\bar{5}1]$ and corresponds to quite simple Miller indices. It is fairly probable.

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143 Whitlock, H. P., S. of M. Quarterly, **31**, 231 (1910).
144 Flink, Arkiv. f. Kemi. Min. Geol., **3**, Fig. 155, 156 (1910).
145 Flink, Arkiv. f. Kemi. Min. Geol., **3**, Fig. 174 (1910).

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g' $(21\cdot8\cdot\bar{2}\bar{9}\cdot5) = (55\cdot8\cdot\bar{3}\bar{2})$. This scalenohedron was noted by Flink [146] on crystals from Långbån, Sweden. As shown by the figure cited, the observed planes lie in zone $[50\bar{5}1 : 0\cdot10\cdot\bar{1}\bar{0}\cdot1]$ the form $(50\bar{5}1)$ being present on the combination. The zonal relations of this form in general are not good and it is to be noted that the hypothetical scalenohedron $(25\cdot10\cdot\bar{3}\bar{5}\cdot6) = (22\cdot3\cdot\bar{1}\bar{3})$ which is also in the zone $[50\bar{5}1 : 0\cdot10\cdot\bar{1}\bar{0}\cdot1]$ has simpler Miller indices, better general zonal relations and differs from $(21\cdot8\cdot\bar{2}\bar{9}\cdot5)$ in value of the φ angle by 35′, the ρ angles corresponding to within 2′. The hypothetical scalenohedron $(25\cdot10\cdot\bar{3}\bar{5}\cdot6)$ is a possible substitution.

Negative Scalenohedrons not Previously Included.

Γ $(6\cdot8\cdot\bar{1}\bar{4}\cdot3) = (23\cdot5\cdot\bar{1}\bar{9})$. This scalenohedron was recorded by Sansoni [147] on crystals from Andreasberg. The form marks the intersection of a number of well marked zones and should be classed as probable.

$\mathfrak{r}$ $(4\cdot9\cdot\bar{1}\bar{3}\cdot6) = (23\cdot11\cdot\bar{1}\bar{6})$. This scalenohedron was observed by Schnoor [148] on crystals from Neumark, Sweden. Although not presenting as good zonal relations as the preceding form, this scalenohedron marks the intersection of the zones $[03\bar{3}2 : 10\bar{1}0]$, $[01\bar{1}1 : 45\bar{9}2]$ and $[05\bar{5}3 : 13\cdot0\cdot\bar{1}\bar{3}\cdot2)$. The form should be classed as probable.

$\mathfrak{f}$ $(2\cdot20\cdot\bar{2}\bar{2}\cdot21) = (15\cdot13\cdot\bar{7})$. This scalenohedron was observed by Cesàro [149] on crystals from Rhisnes, Belgium, and the indices $(2\cdot20\cdot\bar{2}\bar{2}\cdot21)$ substituted for $(2\cdot19\cdot\bar{2}\bar{1}\cdot20) = (43\cdot37\cdot\bar{2}\bar{0})$ given in a former paper on the same occurrence. Cesàro points out the fact that the form $(2\cdot20\cdot\bar{2}\bar{2}\cdot21)$ lies at the intersection of the zones $[01\bar{1}1 : 2\bar{1}\bar{1}0]$, $[44\bar{8}3 : 17\cdot8\cdot\bar{2}\bar{5}\cdot3]$ and $[40\bar{4}7 : 08\bar{8}7]$. This would seem to be a good zonal relation, the form, however, lies quite close to $(01\bar{1}1)$ and may possibly be vicinal to the rhombohedron. It is somewhat doubtful.

$\mathfrak{c}$ $(4\cdot20\cdot\bar{2}\bar{4}\cdot17) = (15\cdot11\cdot\bar{9})$. This scalenohedron was noted by Sansoni [150] on crystals from Utö, Sweden. The form was observed in combination with $(08\bar{8}7)$ thus emphasizing the zone $[08\bar{8}7 : 62\bar{8}1]$,

146 Flink, Arkiv. f. Kemi. Min. Geol., **3**, Fig. 156 (1910).
147 Sansoni, F., loc. cit., Fig. 10.
148 Schnoor, loc. cit.
149 Cesàro, loc. cit.
150 Sansoni, F., Giorn. d. Min., **1** (2), 129 (1890).

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a well marked zone containing several important forms. Palache [151] also observed this form on several crystals from the Lake Superior region, on one of which the above mentioned zone is quite strongly emphasized. The form also lies in zones [04$\bar{4}$3 : 20$\bar{2}$1] and [05$\bar{5}$4: 40$\bar{4}$1]. It should be classed as well established.

δ (4·36·$\bar{4}\bar{0}$·31) = (25·21·$\bar{1}\bar{5}$). This scalenohedron was also observed by Palache on the same combination noted above, also falling in the zone [08$\bar{8}$7 : 62$\bar{8}$1]. The form also falls at the intersection of the zones [40$\bar{4}$7 : 03$\bar{3}$2] and [10$\bar{1}$1 : 04$\bar{4}$3]. The form should be classed as probable.

R (8·32·$\bar{4}\bar{0}$·21) = (23·15·$\bar{1}\bar{7}$). This scalenohedron was noted by Sansoni [152] on one crystal from Andreasberg. The combination figured by Sansoni does not suggest any zonal relations between this scalenohedron and the other forms present on the combination, nor does the form lie in any well marked zone. On the other hand the hypothetical scalenohedron (4·15·$\bar{1}\bar{9}$·10) = (11·7·$\bar{8}$), the pole of which lies very close to (8·32·$\bar{4}\bar{0}$·21) marks the intersection of several well defined zones, viz: [01$\bar{1}$2 : 13$\bar{4}$1], [02$\bar{2}$1 : 41$\bar{5}$3], [07$\bar{7}$2 : 41$\bar{5}$6] and [61$\bar{7}$4 : 31$\bar{5}$1]. The agreement between the calculated angles of the hypothetical form and Sansoni's measurements is shown below:

	Measured	Calculated (8·32·$\bar{4}\bar{0}$·21)	Calculated (4·15·$\bar{1}\bar{9}$·10)
10$\bar{1}$0 : *hkil*	55° 18′	55° 44′	55° 7′
01$\bar{1}$0 : "	31 51	31 16	31 17½
08$\bar{8}$7 : "	13 49	13 58	14 47½
10$\bar{1}$1 : "	40 50	40 39	40 30

From the above table it would appear that in 3 out of the 4 angles measured the form (4·15·$\bar{1}\bar{9}$·10) agrees with the measured angles within the limits of fair observation and in one angle (4·15·1$\bar{9}$·10) is closer to the observed value than (8·32·$\bar{4}\bar{0}$·21). The form should be classed as doubtful.

C (1·13·$\bar{1}\bar{4}$·10) = (25·22·$\bar{1}\bar{7}$). This scalenohedron was observed by Whitlock [153] on crystals from West Paterson, N. J. In spite of the fact that the Miller indices for this form are somewhat irrational the zonal relations between this scalenohedron and other forms present

151 Palache, C., loc. cit., Fig. 12.
152 Sansoni, F., Atti. d. Lincei. Mem., **14**, Fig. 8 (1884).
153 Whitlock, H. P., Am. Jour. Sci., **24**, 426 (1907).

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on the combination are fairly good. The pole of the form marks the intersection of the zones [04$\bar{4}$3 : 40$\bar{4}$1] and [06$\bar{6}$5 : 11$\bar{2}$0]. There is, however, a disagreement between the observed and calculated angles which is somewhat greater than the limit of fair observation and the form should be classed as rather doubtful.

K (1·11·$\bar{1}$2·8) = (76$\bar{5}$). This scalenohedron which was observed by Palache [154] on one crystal from the Lake Superior region is regarded by him as very characteristic of the region, occurring on a number of crystals as rounded and broken planes. On the combination cited the pole of this form marks the intersection of the zones [21$\bar{3}$1 : 35$\bar{8}$4] and [3·5·$\bar{8}$·13 : 02$\bar{2}$1], all the above zone-indicating forms being present. The form may be regarded as established.

ι (3·10·$\bar{1}$3·3) = (19·10·$\bar{2}$0). This scalenohedron was observed by Schaller [155] on two crystals from New Mexico. The combination cited is of prismatic habit, the form (3·10·$\bar{1}$3·3) lying in the zone [10$\bar{1}$1 : 01$\bar{1}$0], both zone-indicating forms present. The form, furthermore marks the intersection of the zones [07$\bar{7}$2 : 33$\bar{6}$1], [05$\bar{5}$1 : 30$\bar{3}$1] and [3$\bar{2}$$\bar{1}$1 : 34$\bar{7}$2]. It should be classed as probable.

ϑ (3·61·$\bar{6}$4·26) = (31·28·$\bar{3}$3). This scalenohedron was observed by Cesàro [156] on crystals from Rhisnes, Belgium. The Miller indices corresponding to this form are quite complex and the zonal relations poor. Moreover the form was observed as a multiple series of planes producing a number of reentrant angles, a condition unfavorable to exact measurement. The form should be classed as doubtful.

u (16·59·$\bar{7}$5·17) = (36·20·$\bar{3}$9). This scalenohedron was recorded by Gonnard [157] on crystals from Couzon, Rhône, France. Like the preceding form this scalenohedron does not reduce to very rational Miller indices, nor does it fall in any well marked zone. The pole of the hypothetical scalenohedron (9·35·$\bar{4}$4·10) = (21·12·$\bar{2}$3) lies within 31′ of φ angle and within 2′ of ρ angle of that of (16·59·$\bar{7}$5·17). The above hypothetical form also marks the intersection of the zones [07$\bar{7}$2 : 10$\bar{1}$0], [31$\bar{4}$2 : 13$\bar{4}$1], [30$\bar{3}$1 : 05$\bar{5}$1] and [$\bar{1}$4$\bar{3}$1 : 16·7·$\bar{2}$3·3]. It is possible that the indices (9·35·$\bar{4}$4·10) should be substituted for (16·59·$\bar{7}$5·17).

η (3·16·$\bar{1}$9·2) = (8·5·$\bar{1}$1). This scalenohedron was observed by

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154 Palache, C., loc. cit., Fig. 13.
155 Schaller, W. T., Zeit. f. Kryst., **44**, 325.
156 Cesàro, G., loc. cit., page 272, Fig. 52.
157 Gonnard, F., Bull. Soc. fr. Min., **20**, 330 (1897).

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Melczer[158] on crystals from Budapest. Both the Miller indices and the good zonal relations of this form should class it as well established.

ȝ $(1\cdot13\cdot\bar{1}\bar{4}\cdot1) = (16\cdot13\cdot\bar{2}\bar{6})$. This scalenohedron was observed by Sansoni[159] on crystals from Freiberg. In spite of the extreme steepness of this form the zonal relations are fairly good; the pole of $(1\cdot13\cdot1\bar{4}\cdot1)$ marks the intersection of the zones $[10\bar{1}1 : 01\bar{1}0]$, $[4\bar{2}\bar{2}1 : 33\bar{6}1]$ and $[51\bar{6}1 : 44\bar{8}1]$. The form should be classed as fairly probable.

r $(3\cdot15\cdot\bar{1}\bar{8}\cdot2) = (23\cdot14\cdot\bar{3}\bar{1})$. This scalenohedron was noted by Whitlock[160] on crystals from two localities in the New York State, i. e., Union Springs and Rondout. In spite of the steepness of this form and the fact that the Miller indices corresponding to it are somewhat irrational the zonal relations are fair. The pole of the form falls at the intersection of the zones $[54\bar{9}1 : 27\bar{9}1]$, $[31\bar{4}2 : 01\bar{1}0]$ and $[40\bar{4}1 : 33\bar{6}1]$. Like the preceding, this form should be classed as fairly probable.

$\mathfrak{T}$ $(6\cdot13\cdot\bar{1}\bar{9}\cdot4) = (29\cdot11\cdot\bar{2}\bar{8})$. This scalenohedron was observed by Whitlock[161] on crystals from Kelley's Island, Ohio. This form, which was observed on both types from this locality, was identified from a good series of measurements taken from smooth and well developed planes. The form lies in the zone $[8\cdot8\cdot\bar{1}\bar{6}\cdot3 : \bar{8}\cdot16\cdot\bar{8}\cdot3]$ a relation which was checked by observation, and also marks the intersection of the zones $[3\bar{3}02 : 01\bar{1}0]$ and $[0\cdot13\cdot\bar{1}\bar{3}\cdot4 : 10\bar{1}0]$. The form may be regarded as probable.

h^5 $(6\cdot13\cdot\bar{1}\bar{9}\cdot8) = (11\cdot5\cdot\bar{8})$. This scalenohedron was observed by Flink[162] on crystals from Utö, Sweden. The combination as drawn by Flink shows the planes of this form lying in zone with those of $(5\cdot12\cdot\bar{1}\bar{7}\cdot8)$, that is in zone $[5\cdot12\cdot\bar{1}\bar{7}\cdot8 : \bar{5}\cdot17\cdot\bar{1}\bar{2}\cdot8]$ a relation which is proved untrue when tested by zonal equations. Moreover the pole of $(6\cdot13\cdot\bar{1}\bar{9}\cdot8)$ lies close to the intersection of Z_6 Z_9 which intersection is marked by the pole $(25\bar{7}3)$. The form appears to be somewhat doubtful.

h_6 $(5\cdot12\cdot\bar{1}\bar{7}\cdot8) = (10\cdot3\cdot\bar{7})$. This scalenohedron was observed by Flink[162] on the same combination as the preceding. The same objection to the form as drawn as that given under $(6\cdot13\cdot\bar{1}\bar{9}\cdot8)$ applies

158 Melczer, G., Föld. Közlöny, **26**, 79 (1896).
159 Sansoni, F., Giorn. Min., J. 72 (1894).
160 Whitlock, H. P., N. Y. State Mus. Mem., **13**, Plate XVIII, Fig. 7 and Plate XXVI, Fig. 1 (1910).
161 Whitlock, H. P., S. of M. Quarterly, **31**, 231 (1910).
162 Flink, Arkiv. f. Kemi. Min. Geol., **31**, 174 (1910).

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in this instance. The pole of $(5\cdot12\cdot\bar{1}\bar{7}\cdot8)$ lies at the intersection of the zones $[51\bar{6}4 : 0\cdot11\cdot\bar{1}\bar{1}\cdot4]$ and $[11\bar{2}3 : 13\bar{4}1]$ both of which are well defined. The form is considerably more probable than the preceding.

General Conclusions.

In reviewing the foregoing discussion several facts are brought out with more or less clearness.

1.— The forms whose poles are marked by three zone intersections are those which a student of the species would recognize as common forms. An apparent exception to this statement is to be found in the negative rhombohedron $(05\bar{5}1)$. The expression "apparent" is used because in the experience of the writer this form is far more common on calcite crystals than is usually conceded.

2.— It will be noted that the zone of the second order pyramids Z_2 is marked by a fair number of zone intersections and that where the poles of the second order pyramids, particularly those of the $(11\bar{2}3)$, $(22\bar{4}3)$, $(44\bar{8}3)$ etc., series fall in other zones they occur in relatively low normal series. It would be seen that this series of pyramids, which has been regarded as rare up to a comparatively recent date [163] is considerably more common than has been hitherto recognized. This again coincides with observation, the pyramid $(8\cdot8\cdot\bar{1}\bar{6}\cdot3)$ having been the dominant habit in at least four new occurrences of calcite falling under the writer's notice within the last five years.[164]

3.— That portion of the principal zone, Z_4, lying between the poles p $(10\bar{1}1)$ and P: $(32\bar{5}1)$ shows a completeness with respect to recorded forms which argues strongly against subsequent additions to this part of the zonal field. Especial care should be exercised by future observers and tentative interpolated forms should be strictly criticized and if possible referred to established indices.

4.— Although zones Z_5, Z_6 and Z_7 are strongly marked there are several gaps in normal series N_3 some of which will undoubtedly be filled by future observation. The investigator of calcite should keep these well in mind and weigh carefully the data concerning forms whose poles fall sufficiently near them to be within the limits of goniometrical measurement.

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163 See Penfield and Ford, Am. Jour. Sci., **10**, 238 (1900).

164 These occurrences are Kelley's Island, Wisby and two undescribed Canadian localities.

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RECORDS OF MEETINGS, 1914-15.

OFFICERS AND COMMITTEES FOR 1915-16.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

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RECORDS OF MEETINGS.

One thousand and thirty-seventh Meeting.

OCTOBER 14, 1914.—STATED MEETING.

The Academy met at its House.

The PRESIDENT in the Chair.

There were one hundred and twenty-five Fellows and Ladies present.

The President announced that the Council had appointed Professor H. W. Tyler acting Corresponding Secretary during the absence of Professor Hall, and that Professor A. G. Webster was appointed to succeed Professor Tyler as Librarian.

Professor C. R. Cross, Chairman of the Rumford Committee, stated the grounds for the award of the Rumford Medals to Professor Joel Stebbins, and to Dr. William David Coolidge, in substance as follows:

"At its Annual Meeting of May 14, 1913, the Academy voted on the recommendation of the Rumford Committee to award the Rumford Premium to Professor Joel Stebbins of the University of Illinois, for his development of the selenium photometer and its application to astronomical problems.

"Considerably over thirty years ago, the remarkable discovery was made by Bidwell that the substance selenium under certain conditions, sustained a remarkable lowering of its electrical resistance when subjected to the action of light so that if a selenium 'cell' as it was called, was placed in circuit with a battery and galvanometer, a marked deflection of the galvanometer would be observed if the light from a lamp or other source of illumination was thrown upon the cell.

"Very naturally the idea suggested itself that the selenium cell

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could be used as a photometer and such cells of which I have one at the Institute of Technology were put upon the market. They did not prove however, to be of practical value since their indications were uncertain and the results unreliable.

"A few years since however, Professor Stebbins undertook a new investigation of the selenium cell with a view to its applications to stellar photometry. By a most careful and painstaking investigation, he was able to discover the necessary conditions of success, finding among other things, that the cell must be kept at a low temperature.

"As a result of these investigations, he finally succeeded in constructing a stellar photometer of exquisite sensitiveness, the accuracy of whose results much exceeded those given by visual observation.

"He has applied the instrument to the study of the light variations of variable stars. In one case, that of the well known variable Algol, the "light curve" determined with the selenium photometer showed a secondary minimum which had not been detected by visual observation. He has also applied the instrument to the study of short period spectroscopic binaries with very important results. By the same method, he has detected and studied the variations in brightness of the pole star."

The President then presented the Medals to Professor Stebbins, who responded, giving a brief account of his researches.

In stating the grounds for the award of the Rumford Medals to Dr. William David Coolidge, Professor Cross said in part, "At its last Annual Meeting, May 13, 1914, the Academy acting on the recommendation of the Rumford Committee, voted to award the Rumford Premium to William David Coolidge for his invention of ductile tungsten and its application in the production of radiation.

"Lamps with carbon filaments were the standard form for many years. The efforts of inventors were unremitting however, to secure a filament which should stand the strong current and high temperature needed to give a satisfactory white light and especially a higher efficiency in operation.

"Wires of tantalum gave quite satisfactory results but the substance tungsten seemed to possess certain advantages. It was

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however, an exceedingly brittle metal and hence apparently entirely unsuited for use as the filament of an incandescent lamp.

"Attempts were made however, by various persons abroad and in this country, Dr. Coolidge among them. Their efforts resulted in the production of a filament of tungsten made however, by a paste process giving a filament which stood the action of the current and produced a very white light with a remarkable economy of electrical power. This filament however, was very fragile so that it was difficult to transport such lamps without breaking them and even moderate jarring in the case of the already installed lamps oftentimes destroyed it. It was also impossible accurately to adjust individual lamps to a particular voltage.

"It was to remedy these difficulties that Dr. Coolidge, working in the Research Laboratory of the General Electric Co. at Schenectady, bent his energies.

"He had observed in some experiments that under certain circumstances, the brittle tungsten was so altered as to show some signs of becoming ductile and by a long continued and tedius series of experiments, in which the tungsten was heated and mechanically treated, he was finally able to devise a process by which this typically brittle metal could be rendered ductile so that it could be drawn into a wire of any length and fineness desired. A wire thus made was found to be admirably suited for the purpose of the filament of an incandescent lamp. It could be run at a very high temperature, thus giving a very white light and securing high efficiency. It was not fragile so that no difficulty existed on this score. It could be produced of uniform length and section so that the individual lamps could all be adjusted to run at any desired voltage. The result of this invention was the immediate and universal introduction of the tungsten lamp which we now see everywhere.

"Various other applications of mechanically treated tungsten have been made. Thus it furnishes a reliable and comparatively inexpensive material for use as the electrodes of the spark apparatus of automobiles, and also for use as the target in X-Ray tubes.

"Dr. Coolidge has produced a greatly improved form of X-Ray tube in which both the cathode and the target are of tungsten. Most important however, in the tube is the manner in which the

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stream of electrons which constitute the cathode rays is produced. A fine tungsten wire is placed at the cathode with exterior terminals so that this can be heated to any desired temperature. It is a well known fact that a wire thus heated liberates electrons so that when these are set free within the tube at the cathode, the electric force has only to direct and propel the freed electrons to the target without having to do the work of producing them. This process allows of the production of an excessively strong cathode stream and also allows the strength and character of the stream to be regulated by varying the temperature of the tungsten wire.

"The work of Dr. Coolidge has been carried out and brought to success in the Research Laboratory of the General Electric Co.

"It is pleasing to note that in the awards of the Rumford Premium to Professor Stebbins and to Dr. Coolidge respectively, the Academy has recognized both classes of achievement, for the recognition of which, the Rumford endowment was originally made; for purely scientific work of research in light and heat on the one hand; and on the other, for the advancement of the industrial arts within the borders of these branches of science."

The Medals were then presented by the President to Dr. Coolidge, who responded, giving an account of his researches.

It was

Voted, To meet on adjournment, November 11, 1914.

A reception to the recently elected Fellows then took place in the Reading room, where a collation was served.

One thousand and thirty-eighth Meeting.

NOVEMBER 11, 1914.— ADJOURNED STATED MEETING.

The Academy met at its House.

The PRESIDENT in the Chair.

There were forty-five Fellows present.

The following correspondence was presented: — from F. G. Allinson, Maurice Bloomfield, M. T. Bogert, H. A. Bumstead, C. V. Chapin, G. P. Clinton, E. G. Conklin, James DeNormandie, W. J. Drisko, William Duane, G. A. Gordon, L. C. Graton, J. W. Hammond, Alfred Hemenway, B. H. Hill, A. C. Humphreys,

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C. C. Hutchins, H. S. Jennings, F. D. Lambert, R. S. Lillie, B. E. Livingston, Jacques Loeb, E. G. Merritt, D. C. Miller, R. A. Millikan, H. N. Morse, H. V. Neal, T. B. Osborne, E. D. Peters, S. C. Prescott, Alfred Rehder, G. A. Reisner, R. G. D. Richardson, M. A. Rosanoff, Ellery Sedgwick, F. C. Shattuck, F. H. Verhoeff, W. C. Wait, Eugene Wambaugh, G. C. Whipple and Owen Wister, accepting Fellowship; from J. H. Wigmore, declining Fellowship; from Otto Backlund, John Briquet, Viktor Goldschmidt, Fritz Haber, Sir Sidney Lee, Max Planck, Ignatz Urban and Eugene Warming, accepting Foreign Honorary Membership; from the family of Hugo Kronecker, announcing his death; from the Secretary of the Smithsonian Institution, announcing the death of F. W. True, the Assistant Secretary; from the nineteenth International Congress of Americanists, to be held in La Paz, the capital of Bolivia, in November.

The Chair announced the following deaths: — Abner Cheney Goodell, Fellow in Class III., Section 3; Francis Humphreys Storer, Fellow in Class I., Section 3; Gardiner Martin Lane, Fellow in Class III., Section 4; Hugo Kronecker, Foreign Honorary Member in Class II., Section 3.

The following letter from Senator Lodge and others, in reply to the letter of the Academy, dated February 4, 1913, addressed to the Senate and House of Representatives of the United States, which was read to the Academy at the meeting of March 12, 1913, was presented: —

To the Council of the American Academy of Arts and Sciences,
American Academy of Arts and Sciences,
28 Newbury St., Boston.

Gentlemen:

The undersigned, members in common of the American Academy of Arts and Sciences and of the American Academy of Arts and Letters, have learned with regret that the former organization has protested to Congress against the bill, now pending, providing for a charter to the latter.

We beg to remind you, very respectfully, that the two institutions are essentially different, covering only in small part similar ground:

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the former being concerned, by a very large preponderance of its distinguished membership, its notable library and its useful work, with various aspects of science, a world of intellectual activity with which the latter is not at all concerned. In our judgment the words "Sciences" and "Letters" are appropriately and abundantly descriptive of the chief differences between the two. As for the word "Arts," at the time of the founding of the older body — an institution of long and honorable record — this word was intended, probably, to convey the meaning of the Applied Arts, whereas it is now generally accepted as indicating the Fine Arts, so largely represented in the newer body.

Both Academies include addresses in their proceedings, but there is a wide divergence in subject and character. With this exception, there is not likely to be any aspect of the work of the older institution that will be trenched upon or even paralleled by the newer, which it is expected will devote itself in new and practical ways to the maintenance and dissemination of standards of creative work in literature, painting, sculpture, architecture and music.

We submit that such similarity of names as now exists is natural and not unusual, and is more than compensated for by the discriminating words above mentioned. We feel sure that there is not likely to be any important confusion of the two organizations.

In view of these considerations, we respectfully and earnestly request that as soon as may conveniently be possible, in the interest of the comity that should exist between important agencies of intellectual progress, you will reconsider the action of your body, with a view to the withdrawal of the protest which has been made to Congress.

We have the honor to remain,

Sincerely yours,

HENRY CABOT LODGE
JAMES FORD RHODES.

I agree to the substance and object of the foregoing,

WOODROW WILSON
WM. M. SLOANE
A. T. HADLEY
THOMAS R. LOUNSBURY
DANIEL C. FRENCH
A. T. MAHAN
ANDREW D. WHITE

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B. L. GILDERSLEEVE
GEORGE E. WOODBERRY
GEORGE W. CHADWICK
CHARLES FRANCIS ADAMS

I agree to the substance and object of the foregoing,

A. LAWRENCE LOWELL.

On motion of the Treasurer, it was

Voted, (1) That a Committee, consisting of Hon. Charles W. Eliot and Professor John Trowbridge, be appointed to draft and send a reply to the letter of Senator Lodge and others, declining to withdraw the protest of this Academy against the incorporation by the United States of the "American Academy of Arts and Letters."

(2) That the above Committee be requested to insist on the protest of this Academy against the incorporation of the above-mentioned Society, before any Committee of Congress, and to appear before that Committee in person or by counsel, as may seem to them advisable.

(3) That the Treasurer be requested to advance the sum of one hundred and fifty ($150.) dollars, or such part thereof as may be called for, for the use of the above Committee.

On motion of A. G. Webster, it was

Voted, That a Committee on the Revision of the Statutes be appointed.

The President appointed Charles P. Bowditch and John Trowbridge as a Committee to revise the Statutes.

The following Communication was given: — "The Zeppelin in Peace and War." By Dr. Louis Bell.

One thousand and thirty-ninth Meeting.

NOVEMBER 23, 1914.— SPECIAL MEETING.

The Academy met at its House, for the purpose of receiving a scientific communication.

The PRESIDENT in the Chair.

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There were thirty-three Fellows and twelve guests present.

The following communication was given by Professor William Henry Bragg, F. R. S., of the University of Leeds: — "X-Rays and Crystals."

After the paper, adjournment was made to the Reading Room, where the usual collation was served.

One thousand and fortieth Meeting.

DECEMBER 9, 1914.

The Academy met at its House.

The PRESIDENT in the Chair.

There were fifty-one Fellows and many guests, including Ladies, present.

The following communication was given: — Professor William M. Davis, "A Shaler Memorial Study of Pacific Coral Reefs."

Following the communication, adjournment was made to the third floor where a reception was held, given by Professor John E. Wolff, in honor of Professor Davis on his return from a voyage across the Pacific.

One thousand and forty-first Meeting.

JANUARY 13, 1915.— STATED MEETING.

The Academy met at its House.

The PRESIDENT in the Chair.

There were fifty-four Fellows present.

The following letters were presented by the acting Corresponding Secretary: — from G. R. Lyman, Nathan Matthews, and Erwin F. Smith, accepting Fellowship.

The following deaths were announced by the Chair: Charles Sedgwick Minot, Fellow in Class II., Section 3; Alfred Thayer Mahan, Fellow in Class III., Section 3.

The Committee consisting of the President and Dr. C. W. Eliot, reported that a letter was sent to Senator Lodge in answer to the one signed by him and others, which was read to the Academy at the November 11th Meeting, as follows: —

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November 28, 1914.

Dear Senator Lodge,

The American Academy of Arts and Sciences has received a letter, dated October 10, 1914, from members of that Academy who are also members of the society desiring national incorporation under the title of the American Academy of Arts and Letters, stating their conviction that the aims and purposes of the latter body do not conflict with those of the older body.

At a large meeting of the American Academy of Arts and Sciences, we were appointed a committee to reply to this letter, and since your name heads the list of signatures, we send this reply to you, reaffirming the protest of this Academy against the use of its name by a new organization with the change of only one word.

The American Academy of Arts and Sciences would not wish to oppose the incorporation of any scientific or literary society, whether the aims of the new body do or do not conflict with those of the older. It does, however, strongly object to the use, in the title of the new body, of five words out of six in the title — American Academy of Arts and Sciences — under which the older body has won prestige and honor during more than one hundred and thirty years, both in this country and in Europe, through its membership, Proceedings and Memoirs.

The reputation and influence of the older body might be confounded with those of the new, and confusion might easily arise and in fact, has already arisen, in correspondence.

We suggest that the new organization use the words "Institute" and "United States," instead of "American"; and we venture to think that the adoption of these changes would furnish a good illustration of the amity and comity which should always exist among learned societies.

Very truly yours,

(Signed) CHARLES W. ELIOT.
(Signed) JOHN TROWBRIDGE.

A letter of protest, which quoted the above letter and also gave a list of past and present distinguished members of the Academy, in the section of Arts and Letters, was sent to the Library Committee of the House of Representatives.

The following letter from Senator Lodge was received by the committee: —

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December 2, 1914.

My dear Mr. Eliot:

I have the letter of November 28, signed by you and Mr. Trowbridge, in regard to the name of the American Academy of Arts and Letters. I was elected a member of that Academy many years ago, and my name simply appears as one of its members. I have also been for many years a member of the American Academy of Arts and Sciences, and I confess it never occurred to me that there could be any harm in the name "American Academy of Arts and Letters." We have a National Academy of Sciences and I do not see why any of them should conflict. I fear it is too late now to change the name, for the bill has already passed the Senate, and the name has now been held so long by the American Academy of Arts and Letters that it would be very difficult for them to alter it.

Very truly yours,

(Signed) H. C. Lodge.

The following gentlemen were elected Fellows of the Academy:—

Class I., Section 1 (Mathematics and Astronomy): —

Edward Skinner King, of Cambridge; Carl Otto Lampland, of Flagstaff.

Class I., Section 4 (Technology and Engineering): —

Harrison Prescott Eddy, of Boston; Mordecai Thomas Endicott, of Washington; Hector James Hughes, of Cambridge; Leonard Metcalf, of Boston; Charles Francis Park, of Boston; Frederick Winslow Taylor, of Philadelphia; Joseph Ruggles Worcester, of Boston.

Class II., Section 2 (Botany): —

Irving Widmer Bailey, of Cambridge; Joseph Young Bergen, of Cambridge.

Class II., Section 3 (Zoölogy and Physiology): —

Gilman Arthur Drew, of Woods Hole; Albert Davis Mead, of Providence.

Class III., Section 1 (Theology, Philosophy and Jurisprudence):

William Harrison Dunbar, of Cambridge; Edward Henry Warren, of Boston.

The following gentlemen were elected Foreign Honorary Members: —

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Class I., Section 4 (Technology and Engineering): —
Vsevolod Jevgenjevic Timonoff, of Petrograd.
Class III., Section 2 (Philology and Archaeology): —
Arthur Sampson Napier, of Oxford.
Class III., Section 4 (Literature and the Fine Arts): —
Léon Bonnat, of Paris.

The following communications were given: —

Professor Percival Lowell. "A Trans-Neptunian Planet."

Mr. Desmond FitzGerald. "Wanderings in the Philippines."

The following papers were presented by title: —

"A Revision of the Atomic Weight of Praseodymium. The Analysis of Praseodymium Chloride." By G. P. Baxter and O. J. Stewart.

"Cell Division in Trichomonas and Allied Flagellates." By C. A. Kofoid and Olive Swezy.

"Synopsis of the Chinese Species of Pyrus." By Alfred Rehder.

"Planck's Radiation Formula and the Classical Electrodynamics." By D. L. Webster. Presented by G. W. Pierce.

One thousand and forty-second Meeting.

FEBRUARY 10, 1915.

The Academy met at its House.

The PRESIDENT in the Chair.

There were twenty-one Fellows present.

The following letters were presented by the acting Corresponding Secretary: from I. W. Bailey, G. A. Drew, W. H. Dunbar, H. J. Hughes, E. S. King, C. F. Park, F. W. Taylor, E. H. Warren, and J. R. Worcester, accepting Fellowship.

A letter from Edward C. Pickering, in regard to improving methods of securing expert testimony was laid on the table until the next stated meeting.

The Corresponding Secretary announced the transfer by the Council, of Harry Ellsworth Clifford, at his own request, from Section 2, to Section 4, of Class I.

The following Communication was given: — Mr. Andrew McFarland Davis, "Certain Specimens of old Chinese Paper Money dating from 850 A.D. to 1425 A.D."

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One thousand and forty-third Meeting.

FEBRUARY 24, 1915.

The Academy met at its House.

The PRESIDENT in the Chair.

There were thirty-six present including ladies and guests.

The following Communication was given: — Professor George H. Parker, "The Fur-seal Herds of the Pribilof Islands."

One thousand and forty-fourth Meeting.

MARCH 10, 1915. — STATED MEETING.

The Academy met at its House.

The PRESIDENT in the chair.

There were twenty-one Fellows present.

The following letters were presented: — from J. Y. Bergen, C. O. Lampland, and A. D. Mead, accepting Fellowship; from A. S. Napier, accepting Foreign Honorary Membership; from Professor E. C. Pickering, Chairman of a committee appointed by the American Association for the Advancement of Science, requesting coöperation with the Committee in regard to improved methods of securing expert testimony.

The following resolution was adopted: —

RESOLVED, That the American Academy of Arts and Sciences recognizes the urgent need in the United States of reform in the methods of securing evidence of expert opinion in judicial procedure;

That the Academy approves the efforts of the American Association for the Advancement of Science in this behalf; and,

That the Council is hereby authorized and directed to coöperate with the committee of the American Association for the Advancement of Science in the endeavor to bring about such reform.

The Chair announced the death of John Chipman Gray, Fellow in Class III., Section 1, Vice-President of Class III, from 1902–1911.

On the recommendation of the Council, the following appropriations were made for the ensuing year: —

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From the General Fund, $5300. to be used as follows: —

for House expenses	$1600.
for Library expenses	1500.
for Books, periodicals and binding	1000.
for Meeting expenses	250.
for Treasurer's office	550.
for General expenses	400.

From the income of the Publication Fund, $3000. to be used for publication.

From the income of the Rumford Fund $2910.10 to be used as follows: —

for Research	$1000.
for Books, periodicals and binding	200.
for Publication	600.
to be used at the discretion of the Committee	1110.10

Also whatever available balance remains unappropriated, for 1914–15.

From the Warren Fund, $550. to be used at the discretion of the Committee.

An appropriation of $525.64 was made from the Publication Fund for publication during the present year.

An appropriation of $50. was made from the income of the General Fund for Expense of Meetings during the present year.

The Committee on the Revision of the Statutes reported as follows: —

The Committee recommends the following changes in Chapter II, Article 4.

Line 2. Omit "two" and insert "nine."
" 3. Omit "at that time," and insert "at the time of notification" after the word "Commonwealth."
" 5. Omit "twelve," and insert "nine."
" 21. Omit "six," and insert "nine."

The first and second paragraphs of Chapter II., Article 4, will then read as follows: —

"If any person after being notified of his election as Fellow, shall neglect for nine months to accept in writing and to pay his Admission Fee (unless he be absent from the Commonwealth at the

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time of his notification) his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for nine months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time."

"If any person elected a Foreign Honorary Member shall neglect for nine months after being notified of his election to accept in writing, his election shall be void."

Add the following paragraph to Chapter III, Article 2: —

"Notice shall be sent to every Fellow, not later than September 30th, and January 31st, of each year, calling attention to the fact that the limit of time for sending nominations to the Corresponding Secretary will expire on the fifteenth of the following month."

In Chapter IV, Article 1, lines 9–11. Omit the sentence beginning "At the Annual," and ending with "four years."

Line 12: Omit "subsequent."

The paragraph will then read as follows: —

"There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms."

In Chapter XI, Article 1.

Line 1. Omit "four," and insert "eight."

" 2–3. Omit "January, March, May and October," and insert "October, November, December, January, February, March, April, and May."

" 14. Omit "the second or."

" 15. Omit "or both."

" " Omit "not appointed for Stated Meetings."

" 16. Omit "any meeting which may be held on the fourth Wednesday," and insert "at said meetings."

Paragraphs 1 and 4 of Chapter XI, Article 1, will then read as follows:

"There shall be annually eight Stated Meetings of the Academy, namely on the second Wednesday of October, November, December, January, February, March, April and May. Only at these

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meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made, or amendments of the Statutes or Standing Votes be effected."

"A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August and September, but no business shall be transacted at said meetings."

In Chapter II., Article 5, paragraph 2, line 4, insert after the word "due," the words, "except in the case of Fellows elected at the January meetings, who shall be obliged to pay but one half of such Annual Dues."

On motion, it was,

Voted, That the amendments proposed by the Committee on the Amendments of the Statutes be referred back to the Committee, with instructions to report at a special meeting to be called for the purpose of considering and acting on the amendments, on April 14, 1915.

The Chair appointed the following Councillors to act as Nominating Committee: —

ARTHUR G. WEBSTER, of Class I.,
MERRITT L. FERNALD, of Class II.,
GEORGE F. MOORE, of Class III.

The following communications were given: —

Professor George D. Birkhoff. "The Restricted Problem of Three Bodies."

Professor John Trowbridge. "Note on a Prophecy in Regard to the Future of the English Language."

The following paper was presented by title: —

"Geometry whose Element of Arc is a Linear Differential Form, with Application to the Study of the Minimum Developable." By C. L. E. Moore.

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One thousand and forty-fifth Meeting.

MARCH 24, 1915.

The Academy met in conjunction with the Lawrence Scientific Association at the Franklin Union Hall, 41 Berkeley Street, Boston.

The PRESIDENT in the Chair.

There were sixteen Fellows and many members and guests of the Lawrence Scientific Association present.

The following communication was given: "The Gyroscope in Service." By Mr. Elmer A. Sperry.

One thousand and forty-sixth Meeting.

APRIL 14, 1915 — 7.45 P.M. — SPECIAL MEETING.

The Academy met at its House at a Special Meeting called for the purpose of amending the Statutes.

VICE-PRESIDENT Walcott in the Chair.

There were twenty-three Fellows present.

The Committee on the Amendment of the Statutes reported, advising the adoption of all the proposed amendments: —

On motion, it was

Voted, To amend the Statutes as follows: —

Chapter II., Article 4, Paragraph 1, to read: —

"If any person, after being notified of his election as Fellow, shall neglect for six months to accept in writing and to pay his Admission Fee (unless he be absent from the Commonwealth at the time of his notification) his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time."

Chapter II., Article 5, Paragraph 2, to read: —

"Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when

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they shall become due, except in the case of Fellows elected at the January meetings, who shall be obliged to pay but one half of such Annual Dues in the year in which they are elected;"

Chapter III., Article 2, Paragraph 3, to be added: —

"Notice shall be sent to every Fellow having the right to vote, not later than the thirtieth of September or the thirty-first of January, of each year, calling attention to the fact that the limit of time for sending nominations to the Corresponding Secretary will expire on the fifteenth of the following month."

Chapter IV., Article 1, Paragraph 2, to read: —

"There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms."

Chapter XI., Article 1, Paragraph 1, to read: —

"There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made, or amendments of the Statutes or Standing Votes be effected."

Chapter XI., Article 1, Paragraph 4, to read: —

"A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August and September, but no business shall be transacted at said meetings."

The meeting then adjourned.

One thousand and forty-seventh Meeting.

APRIL 14, 1915 — 8 P. M. — STATED MEETING.

The Academy met at its House.

VICE-PRESIDENT Walcott in the Chair.

There were twenty-five Fellows present.

The following letters were presented by the acting Correspond-

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ing Secretary: — from V. J. Timonoff, accepting Foreign Honorary Membership; from Leonard Metcalf, declining Fellowship for himself and H. P. Eddy; from F. H. Newell, declining Fellowship; a circular from C. R. Cross, chairman of Sub-Committee on Research Funds, asking information regarding Research Funds of the Academy.

The following deaths were announced by the Chair: — Charles Francis Adams, Fellow in Class III., Section 3; Frederick Winslow Taylor, Fellow in Class I., Section 4; Thomas Raynesford Lounsbury, Fellow in Class III., Section 2.

The following Communication was given: — "The Effects of Radiation on the Eye." By Dr. F. H. Verhoeff and Dr. Louis Bell.

The following papers were presented by title: — "The Mechanics of Telephone-Receiver Diaphragms, as derived from their Motional-Impedance Circles." By A. E. Kennelly and H. A. Affel.

"A Critical Discussion of the Crystal Forms of Calcite." By H. P. Whitlock.

One thousand and forty-eighth Meeting.

APRIL 28, 1915.— SPECIAL MEETING.

The Academy met at its House.
The PRESIDENT in the Chair.
There were fifteen Fellows and four guests present.
The following communication was given: —
Col. W. R. Livermore, "The Turks and Their Predecessors."

One thousand and forty-ninth Meeting.

MAY 12, 1915.— ANNUAL MEETING.

The Academy met at its House.
VICE-PRESIDENT Thomson in the Chair.
There were fifty Fellows present.
The following invitation was presented: from the Johns Hopkins University, requesting the presence of a representative of

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the Academy at the inauguration of Frank Johnson Goodnow, LL.D., as President, May 20, 1915.

The following report of the Council was presented: —

Since the last report of the Council there have been reported the deaths of nine Fellows: — Abner Cheney Goodell, Francis Humphreys Storer, Gardiner Martin Lane, Charles Sedgwick Minot, Alfred Thayer Mahan, John Chipman Gray, Charles Francis Adams, Frederick Winslow Taylor, Thomas Raynesford Lounsbury; and of one Foreign Honorary Member: — Hugo Kronecker.

Sixty Fellows have been elected, of which number four have declined Fellowship, two have not yet accepted Fellowship.

Twelve Foreign Honorary Members have been elected, of which number, two have not yet accepted Membership.

The roll now includes 434 Fellows and 64 Foreign Honorary Members.

The annual report of the Treasurer was read, of which the following is an abstract: —

GENERAL FUND.

Receipts.

Balance, April 1, 1914	$219.10	
Investments	2,923.46	
Assessments	2,920.00	
Admissions	570.00	
Sundries	70.67	$6,703.23

Expenditures.

Expenses of Library	$2,671.81	
Expense of House	1,578.74	
Expense of Meetings	226.79	
Treasurer	207.03	
Insurance	345.14	
General Expense of Society	400.81	
Charged to cancel premium on Bonds	25.00	
Income transferred to principal	246.72	$5,702.04
Balance, April 1, 1915		1,001.19
		$6,703.23

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Rumford Fund.

Receipts.

Balance April 1, 1914	$1,763.83	
Investments	3,148.01	
Sale of Publications	7.50	
Grant returned	250.00	$5,169.34

Expenditures.

Research	$700.00	
Books, Periodicals and Binding	187.86	
Publication	332.09	
Medals	456.00	
Sundries	3.50	
Income transferred to principal	155.41	$1,834.86
Balance, April 1, 1915		3,334.48
		$5,169.34

C. M. Warren Fund.

Receipts.

Balance, April 1, 1914	$1,391.02	
Investments	1,388.19	$2,779.21

Expenditures

Research	$553.00	
Vault, rent, part	4.00	
Income transferred to principal	32.57	$589.57
Balance, April 1, 1915		2,189.64
		$2,779.21

Publication Fund.

Receipts.

Balance, April 1, 1914	$5.06	
Appleton Fund Investments	819.33	
Centennial Fund investments	2,380.65	
Sale of Publications	621.84	$3,826.88

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Expenditures.

Publications	$1,982.50	
Vault, rent, part	12.50	
Income transferred to principal	161.84	$2,156.84
Balance, April 1, 1915		1,670.04
		$3,826.88

May 12, 1915.

The following reports were also presented: —

Report of the Library Committee.

The Librarian begs to submit the following report: —

During the year, 68 books have been borrowed from the Library by 24 persons, including 11 Fellows and 3 libraries. All but 5 books have been returned for examination, or satisfactorily accounted for.

The number of bound volumes on the shelves at the time of the last report was 33,675. 612 volumes have been added during the past year, making the number now on the shelves, 34,287. This includes 81 purchased from the income of the General Fund, 28 from that of the Rumford Fund, and 503 received by gift or exchange. The pamphlets added during the year number 479.

The expenses charged to the Library during the financial year are: —

Salaries	$1,644.12
Binding: —	
General Fund	553.90
Rumford Fund	54.75
Purchase of periodicals and books: —	
General Fund	439.07
Rumford Fund	133.11
Miscellaneous	34.72
Total	$2,859.67

A. G. Webster, *Librarian.*

May 13, 1915.

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Report of the Rumford Committee.

During the present year grants have been made in aid of research as follows: —

November 11, 1914, to Professor P. W. Bridgman, in aid of his researches on the "Thermal Effects of High Pressures." (additional)	$150
To Professor Frederick A. Saunders, in aid of his research "On the Spectra of Metallic Vapors."	100
To Professor Frederick Palmer, Jr., in aid of his research on the "Properties of Light of Extremely Short Wave Lengths."	200
To Professor Henry Crew, in aid of his research on the "Specific Heat of Liquids."	200
May 12, 1915, to Professor Joel Stebbins, in aid of his research with his improved photo-electric cell photometer upon variable stars.	140

December 9, 1914. It was voted to appropriate $300 for the purchase of a refrigerating apparatus for the Academy, the same to be loaned to Professor C. A. Kraus, for his research on "Solutions in Liquid Ammonia" for the term of three years.

It was also voted that $300 be appropriated for the purchase of a motor generator for the Academy, the same to be loaned to Dr. H. P. Hollnagel, for his research "An Investigation of the Extreme Infra-Red Portion of the Spectrum" for the term of three years.

January 13, 1915. It was voted "That it is the opinion of the Rumford Committee that in case of the purchase of standard apparatus which may be of use in various researches such apparatus should be purchased by the Academy and loaned to the grantee for such period as may be desirable."

February 10, 1915. It was voted to adopt the following circular which is to be sent in print to all persons to whom grants shall be made.

American Academy of Arts and Sciences.

Rumford Fund.

The following regulations regarding grants from the Rumford Fund have been made by the Rumford Committee.

All apparatus purchased through appropriations from the Rumford

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Fund is the property of the American Academy of Arts and Sciences and is to be returned to it when the research for which it was provided is completed, unless otherwise directed by the Committee.

In case of the purchase of standard apparatus which might be of use in various researches, it is the policy of the Committee to purchase it directly and to loan it to the grantee for such period as may be desirable.

Recipients of grants for investigations are expected to report annually to the Committee as to the progress of the work for which the grant was made.

Researches carried on with aid from the Rumford Fund may be published in any place or form with the proviso that due recognition be made of the grant as from the Rumford Fund of the American Academy of Arts and Sciences. A complete copy of every publication shall be presented to the Academy.

Persons making application for grants from the Rumford Fund are expected to inform the Committee of similar applications made by them for grants from other funds in aid of the same research or of related researches.

Applications for grants should be made to the Chairman of the Rumford Committee, Care of the American Academy of Arts and Sciences, 28 Newbury St., Boston.

February 10, 1915.

The replicas which remain to be made of Rumford Medals awarded prior to the date at which the striking of a bronze replica was instituted are under construction and it is expected that they will all be finished during the summer.

Reports of progress in their several researches have been received from the following persons: Messrs. C. G. Abbot, P. W. Bridgman, W. W. Campbell, A. L. Clark, D. F. Comstock, H. Crew, E. B. Frost, H. P. Hollnagel, L. R. Ingersoll, N. A. Kent, F. G. Keyes, L. V. King, C. A. Kraus, A. B. Lamb, G. N. Lewis, E. L. Nichols, C. L. Norton, F. Palmer, Jr., J. A. Parkhurst, T. W. Richards, G. W. Ritchey, F. A. Saunders, W. O. Sawtelle, A. W. Smith, F. W. Very.

Nine grantees have failed to respond to the request of the Chairman of the Committee for the usual report of progress.

The following papers have been published in Volume 50 of the Proceedings of the Academy with aid from the Rumford Fund since the last Annual Meeting.

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No. 1. "Types of Abnormal Color Vision," by Louis Bell.

No. 4. "On Electric Conduction and Thermoelectric Action in Metals," by Edwin H. Hall.

No. 6. "Planck's Radiation Formula and the Classical Electrodynamics," by David L. Webster.

At the meeting of February 10, 1915, it was unanimously voted for the first time and at the meeting of March 10, 1915, for the second time, to recommend to the Academy the award of the Rumford Premium to Charles G. Abbot for his researches on Solar Radiation.

CHARLES R. CROSS, *Chairman.*

May 12, 1915.

REPORT OF THE C. M. WARREN COMMITTEE.

The C. M. Warren Committee begs to submit the following report: At the beginning of the year 1914–1915, the Committee had at its disposal a balance of $585 from previous appropriations by the Academy, and the sum of $800 was appropriated for the use of the Committee at the meeting of March, 1914.

During the year grants have been made as follows: —

To Professor George H. Burrows, for a study of equilibria in certain organic reactions, including the determination of free energy of formation of the compounds involved $250

To Professor Roger F. Brunel, for the purchase of a polarimeter, with accessories, to be used in connection with a study of the equilibria between organic compounds, with special reference to the affinities of certain organic radicals (not to exceed) . . 400

To Professor Charles A. Kraus, for the purchase of a special high grade balance to be used in an investigation of the properties of very dilute solutions 300

The two last-named grants for the purchase of apparatus have been made with the provision that these pieces of apparatus are to be the property of the Academy, and are loaned to the investigators in question for the period of three years, or for such longer or shorter period as may be shown to be required for the completion of the investigations for which they were purchased. This policy is in line with that adopted by other Committees or Trustees having charge of research funds, and it is believed that it will tend to increase the effectiveness of the Warren Fund, and will also bring about a larger

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sense of responsibility on the part of the investigators with respect to the care and efficient use of such apparatus, without in any way hindering the progress of their work.

The Committee has also made an appropriation of $133.50 for the reprinting of two hundred copies of each of five papers covering investigations by Professor Charles F. Mabery, for which grants have previously been made from the Warren Fund. The supply of these papers has been exhausted, and requests are still made to the Librarian for them, either separately or to complete sets of the Proceedings.

At the meeting of the Academy in March, 1915, a further appropriation of $550 was made for the use of the Committee. The balance in its hands at the present time is $651.50.

Reports have been received from most of those who have unfinished investigations under way for which grants have been made. No papers are reported as having been published during the year, but several investigations are apparently nearing completion.

Respectfully,

H. P. TALBOT, *Chairman.*

May 12, 1915.

REPORT OF THE PUBLICATION COMMITTEE.

The Committee on Publication has the honor to report as follows: —

Between April 1, 1914, and April 1, 1915, there were published one number of Volume XLIX (No. 12) and eight numbers of Volume L, of the Proceedings, aggregating 272 pages. This is much below the average number of pages usually printed per year. There are, however, four papers of considerable length in press, and there is also a large quantity of manuscript on hand, which will bring the present volume about up to the standard size.

There was available for the use of the Publication Committee, an unexpended balance from last year of $5.06, an appropriation of $2500, an additional appropriation of $525.64, and a sum of $294.46 from the sales of publications to March 2 — in all $3,325.16 from the Publication Fund and sales. Bills against this appropriation to the amount of $1,982.50 have been approved by the Chairman. This leaves an unexpended balance of $1,342.66 plus the amount on hand for sales of publications from March 2 to April 1.

Bills aggregating $332.09 for the publication of papers on light and

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heat have been submitted to the Rumford Committee for their approval in accordance with authorization of the Chairman of the Rumford Committee.

Respectfully,

G. W. PIERCE, *Chairman.*

May 12, 1915.

REPORT OF THE HOUSE COMMITTEE.

The House Committee submits the following report for the year 1914–15: —

The Committee had at its disposal at the beginning of the year a balance of $28.86. The appropriation by the Academy for the year was $1,800, and there has been received, for the use of the rooms, from the Colonial Society $15, from the Boston Board of Trade $36, and from the American Oriental Society $5, making the total amount at the disposal of the Committee $1,884.86.

The total expenditure for the year is $1,578.74, leaving an unexpended balance of $306.12. The expenditures may be summarized as follows: —

Janitor		$750.00
Electricity	A. Light	69.20
	B. Power	49.60
Gas		10.64
Water		8.00
Telephone		64.03
Coal	Furnace	350.00
	Water Heat	30.50
Ash tickets		7.00
Care of Elevator		35.03
Ice		14.40
Janitor's materials		13.85
Furnishings		44.57
Upkeep		131.92
Total expenditure		$1,578.74

Of the expenditure for "Up-keep" $50 was paid out for changes in the elevator-well, made at the demand of the State officials, in compliance with recent legislative enactments, which are more stringent

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than formerly with respect to the protection of laborers who might be employed in connection with the repairs or maintenance of the elevator. Your Committee believes that the Academy Building is in good condition at present.

During the year the Academy has held twelve meetings in the Building. Other meetings have been held as follows: The Colonial Society, four; the Harvard Biblical Club, six; the Harvard-Technology Chemical Club, three; and the Boston Board of Trade has had the use of the hall only, for twelve lectures on agricultural subjects. There has been the usual use of the small rooms by the Council and Committees.

The arrangement of the two previous years with respect to the services of the janitor has been continued, and the building has been open during the same hours each week as during the past years.

The Committee takes much pleasure in expressing once more its sense of obligation to Mrs. Holden, the Assistant Librarian, for her constant and efficient co-operation in all that the Committee has undertaken.

Respectfully,

H. P. TALBOT, *Chairman.*

May 12, 1915.

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to Charles Greeley Abbot, of Washington, D. C., for his researches on Solar Radiation.

On motion of the Treasurer, it was

Voted, That the Annual Assessment be ten ($10) dollars.

On the recommendation of the C. M. Warren Committee, it was

Voted, To make an additional appropriation of $800 from the income of the C. M. Warren Fund for the use of the Committee.

Whereas it was deemed advisable by the executors under the will of Francis Amory, after consultation with the representatives of the residuary legatee and the other principal legatees mentioned in the will, to settle with the heirs of the said Amory in order to avoid a contest of the will; and whereas a settlement has been effected with the said heirs upon terms satisfactory to the said legatees and said executors; it was

Voted, That the American Academy of Arts and Sciences contribute to the amount of the settlement made with the heirs of the

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late Francis Amory and expenses connected therewith, twenty-five hundred dollars ($2500.), by accepting twenty-two thousand five hundred dollars ($22,500.) in lieu of the legacy of twenty-five thousand dollars ($25,000.) given to it by the will of the said Amory. It was also

Voted, That the Treasurer of the Academy be authorized to receive and receipt for said legacy.

The annual election resulted in the choice of the following officers and committees: —

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The following gentlemen were elected Fellows of the Academy:—

Class I., Section 1 (Mathematics and Astronomy): —

Leonard Eugene Dickson, of Chicago Ill.; Philip Fox, of Evanston, Ill.; Frank Lauren Hitchcock, of Boston; Dunham Jackson, of Cambridge.

Class I., Section 2 (Physics): —

Willian David Coolidge, of Schenectady, N. Y.

Class I., Section 3 (Chemistry): —

George Shannon Forbes, of Cambridge; Charles August Kraus, of Worcester; Warren Kendall Lewis, of Newton; Miles Standish Sherrill, of Brookline.

Class I., Section 4 (Technology and Engineering): —

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Class II., Section 2 (Botany): —

Thomas Jonathan Burrill, of Urbana, Ill.; Lincoln Ware Riddle, of Wellesley.

Class II., Section 3 (Zoölogy and Physiology): —

Glover Morrill Allen, of Boston; Charles Thomas Brues, of Boston; Hermon Carey Bumpus, of Tufts College; Hubert Lyman Clark, of Cambridge; Alexander Forbes, of Milton; Ernest Gale Martin, of Cambridge; John Charles Phillips, of Wenham; Arthur Wisswald Weysse, of Boston; Frederick Adams Woods, of Brookline; Robert Mearns Yerkes, of Cambridge.

Class II., Section 4 (Medicine and Surgery): —

Reid Hunt, of Boston.

Class III., Section 1 (Theology, Philosophy and Jurisprudence):

Edward Staples Drown, of Cambridge; William Dewitt Hyde, of Brunswick, Me.; William Caleb Loring, of Boston; Samuel Walker McCall, of Winchester; John Winthrop Platner, of Cambridge.

Class III., Section 2 (Philology and Archaeology): —

Edward Washburn Hopkins, of New Haven, Conn.; Aleš Hrdlička, of Washington, D. C.; Carl Newell Jackson, of Cambridge; Kirsopp Lake, of Cambridge; Henry Roseman Lang, of New Haven, Conn.; Bernadotte Perrin, of New Haven, Conn.

Class III., Section 3 (Political Economy and History): —

Richard Henry Dana, of Cambridge; Edward Bangs Drew, of Cambridge; William Roscoe Thayer, of Cambridge.

Class III., Section 4 (Literature and the Fine Arts):—

James Phinney Baxter, of Portland, Me.; James Kendall Hosmer, of Minneapolis, Minn.; Samuel Eliot Morison, of Boston.

The following gentlemen were elected Foreign Honorary Members: —

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Class I., Section 1 (Mathematics and Astronomy): —

Charles Jean de la Vallée Poussin, of Louvain, Belgium; Joseph Norman Lockyer, London, England.

Class I., Section 2 (Physics): —

Ernest Rutherford, of Manchester, England.

Class I., Section 4 (Technology and Engineering): —

Guglielmo Marconi, of Rome, Italy.

Class II., Section 1 (Geology, Mineralogy and Physics of the Globe): —

Charles Barrois, of Lille, France; Johan H. L. Vogt, of Trondhjem, Norway.

The following communication was given: —

Professor A. C. Lane, "Explanation of the Masses of Early Non-Fossiliferous Strata."

The following papers were presented by title: —

"The Glacial-control Theory of Coral Reefs." By R. A. Daly.

"New Indo-Malayan Laboulbeniales." By Roland Thaxter.

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(Corrected to July 1, 1915.)

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(Number limited to six hundred.)

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Section I. — *Mathematics and Astronomy.*— 40.

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Thomas Chrowder Chamberlin . . . . . Chicago, Ill.
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Elliott Proctor Joslin	Boston
William Williams Keen	Philadelphia, Pa.
Frank Burr Mallory	Brookline
Samuel Jason Mixter	Boston
Edward Hall Nichols	Boston
Sir William Osler	Oxford, Eng.
Theophil Mitchell Prudden	New York, N. Y.
William Lambert Richardson	Boston
Milton Joseph Rosenau	Boston
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Elmer Ernest Southard	Boston
Richard Pearson Strong	Boston
Ernest Edward Tyzzer	Boston
Frederick Herman Verhoeff	Boston
Henry Pickering Walcott	Cambridge
John Collins Warren	Boston
William Henry Welch	Baltimore, Md.
Francis Henry Williams	Boston
Simeon Burt Wolbach	Boston
Horatio Curtis Wood	Philadelphia, Pa.
James Homer Wright	Boston

CLASS III.— *Moral and Political Sciences.*— 146.

SECTION I.— *Theology, Philosophy and Jurisprudence.*— 41.

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Joseph Henry Beale	Cambridge
Melville Madison Bigelow	Cambridge
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Williston Walker . . . . . New Haven, Conn.
Eugene Wambaugh . . . . . Cambridge
Edward Henry Warren . . . . . Boston
Samuel Williston . . . . . Belmont
Woodrow Wilson . . . . . Washington, D. C.

CLASS III., SECTION II.—*Philology and Archæology.*—45.

Francis Greenleaf Allinson . . . . . Providence, R. I.
William Rosenzweig Arnold . . . . . Cambridge
Maurice Bloomfield . . . . . Baltimore, Md.

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Franz Boas . . . . . New York, N. Y.
Charles Pickering Bowditch . . . . . Jamaica Plain
Franklin Carter . . . . . Williamstown
George Henry Chase . . . . . Cambridge
Roland Burrage Dixon . . . . . Cambridge
William Curtis Farabee . . . . . Cambridge
Jesse Walter Fewkes . . . . . Washington, D. C.
Jeremiah Denis Mathias Ford . . . . . Cambridge
Basil Lanneau Gildersleeve . . . . . Baltimore, Md.
Charles Hall Grandgent . . . . . Cambridge
Charles Burton Gulick . . . . . Cambridge
William Arthur Heidel . . . . . Middletown, Conn.
Bert Hodge Hill . . . . . Athens, Greece
Edward Washburn Hopkins . . . . . New Haven, Conn.
Albert Andrew Howard . . . . . Cambridge
Aleš Hrdlička . . . . . Washington, D. C.
Carl Newell Jackson . . . . . Cambridge
Hans Carl Gunther von Jagemann . . . . . Cambridge
James Richard Jewett . . . . . Cambridge
Alfred Louis Kroeber . . . . . Berkeley, Cal.
Kirsopp Lake . . . . . Cambridge
Henry Roseman Lang . . . . . New Haven, Conn.
Charles Rockwell Lanman . . . . . Cambridge
David Gordon Lyon . . . . . Cambridge
Clifford Herschel Moore . . . . . Cambridge
George Foot Moore . . . . . Cambridge
Hanns Oertel . . . . . New Haven, Conn.
Charles Pomeroy Parker . . . . . Cambridge
Bernadotte Perrin . . . . . New Haven, Conn.
Frederick Ward Putnam . . . . . Cambridge
Edward Kennard Rand . . . . . Cambridge
George Andrew Reisner . . . . . Cambridge
Edward Robinson . . . . . New York, N. Y.
Fred Norris Robinson . . . . . Cambridge
Edward Stevens Sheldon . . . . . Cambridge
Herbert Weir Smyth . . . . . Cambridge
Franklin Bache Stephenson . . . . . Claremont, Cal.
Charles Cutler Torrey . . . . . New Haven, Conn.
Alfred Marston Tozzer . . . . . Cambridge
Andrew Dickson White . . . . . Ithaca, N. Y.
John Williams White . . . . . Cambridge
James Haughton Woods . . . . . Cambridge

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CLASS III., SECTION III.— *Political Economy and History.*— 29.

Henry Adams	Washington, D. C.
Charles Jesse Bullock	Cambridge
Thomas Nixon Carver	Cambridge
Archibald Cary Coolidge	Boston
Richard Henry Dana	Cambridge
Andrew McFarland Davis	Cambridge
Davis Rich Dewey	Cambridge
Edward Bangs Drew	Cambridge
Ephraim Emerton	Cambridge
Henry Walcott Farnam	New Haven, Conn.
Irving Fisher	New Haven, Conn.
Worthington Chauncey Ford	Boston
Edwin Francis Gay	Cambridge
Arthur Twining Hadley	New Haven, Conn.
Charles Homer Haskins	Cambridge
Henry Cabot Lodge	Nahant
Abbott Lawrence Lowell	Cambridge
Roger Bigelow Merriman	Cambridge
William Bennett Munro	Cambridge
James Ford Rhodes	Boston
William Mulligan Sloane	New York, N. Y.
Charles Card Smith	Boston
Henry Morse Stephens	Berkeley, Cal.
Frank William Taussig	Cambridge
William Roscoe Thayer	Cambridge
Frederick Jackson Turner	Cambridge
Thomas Franklin Waters	Ipswich
George Grafton Wilson	Cambridge
George Parker Winship	Providence, R. I.

CLASS III., SECTION IV.— *Literature and the Fine Arts.*— 31.

James Burrell Angell	Ann Arbor, Mich.
George Pierce Baker	Cambridge
Arlo Bates	Boston
William Sturgis Bigelow	Boston
Le Baron Russell Briggs	Cambridge
George Whitefield Chadwick	Boston

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Samuel McChord Crothers . . . . . Cambridge
Wilberforce Eames . . . . . New York, N. Y.
Henry Herbert Edes . . . . . Cambridge
Arthur Fairbanks . . . . . Cambridge
Arthur Foote . . . . . Brookline
Kuno Francke . . . . . Cambridge
Daniel Chester French . . . . . Stockbridge
Robert Grant . . . . . Boston
Henry Lee Higginson . . . . . Boston
James Kendall Hosmer . . . . . Minneapolis, Minn.
Mark Antony DeWolfe Howe . . . . . Boston
George Lyman Kittredge . . . . . Cambridge
William Coolidge Lane . . . . . Cambridge
Albert Matthews . . . . . Boston
Samuel Eliot Morison . . . . . Boston
William Allan Neilson . . . . . Cambridge
Bela Lyon Pratt . . . . . Boston
Herbert Putnam . . . . . Washington, D. C.
Denman Waldo Ross . . . . . Cambridge
John Singer Sargent . . . . . London, Eng.
Ellery Sedgwick . . . . . Boston
Herbert Langford Warren . . . . . Cambridge
Barrett Wendell . . . . . Boston
Owen Wister . . . . . Philadelphia, Pa.
George Edward Woodberry . . . . . Beverly

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FOREIGN HONORARY MEMBERS.— 68.

(Number limited to seventy-five).

Class I.— *Mathematical and Physical Sciences.*— 23.

Section I.— *Mathematics and Astronomy.*— 6.

Arthur Auwers	Berlin
Johann Oskar Backlund	Petrograd
Felix Klein	Göttingen
Sir Joseph Norman Lockyer	London
Émile Picard	Paris
Charles Jean de la Vallée Poussin	Louvain

Class I., Section II — *Physics.*— 9.

Svante August Arrhenius	Stockholm
Oliver Heaviside	Torquay
Sir Joseph Larmor	Cambridge
Hendrik Antoon Lorentz	Leyden
Max Planck	Berlin
Augusto Righi	Bologna
John William Strutt, Baron Rayleigh	Witham
Sir Ernest Rutherford	Manchester
Sir Joseph John Thomson	Cambridge

Class I., Section III.— *Chemistry.*— 5.

Adolf, Ritter von Baeyer	Munich
Emil Fischer	Berlin
Fritz Haber	Berlin
Wilhelm Ostwald	Leipsic
Sir Henry Enfield Roscoe	London

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CLASS I., SECTION IV.— *Technology and Engineering.*— 3.

Heinrich Müller-Breslau . . . . . Berlin
Vsevolod Jevgenjevic Timonoff . . . . . Petrograd
William Cawthorne Unwin . . . . . London

CLASS II.— *Natural and Physiological Sciences.*— 21.

SECTION I.— *Geology, Mineralogy, and Physics of the Globe.*— 6.

Waldemar Christofer Brögger . . . . . Christiania
Sir Archibald Geikie . . . . . Haslemere, Surrey
Viktor Goldschmidt . . . . . Heidelberg
Julius Hann . . . . . Vienna
Albert Heim . . . . . Zürich
Johan Herman Lie Vogt . . . . . Trondhjem

CLASS II., SECTION II.— *Botany.*— 6.

John Briquet . . . . . Geneva
Adolf Engler . . . . . Berlin
Wilhelm Pfeffer . . . . . Leipsic
Hermann, Graf zu Solms-Laubach . . . . . Strassburg
Ignatz Urban . . . . . Berlin
Eugene Warming . . . . . Copenhagen

CLASS II., SECTION III.— *Zoölogy and Physiology.*— 4.

Ludimar Hermann . . . . . Königsberg
Sir Edwin Ray Lankester . . . . . London
Elie Metchnikoff . . . . . Paris
Magnus Gustav Retzius . . . . . Stockholm

CLASS II., SECTION IV.— *Medicine and Surgery.*— 5.

Emil von Behring . . . . . Marburg
Sir Thomas Lauder Brunton, Bart . . . . . London
Angelo Celli . . . . . Rome
Sir Victor Alexander Haden Horsley . . . . . London
Adam Politzer . . . . . Vienna

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Class III.— *Moral and Political Sciences.*— 24.

Section I.— *Theology, Philosophy and Jurisprudence.*— 4.

Arthur James Balfour . . . . . Prestonkirk
Heinrich Brunner . . . . . Berlin
Albert Venn Dicey . . . . . Oxford
Sir Frederick Pollock, Bart . . . . . London

Section II.— *Philology and Archæology.*— 10.

Ingram Bywater . . . . . London
Friedrich Delitzsch . . . . . Berlin
Hermann Diels . . . . . Berlin
Wilhelm Dörpfeld . . . . . Athens
Henry Jackson . . . . . Cambridge
Hermann Georg Jacobi . . . . . Bonn
Sir Gaston Camille Charles Maspero . . . . . Paris
Alfred Percival Maudslay . . . . . Hereford
Arthur Sampson Napier . . . . . Oxford
Eduard Seler . . . . . Berlin

Section III.— *Political Economy and History.*— 5.

Viscount Bryce . . . . . London
Adolf Harnack . . . . . Berlin
John Morley, Viscount Morley of Blackburn . . . . . London
Sir George Otto Trevelyan, Bart . . . . . London
Pasquale Villari . . . . . Florence

Section IV.— *Literature and the Fine Arts.*— 5.

Georg Brandes . . . . . Copenhagen
Jean Adrien Aubin Jules Jusserand . . . . . Paris
Rudyard Kipling . . . . . Burwash
Sir Sidney Lee . . . . . London
Sir James Augustus Henry Murray . . . . . Oxford

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STATUTES AND STANDING VOTES

STATUTES

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915.

CHAPTER I

The Corporate Seal

Article 1. The Corporate Seal of the Academy shall be as here depicted:

Article 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v. art. 3; chap. vi. art. 2.

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CHAPTER II

Fellows and Foreign Honorary Members and Dues

Article 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

Class I. *The Mathematical and Physical Sciences*
- Section 1. Mathematics and Astronomy
- Section 2. Physics
- Section 3. Chemistry
- Section 4. Technology and Engineering

Class II. *The Natural and Physiological Sciences*
- Section 1. Geology, Mineralogy, and Physics of the Globe
- Section 2. Botany
- Section 3. Zoölogy and Physiology
- Section 4. Medicine and Surgery

Class III. *The Moral and Political Sciences*
- Section 1. Theology, Philosophy, and Jurisprudence
- Section 2. Philology and Archaeology
- Section 3. Political Economy and History
- Section 4. Literature and the Fine Arts

Article 2. The number of Fellows shall not exceed Six hundred, of whom not more than Four hundred shall be residents of Massachusetts, nor shall there be more than Two hundred in any one Class.

Article 3. The number of Foreign Honorary Members shall not exceed Seventy-five. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Twenty-five in any one Class.

Article 4. If any person, after being notified of his election as Fellow, shall neglect for six months to accept in writing and to pay his Admission Fee (unless he be absent from the Commonwealth at the time of his notification) his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime,

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he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii. art. 2.

ARTICLE 5. Every Fellow hereafter elected shall pay an Admission Fee of Ten dollars.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due, except in the case of Fellows elected at the January meetings, who shall be obliged to pay but one half of such Annual Dues in the year in which they are elected; but any Fellow shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any Fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

ARTICLE 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. x. art. 2.

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ARTICLE 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to all the Fellows and Foreign Honorary Members.

ARTICLE 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three fourths of the Fellows present, out of a total attendance of not less than fifty, at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii.; chap. vi. art. 1; chap. ix. art. 1, 7; chap. x. art. 2.

CHAPTER III

ELECTION OF FELLOWS AND FOREIGN HONORARY MEMBERS

ARTICLE 1. Elections of Fellows and Foreign Honorary Members shall be by ballot, and only at the Stated Meetings in January and May. Three fourths of the ballots cast, and not less than twenty, must be affirmative to effect an election.

ARTICLE 2. Candidates must be proposed in writing by two Fellows of the Section for which the proposal is made. These signed nominations shall be sent to the Corresponding Secretary and shall be retained by him until the fifteenth of the following October or February, as the case may be, when all nominations then in his hands shall be immediately sent in printed form to every Fellow having the right to vote, with the names of the proposers in each case, and with a request to send to the Corresponding Secretary written comments on these names not later than the fifth of November or the fifth of March respectively.

All the signed nominations, with the comments thereon, received up to the fifth of November or the fifth of March shall be sent at once to the appropriate Class Committees, which shall report their decisions to the Council at a special meeting to be called to consider nominations, not later than two days before the meeting of the Academy in December and April respectively.

Notice shall be sent to every Fellow having the right to vote, not later than the thirtieth of September or the thirty-first of January, of each year, calling attention to the fact that the limit of time for

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sending nominations to the Corresponding Secretary will expire on the fifteenth of the following month.

ARTICLE 3. All nominations approved by the Council shall be read to the Academy at a meeting in December or in April, or be sent to the Fellows in print with the official notice of the meeting, and shall then be posted in the Hall of the Academy until the balloting.

Not later than two weeks after any nomination is reported to the Academy, the Corresponding Secretary shall send to every Fellow having the right to vote a brief printed account of the nominee.

See Chap. ii.; chap. vi. art. 1; chap. ix. art. 1.

CHAPTER IV

OFFICERS

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), three Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, and a Librarian, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. x. art. 1.

ARTICLE 2. If any office shall become vacant during the year, the vacancy may be filled by the Council in its discretion for the unexpired term.

ARTICLE 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of three Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and cause it to be sent to the Recording Secretary not later than four weeks before the Annual Meeting.

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ARTICLE 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted, together with the list of nominees prepared by the Nominating Committee, in the call therefor, and shall be mailed to all the Fellows.

See Chap. vi. art. 2.

ARTICLE 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V

THE PRESIDENT

ARTICLE 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii. art. 7; chap. iv. art. 1, 3; chap. vi. art. 2; chap. vii. art. 1; chap. ix. art. 6; chap. x. art. 1; 2; chap. xi. art. 1.

CHAPTER VI

THE SECRETARIES

ARTICLE 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which

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have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows and Foreign Honorary Members, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii. art. 7; chap. iii. art. 2, 3; chap. iv. art. 1; chap. ix. art. 6; chap. x. art. 1; chap. xi. art. 1.

ARTICLE 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

He shall post in the Hall a list of the persons nominated for election into the Academy; and after all elections, he shall insert in the Records the names of the Fellows by whom the successful candidates were nominated.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. i.; chap. ii. art. 7; chap. iv. art. 3, 4, 5; chap. ix. art. 6; chap. x. art. 1, 2; chap. xi. art. 1, 3.

ARTICLE 3. The Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

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CHAPTER VII

The Treasurer and the Treasury

Article 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

Article 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows who may be then delinquent in the payment of their Annual Dues.

Article 3. He shall give such security for the trust reposed in him as the Academy may require.

Article 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the funds of the Academy.

Article 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition

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of all the property of the Academy entrusted to him, and the character of the investments.

ARTICLE 6. The Financial Year of the Academy shall begin with the first day of April.

ARTICLE 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without the previous consent and approval of the Council.

See Chap. ii. art. 4, 5; chap. vi. art. 2; chap ix. art. 6; chap. x. art. 1, 2, 3; chap. xi. art.

CHAPTER VIII

THE LIBRARIAN AND THE LIBRARY

ARTICLE 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

ARTICLE 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

ARTICLE 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

ARTICLE 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

ARTICLE 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any,

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shall be delivered to the Fellow so paying, unless such remainder be valuable by reason of association.

ARTICLE 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

ARTICLE 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii. art. 6; chap. x. art. 1, 2.

CHAPTER IX

THE COUNCIL

ARTICLE 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and present to the Academy for action such of these nominations as it may approve by a majority vote of the members present at a meeting, of whom not less than seven shall have voted in the affirmative.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections of the same Class, reporting its action to the Academy.

See Chap. iii. art. 2, 3; chap. x. art. 1.

ARTICLE 2. Seven members shall constitute a quorum.

ARTICLE 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

ARTICLE 4. It shall act upon all resignations of officers, and all resignations and forfeitures of fellowship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation.

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They shall hold their respective positions during the pleasure of the Council.

ARTICLE 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

ARTICLE 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. x. art. 3.

ARTICLE 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a Memoir for publication in the Proceedings.

ARTICLE 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii. art. 4, 5, 8; chap. iv. art. 1, 2; chap. vi. art. 1; chap. vii. art. 1; chap. xi. art. 1, 4.

CHAPTER X

STANDING COMMITTEES

ARTICLE 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii. art. 2.

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ARTICLE 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance,* to consist of three Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and of changing the investments thereof in their discretion.

See Chap. iv. art. 3; chap. vii. art. 1, 4; chap. ix. art. 6.

(ii) *The Rumford Committee,* to consist of seven Fellows, who shall report to the Academy on all applications and claims for the Rumford Premium. It alone shall authorize the purchase of books publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iii) *The Cyrus Moors Warren Committee,* to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv. art. 3; chap. ix. art. 6.

(iv) *The Committee of Publications,* to consist of three Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which the Publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

Two hundred extra copies of each paper accepted for publication in the Proceedings or the Memoirs shall be placed at the disposal of the author without charge.

See Chap. iv. art. 3; chap. vi. art. 1, 3; chap. ix. art. 6.

(v) *The Committee on the Library,* to consist of the Librarian, *ex officio,* as Chairman, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv. art. 3; chap. viii. art. 1, 2; chap. ix. art. 6

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(vi) *The House Committee,* to consist of three Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv. art. 1, 3; chap. ix. art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and three other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv. art. 3; chap. ix. art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv. art. 3; chap. vii. art. 1; chap. ix. art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii. art. 1, 7; chap. ix. art. 6.

CHAPTER XI

MEETINGS, COMMUNICATIONS, AND AMENDMENTS

ARTICLE 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April and May. Only at these meetings, or at adjournments thereof regularly notified, or at Special Meetings called for the purpose, shall appropriations of money be made, or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten

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Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

ARTICLE 2. Twenty Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special Meetings. Fifteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

ARTICLE 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

ARTICLE 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow without the consent of the author; and no report shall in any case be published by any Fellow in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

ARTICLE 5. No Fellow shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after nominations to Fellowship have been read and the result of the balloting for candidates has been declared.

ARTICLE 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

ARTICLE 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two thirds of the ballot cast, and not less than twenty, must be affirmative to effect enactment.

ARTICLE 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two thirds of the members present. They may be supended by a unanimous vote.

See Chap. ii. art. 5, 8; chap. iii.; chap. iv. art. 3, 4, 5; chap. v. art. 1; chap. vi. art. 1, 2; chap. ix. art. 8.

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STANDING VOTES

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. There may be chosen by the Academy, under the same rules by which Fellows are now chosen, one hundred Resident Associates. Not more than forty Resident Associates shall be chosen in any one Class.

The election of Resident Associates shall be for a term of three years with eligibility for reëlection.

Resident Associates shall be entitled to the same privileges as Fellows, in the use of the Academy building, may attend meetings and present papers, but they shall not have the right to vote. They shall pay no Admission Fee, and their Annual Dues shall be one-half that of Fellows residing within fifty miles of Boston.

The Council and Committees of the Academy may ask one or more Resident Associates to act with them in an advisory or assistant capacity.

RUMFORD PREMIUM

In conformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars,

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as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American Islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

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